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Changes in the states of polarization of random electromagnetic beams in atmospheric turbulence

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Taking the random electromagnetic cosh-Gaussian beam as a typical example of random electromagnetic beams, the analytical expressions for the cross-spectral density matrix element of random electromagnetic cosh-Gaussian beams propagating through non-Kolmogorov atmospheric turbulence are derived, and used to study the changes in the states of polarization (degree of polarization, orientation angle and degree of ellipticity) of random electromagnetic cosh-Gaussian beams in non-Kolmogorov atmospheric turbulence. It is shown that the states of polarization of random electromagnetic cosh-Gaussian beams in non-Kolmogorov atmospheric turbulence are different from those in free space. The degree of polarization decreases, and the orientation angle and degree of ellipticity increase with increasing structure constant. The on-axis degree of polarization and the degree of ellipticity appear to have an oscillatory behavior and the orientation angle has a rapid transition for the larger cosh-part parameter of random electromagnetic cosh-Gaussian beams in atmospheric turbulence.

Keywords: non-Kolmogorov atmospheric turbulence, random electromagnetic beams, degree of polarization, orientation angle, degree of ellipticity.

1. Introduction

The propagation of a laser beam through atmospheric turbulence has been of considerable importance in connection with optical communications and laser weapons, *etc.*, for a long time [1–4]. Based on the unified theory of coherence and polarization of random electromagnetic beams [5, 6], the spectrum, spectral degree of coherence, degree of polarization and Stokes parameters of electromagnetic beams propagating through atmospheric turbulence were studied extensively [7–16]. WOLF *et al.* investigated the far-zone behavior of the degree of polarization of electromagnetic Gaussian Schell-model beams propagating through atmospheric turbulence, and pointed out that the degree of polarization of the beam for the long-propagation distance tends to the value in the source plane [7, 8]. The degree of polarization, spectrum and spectral degree of coherence of partially coherent electromagnetic cosh-Gaussian (ChG) and

Hermite–Gaussian beams was reported by XIAOLING JI *et al.* [9, 11]. JIXIONG PU *et al.* analyzed the degree of polarization and the degree of cross-polarization of stochastic electromagnetic beams through atmospheric turbulence, and showed that the degree of cross-polarization is generally unbounded and does not decrease with propagation distance [12, 13]. LI *et al.* studied the changes in the on-axis and transverse spectral Stokes parameters of random electromagnetic vortex beams propagating through atmospheric turbulence [14]. The spectral properties of random electromagnetic partially coherent flat-topped vortex beams in atmospheric turbulence were studied by HAIYAN WANG and XIANMEI QIAN who found that the variations of the spectral properties depend closely on the strength of atmospheric turbulence and the properties of the source beam [16]. However, in analyzing the states of polarization of electromagnetic beams in atmospheric turbulence most of the publications focused on the degree of polarization. In general, not only does the degree of polarization change on propagation, but also the shape and the orientation of the electromagnetic beams will change together [6, 17–19]. As was observed, the power spectrum of atmospheric turbulence in some aspects of the stratosphere and troposphere may exhibit non-Kolmogorov statistics [20, 21]. Therefore, the Kolmogorov model is sometimes incomplete for describing atmospheric turbulence. TOSELLI *et al.* introduced a non-Kolmogorov model to analyze the scintillation index of optical plane wave and angle of arrival fluctuations for a free space laser beam propagating through atmosphere [22, 23].

In this paper, we investigate the changes in the degree of polarization, orientation angle and degree of ellipticity for random electromagnetic cosh-Gaussian (ChG) beams in the non-Kolmogorov atmospheric turbulence. Based on the extended Huygens –Fresnel principle, we also obtain the analytical expressions for the elements of the cross-spectral density matrix of random electromagnetic ChG beams propagating through non-Kolmogorov atmospheric turbulence in Section 2. The changes in the degree of polarization, orientation angle and degree of ellipticity for random electromagnetic ChG beams are illustrated by numerical examples in Section 3. Finally, Section 4 provides some conclusions drawn from the present work.

2. Theoretical formulation

The cross-spectral density matrix of random electromagnetic beams at the source plane $z = 0$ is expressed as [6]

$$\mathbf{W}^{(0)}(\mathbf{s_1}, \mathbf{s_2}, 0, \omega) = \begin{bmatrix} W_{xx}(\mathbf{s_1}, \mathbf{s_2}, 0, \omega) & W_{xy}(\mathbf{s_1}, \mathbf{s_2}, 0, \omega) \\ W_{yx}(\mathbf{s_1}, \mathbf{s_2}, 0, \omega) & W_{yy}(\mathbf{s_1}, \mathbf{s_2}, 0, \omega) \end{bmatrix} \tag{1}$$

where

$$W_{ij}(\mathbf{s_1}, \mathbf{s_2}, 0) = \langle E_i^*(\mathbf{s_1}, 0) E_j(\mathbf{s_2}, 0) \rangle \tag{2}$$

and $i, j = x, y$ unless otherwise stated. The quantities E_x and E_y represent two electric -field components, $\mathbf{s_l} \equiv (s_{lx}, s_{ly})$ $(l = 1, 2)$ is the two-dimensional position vector at the source plane $z = 0$. The * and $\langle\ \rangle$ stand for the complex conjugate and ensemble average, respectively, ω is the frequency and omitted later for brevity.

The elements $W_{ij}(\mathbf{s_1}, \mathbf{s_2}, 0)$ of the cross-spectral density matrix of random electromagnetic ChG beams at the source plane are expressed as [24]

$$W_{ij}(\mathbf{s_1}, \mathbf{s_2}, 0) = A_i A_j B_{ij} \cosh\left[\Omega_0(s_{1x}+s_{1y})\right]\exp\left(-\frac{s_{1x}^2+s_{1y}^2}{w_0^2}\right)$$
$$\times \cosh\left[\Omega_0(s_{2x}+s_{2y})\right]\exp\left(-\frac{s_{2x}^2+s_{2y}^2}{w_0^2}\right)\exp\left[-\frac{(s_{1x}-s_{2x})^2}{2\sigma_{ij}^2}\right]$$
$$\times \exp\left[-\frac{(s_{1y}-s_{2y})^2}{2\sigma_{ij}^2}\right] \tag{3}$$

where A_i and A_j denote the amplitude of the electric field-vector components E_i and E_j, B_{ij} are correlation coefficients between two components E_i and E_j of the electric field -vector at the points $\mathbf{s_1}$ and $\mathbf{s_2}$ in the source plane $z = 0$ [25], w_0 is the waist width, Ω_0 is the parameter associated with the cosh-part, σ_{xx} and σ_{yy} are the auto-correlations length of E_x and E_y field components in the source plane, respectively, σ_{xy} and σ_{yx} are the cross-correlations length of E_x and E_y, which represents the spatial correlation between the x and y components of the electric field vector [26].

Each element of the cross-spectral density matrix, propagating in atmospheric turbulence, obeys the extended Huygens–Fresnel principle [3]

$$W_{ij}(\boldsymbol{\rho_1}, \boldsymbol{\rho_2}, z) = \left(\frac{k}{2\pi z}\right)^2 \iint \mathrm{d}^2\mathbf{s_1} \iint \mathrm{d}^2\mathbf{s_2}\, W_{ij}(\mathbf{s_1}, \mathbf{s_2}, 0)$$
$$\times \exp\left\{-\frac{ik}{2z}\left[(\boldsymbol{\rho_1}-\mathbf{s_1})^2-(\boldsymbol{\rho_2}-\mathbf{s_2})^2\right]\right\}\left\langle \exp\left[\psi^*(\mathbf{s_1}, \boldsymbol{\rho_1})+\psi(\mathbf{s_2}, \boldsymbol{\rho_2})\right]\right\rangle \tag{4}$$

where k is the wave number related to the wavelength λ by $k = 2\pi/\lambda$, $\boldsymbol{\rho_l} \equiv (\rho_{lx}, \rho_{ly})$ is the position vector at the z plane, and $\langle \exp[\psi^*(\mathbf{s_1}, \boldsymbol{\rho_1}) + \psi(\mathbf{s_2}, \boldsymbol{\rho_2})]\rangle$ is given by [6, 27, 28]

$$\left\langle \exp\left[\psi^*(\mathbf{s_1}, \boldsymbol{\rho_1})+\psi(\mathbf{s_1}, \boldsymbol{\rho_1})\right]\right\rangle$$
$$= \exp\left\{-4\pi^2k^2z\int_0^1\int_0^\infty \mathrm{d}\kappa\,\mathrm{d}\xi\,\kappa\varphi_n(\kappa)\left[1-J_0\Big(\kappa\big|(1-\xi)(\boldsymbol{\rho_1}-\boldsymbol{\rho_2})+\xi(\mathbf{s_1}-\mathbf{s_2})\big|\Big)\right]\right\} \tag{5}$$

where J_0 is the zero-order Bessel function and $\varphi_n(\kappa)$ is the spectral density of the refractive index fluctuations of turbulence.

By introducing two new variables of integration $\mathbf{u}$, $\mathbf{v}$

$$\mathbf{u} = \frac{\mathbf{s_1} + \mathbf{s_2}}{2}, \qquad \mathbf{v} = \mathbf{s_1} - \mathbf{s_2} \tag{6}$$

and substituting Eqs. (3) and (5) into Eq. (4), we obtain

$$\begin{aligned} W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = {} & \frac{1}{4} A_i A_j B_{ij} \left(\frac{k}{2\pi z}\right)^2 \exp\left[-\frac{ik}{2z}(\rho_1^2 - \rho_2^2)\right] \exp\left[-T(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2\right] \\ & \times \iint \mathrm{d}^2 u \iint \mathrm{d}^2 v \exp\left[-\frac{2u^2}{w_0^2}\right] \exp\left[\frac{ik}{z}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)\mathbf{u}\right] \exp\left[-\frac{ik}{z}\mathbf{u}\mathbf{v}\right] \\ & \times \exp[-a_{ij}\mathbf{v}^2] \exp[-T(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)\mathbf{v}] \exp\left[\frac{ik}{2z}(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)\mathbf{v}\right] \\ & \times \Big\{ \exp[2\Omega_0(u_x + u_y)] + \exp[\Omega_0(v_x + v_y)] \\ & + \exp[-\Omega_0(v_x + v_y)] + \exp[-2\Omega_0(u_x + u_y)] \Big\} \end{aligned} \tag{7}$$

where

$$a_{ij} = \frac{1}{2w_0^2} + \frac{1}{2\sigma_{ij}^2} + T \tag{8a}$$

$$T(\alpha, z) = \frac{\pi^2 k^2 z}{3} \int_0^\infty \kappa^3 \Phi_n(\kappa, \alpha) \mathrm{d}\kappa \tag{8b}$$

To model the turbulence, the non-Kolmogorov spectrum is used [22, 23]

$$\Phi_n(\kappa) = A(\alpha) \tilde{C}_n^2 \frac{\exp[-(\kappa^2/\kappa_m^2)]}{(\kappa^2 + \kappa_0^2)^{\alpha/2}}, \qquad 0 \le \kappa < \infty, \qquad 3 < \alpha < 4 \tag{9a}$$

$$A(\alpha) = \frac{\Gamma(\alpha - 1)\cos(\alpha\pi/2)}{4\pi^2} \tag{9b}$$

$$\kappa_0 = 2\pi/L_0 \tag{9c}$$

$$\kappa_m = c(\alpha)/l_0 \tag{9d}$$

$$c(\alpha) = \left[\Gamma\left(\frac{5 - \alpha}{2}\right) A(\alpha) \frac{2\pi}{3}\right]^{1/(\alpha - 5)} \tag{9e}$$

where L_0 and l_0 are the outer and inner scales of atmospheric turbulence, respectively, and $\Gamma(\cdot)$ is the Gamma function, α is the generalized exponent, $\tilde{C}_n^2$ is the generalized structure constant with units $\mathrm{m}^{3-\alpha}$ [22, 23, 29]. On substituting Eq. (9) into Eq. (8b), the integral calculations deliver

$$T(\alpha, z) = \frac{\pi^2 k^2 z}{6(\alpha-2)} A(\alpha)\tilde{C}_n^2 \left\{ \exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\kappa_m^{2-\alpha} \right.$$
$$\left. \times \left[(\alpha-2)\kappa_m^2 + 2\kappa_0^2\right]\Gamma\left(2-\frac{\alpha}{2}, \frac{\kappa_0^2}{\kappa_m^2}\right) - 2\kappa_0^{4-\alpha} \right\} \tag{10}$$

Recalling the integral formula [30]

$$\int \exp(-px^2 + 2qx)\,\mathrm{d}x = \sqrt{\frac{\pi}{p}}\exp\left(\frac{q^2}{p}\right) \tag{11}$$

the tedious but straightforward integral calculations lead to the elements of the cross -spectral density matrix of random electromagnetic ChG beams in non-Kolmogorov turbulence, which is given by

$$W_{ij}(\boldsymbol{\rho_1}, \boldsymbol{\rho_2}, z) = \frac{1}{4}A_i A_j B_{ij}\left(\frac{k}{2\pi z}\right)^2 \exp\left[-\frac{ik}{2z}(\rho_1^2 - \rho_2^2)\right]$$
$$\times \exp\left[-T(\boldsymbol{\rho_1} - \boldsymbol{\rho_2})^2\right](M_1 + M_2 + M_3 + M_4) \tag{12}$$

where

$$M_1 = \frac{\pi^2}{a_{ij}g_{ij}}\exp\left(\frac{P_{1x}^2 + P_{1y}^2}{4a_{ij}} + \frac{B_{1x}^2 + B_{1y}^2}{g_{ij}}\right) \tag{13a}$$

$$M_2 = \frac{\pi^2}{a_{ij}g_{ij}}\exp\left(\frac{P_{2x}^2 + P_{2y}^2}{4a_{ij}} + \frac{B_{2x}^2 + B_{2y}^2}{g_{ij}}\right) \tag{13b}$$

$$g_{ij} = \frac{2}{w_0^2} + \frac{k^2}{4z^2 a_{ij}} \tag{13c}$$

$$P_{1x} = \frac{ik}{2z}(\rho_{1x} + \rho_{2x}) - T(\rho_{1x} - \rho_{2x}) \tag{13d}$$

$$B_{1x} = \frac{1}{2}\left[\frac{ik}{z}(\rho_{1x} - \rho_{2x}) + 2\Omega_0 - \frac{ik}{2za_{ij}}P_{1x}\right] \tag{13e}$$

$$P_{2x} = \frac{ik}{2z}(\rho_{1x} + \rho_{2x}) - T(\rho_{1x} - \rho_{2x}) + \Omega_0 \tag{13f}$$

$$B_{2x} = \frac{1}{2}\left[\frac{ik}{z}(\rho_{1x} - \rho_{2x}) - \frac{ik}{2za_{ij}}P_{2x}\right] \tag{13g}$$

Due to the symmetry, P_{1y}, B_{1y}, P_{2y}, B_{2y} can be obtained by the replacement of ρ_{1x}, ρ_{2x} in P_{1x}, B_{1x}, P_{2x}, B_{2x} with ρ_{1y}, ρ_{2y}, respectively. M_3 and M_4 can be obtained by the replacement of Ω_0 in M_2 and M_1 with $-\Omega_0$.

The degree of polarization of random electromagnetic ChG beams through atmospheric turbulence is defined by the formula [6, 17]

$$P(\boldsymbol{\rho}, z) = \sqrt{1 - \frac{4\det[\mathbf{W}(\boldsymbol{\rho}, z)]}{\{\mathrm{Tr}[\mathbf{W}(\boldsymbol{\rho}, z)]\}^2}} \tag{14}$$

where det and Tr denote the determinant and the trace of the cross-spectral density matrix. In general, not only does the degree of polarization change on propagation, but also the shape and the orientation of the beam will change together, which can be specified by the orientation angle θ and the degree of ellipticity ε of the polarization ellipse [6, 18, 19]. The orientation angle θ that the major axis of the polarization ellipse makes with the x direction is given by the formula [6, 17]

$$\theta(\boldsymbol{\rho}, z) = \frac{1}{2}\operatorname{atan}\left\{\frac{2\mathrm{Re}[W_{xy}(\boldsymbol{\rho}, z)]}{W_{xx}(\boldsymbol{\rho}, z) - W_{yy}(\boldsymbol{\rho}, z)}\right\}, \quad -\pi/2 \le \theta \le \pi/2 \tag{15}$$

where Re denotes the real parts. The degree of ellipticity that can describe the shape of the polarization ellipse is given by [6, 17]

$$\varepsilon(\boldsymbol{\rho}, z) = A_{\mathrm{minor}}/A_{\mathrm{major}}, \quad 0 \le \varepsilon \le 1 \tag{16}$$

It is unity for circular polarization and zero for linear polarization. A_{major} and A_{minor} are the major and minor semi-axis of the polarization ellipse. The expressions can be written as

$$A^2_{\mathrm{major}}(\boldsymbol{\rho}, z) = \frac{1}{2}\left\{\sqrt{(W_{xx} - W_{yy})^2 + 4|W_{xy}|^2} + \sqrt{(W_{xx} - W_{yy})^2 + 4[\mathrm{Re}(W_{xy})]^2}\right\} \tag{17a}$$

$$A^2_{\mathrm{minor}}(\boldsymbol{\rho}, z) = \frac{1}{2}\left\{\sqrt{(W_{xx} - W_{yy})^2 + 4|W_{xy}|^2} - \sqrt{(W_{xx} - W_{yy})^2 + 4[\mathrm{Re}(W_{xy})]^2}\right\} \tag{17b}$$

By letting $\boldsymbol{\rho} = 0$ in Eqs. (14)–(17), the on-axis degree of polarization $P(0, z)$, the on-axis orientation angle $\theta(0, z)$ and the on-axis degree of ellipticity $\varepsilon(0, z)$ can be derived for random electromagnetic ChG beams through atmospheric turbulence.

3. Numerical calculations and analyses

Figure 1 gives the on-axis degree of polarization $P(0, z)$, orientation angle $\theta(0, z)$ and degree of ellipticity $\varepsilon(0, z)$ of random electromagnetic ChG beams in free space ($C_n^2 = 0$) and in non-Kolmogorov atmospheric turbulence ($C_n^2 = 10^{-14}$ and 5×10^{-14} m$^{-2/3}$) *vs*. the propagation distance z. The calculation parameters are $\lambda = 1.06$ μm, $w_0 = 3$ cm, $A_x = A_y = 2$, $B_{xx} = B_{yy} = 1$, $B_{xy} = 0.2\exp(i\pi/6)$, $B_{yx} = 0.2\exp(-i\pi/6)$, $\sigma_{xx} = 1$ cm, $\sigma_{yy} = 1.5$ cm, $\sigma_{xy} = \sigma_{yx} = 2$ cm, $\Omega_0 = 30$ m^{-1}, $l_0 = 0.01$ m, $L_0 = 10$ m, $\alpha = 3.2$. The parameters selected meet the realizability conditions [31]. As can be seen, the states of polarization (P, θ, ε) of random electromagnetic ChG beams depend on the structure constant C_n^2 and the propagation distance z. The states of polarization of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence are different from those in free space. Figure 1 implies that the states of polarization vary non-monotonously with increasing propagation distance z, and there exists a maximum for degree of polarization P and a minimum for orientation angle θ and degree of ellipticity ε. At a fixed z, the larger the structure constant C_n^2, the smaller the degree of polarization P; the larger the orientation angle θ and degree of ellipticity ε. For example, at $z = 5$ km, $P(0, 5\text{ km}) = 0.518$,

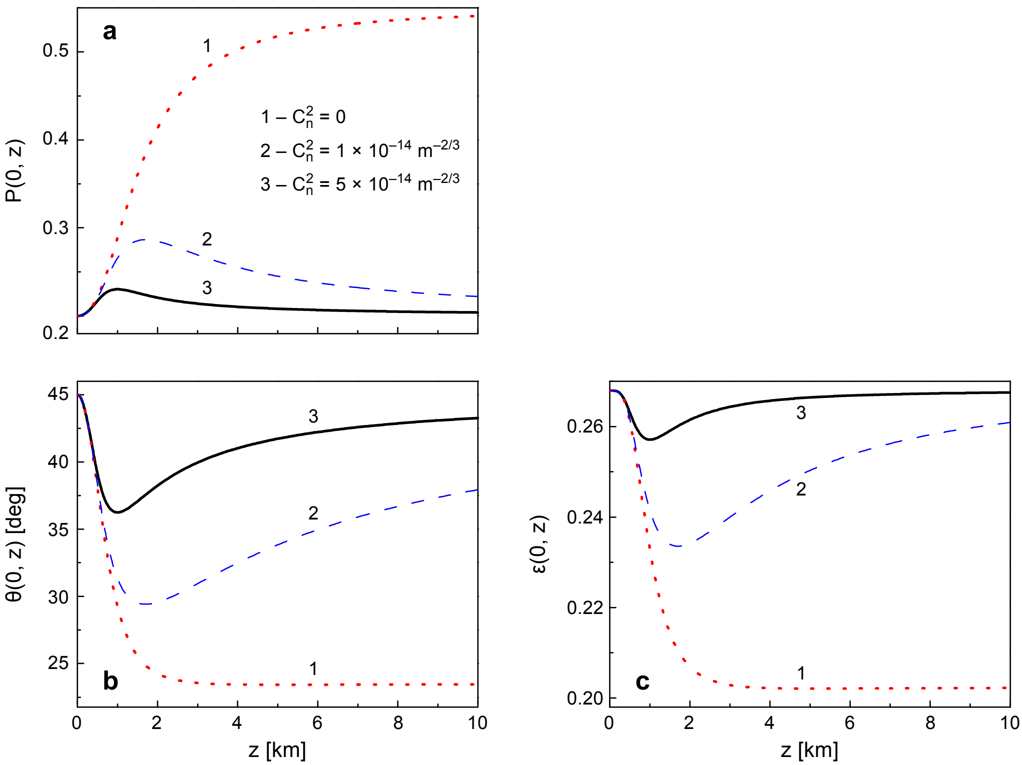

Fig. 1. Changes in the on-axis degree of polarization P (**a**), orientation angle θ (**b**) and degree of ellipticity ε (**c**) of random electromagnetic ChG beams *vs.* the propagation distance z for different values of the C_n^2.

0.245, 0.208, $\theta(0, 5\text{km}) = 24.432$, 33.839, 41.730 deg, $\varepsilon(0, 5\text{ km}) = 0.202$, 0.250, 0.266, for $C_n^2 = 0$, 10^{-14}, 5×10^{-14} m$^{-2/3}$, respectively.

The on-axis degree of polarization $P(0, z)$, orientation angle $\theta(0, z)$ and degree of ellipticity $\varepsilon(0, z)$ of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence *vs*. the propagation distance z are depicted in Fig. 2 for different values of auto-correlations length σ_{yy}, where $C_n^2 = 10^{-14}$ m$^{-2/3}$, $\sigma_{xx} = 1.5$ cm, $\sigma_{xy} = \sigma_{yx}$ = 2 cm. The other calculation parameters are the same as those in Fig. 1. Figure 2**a** demonstrates that the on-axis degree of polarization P decreases with an increase in auto-correlations length σ_{yy}. Figure 2**b** shows that the on-axis orientation angle θ has a minimum or a maximum when $\sigma_{yy} < \sigma_{xx}$ or $\sigma_{yy} > \sigma_{xx}$, respectively. Figure 2**c** shows that the on-axis degree of ellipticity ε is a constant for the case of $\sigma_{yy} = \sigma_{xx}$, and ε increases with an increase in auto-correlations length σ_{yy} for the case of $\sigma_{yy} \neq \sigma_{xx}$.

Figure 3 represents the on-axis degree of polarization $P(0, z)$, orientation angle $\theta(0, z)$ and degree of ellipticity $\varepsilon(0, z)$ of random electromagnetic ChG beams in

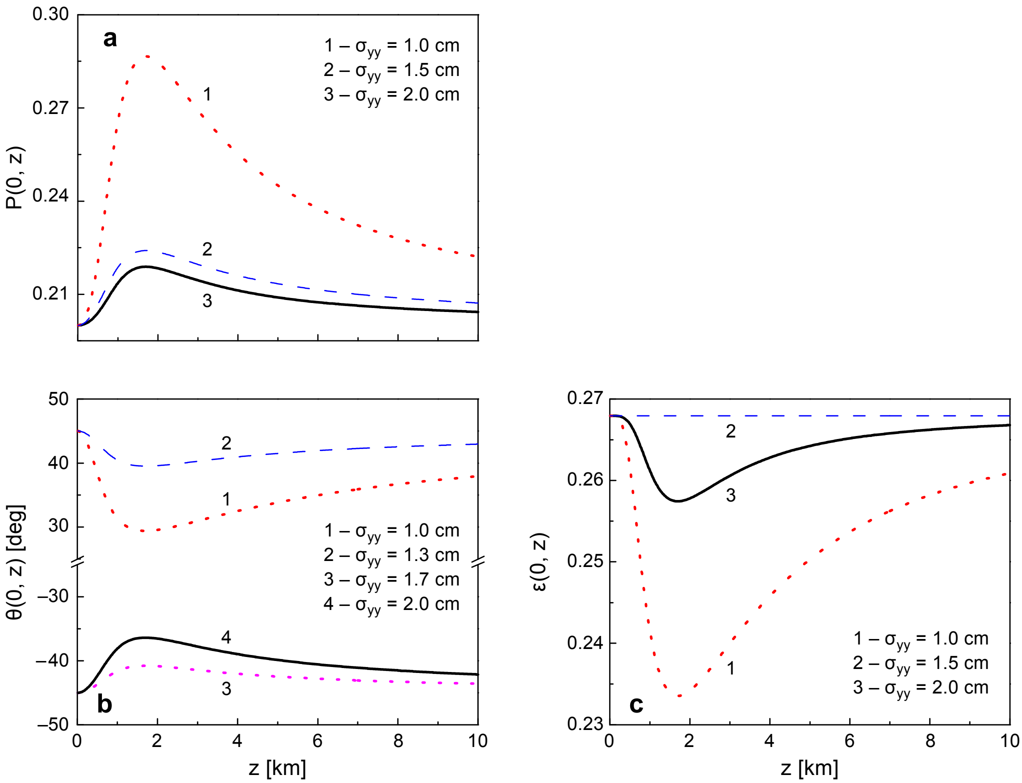

Fig. 2. Changes in the on-axis degree of polarization P (**a**), orientation angle θ (**b**) and degree of ellipticity ε (**c**) of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence for different auto-correlations length σ_{yy}.

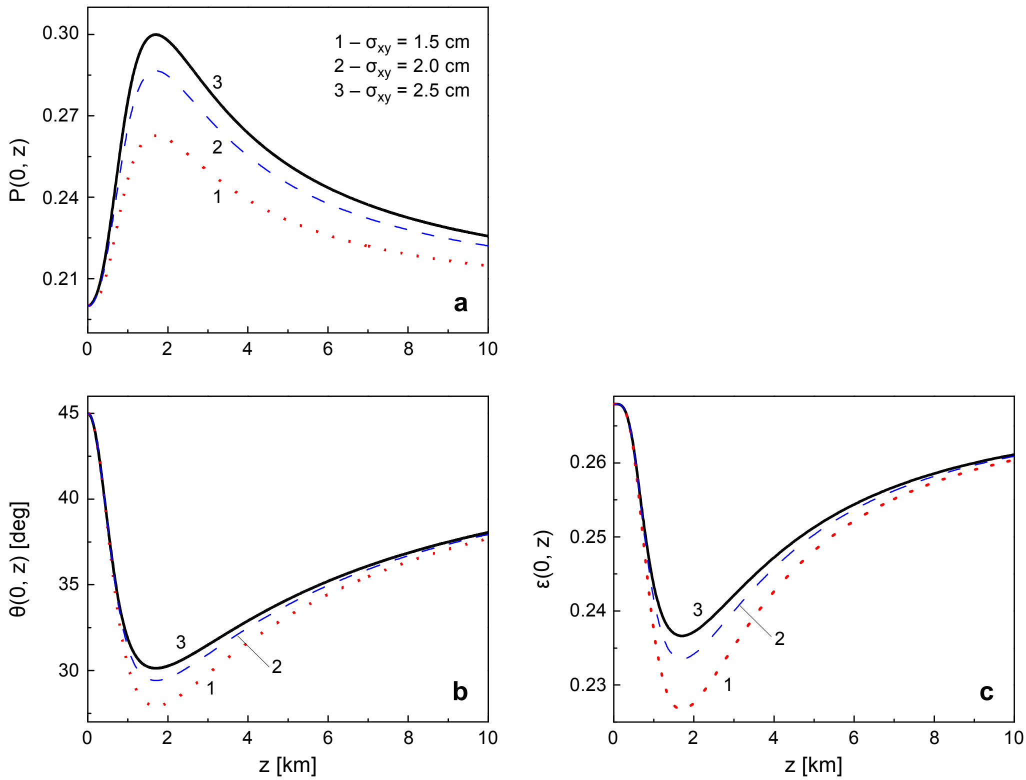

Fig. 3. Changes in the on-axis degree of polarization P (**a**), orientation angle θ (**b**) and degree of ellipticity ε (**c**) of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence for different cross-correlations length σ_{xy} (σ_{yx}).

non-Kolmogorov atmospheric turbulence *vs*. the propagation distance z for different values of cross-correlations length σ_{xy} (σ_{yx}) = 1.5, 2 and 2.5 cm, where σ_{xx} = 1 cm. The other calculation parameters are the same as those in Fig. 1. As can be seen, the larger the cross-correlations length σ_{xy} (σ_{yx}), the larger degree of polarization P, orientation angle θ and degree of ellipticity ε, *i.e.*, the P, θ and ε will increase with an increase in cross-correlations length σ_{xy} (σ_{yx}).

The changes in the on-axis degree of polarization $P(0, z)$, orientation angle $\theta(0, z)$, degree of ellipticity $\varepsilon(0, z)$ of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence *vs*. the propagation distance z for the different values of cosh -part parameter Ω_0 are plotted in Fig. 4, where $C_n^2 = 10^{-14}\ \mathrm{m}^{-2/3}$. The other calculation parameters are the same as those in Fig. 1. From Figs. 4**a** and 4**c** we note that the on-axis degree of polarization P and degree of ellipticity ε appear to have an oscillatory behavior when Ω_0 = 70 and 90 of random electromagnetic ChG beams in atmospheric turbulence, however, the oscillatory behavior disappears for smaller Ω_0 (*e.g.*, $\Omega_0 \le 50$).

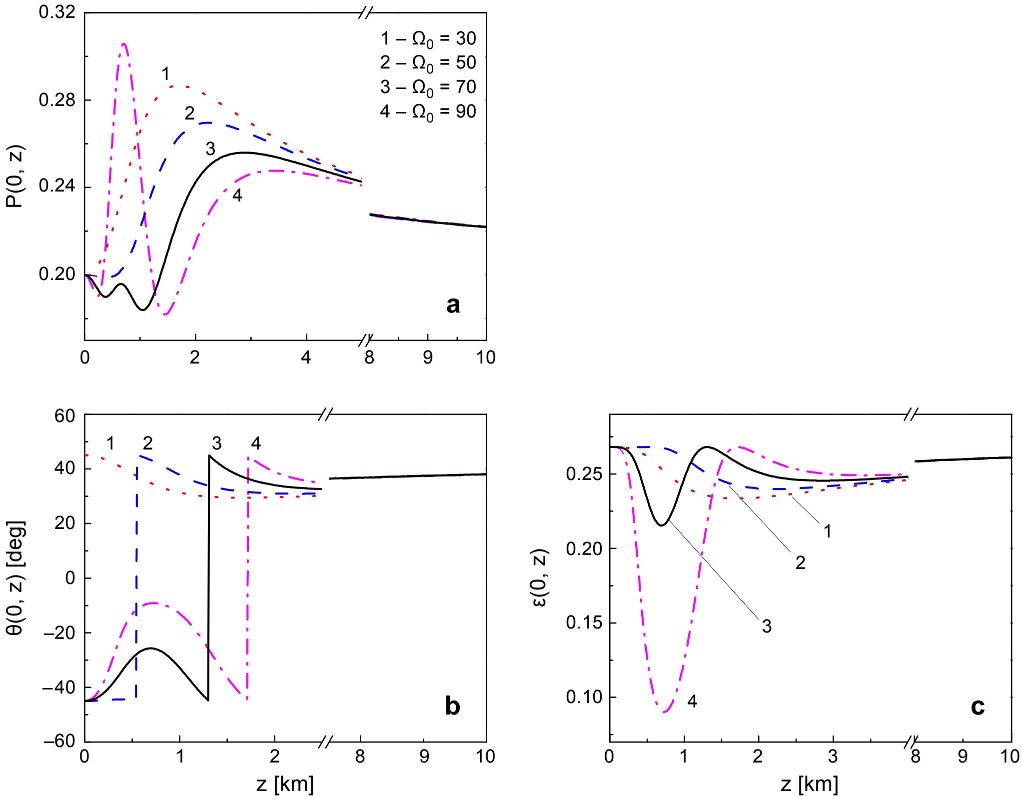

Fig. 4. Changes in the on-axis degree of polarization P (**a**), orientation angle θ (**b**) and degree of ellipticity ε (**c**) of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence for different cosh-part parameter Ω_0.

As Fig. 4**b** suggested, there exists a rapid transition of the on-axis orientation angle θ when $\Omega_0 \geq 50$, the critical position of orientation angle transition increases as the Ω_0 increases, but for $\Omega_0 \leq 30$, the transition will disappear.

4. Conclusion

In this paper, based on the extended Huygens–Fresnel principle, the analytical expression for the elements of the cross-spectral density matrix of random electromagnetic ChG beams propagating through non-Kolmogorov atmospheric turbulence has been derived, and used to study changes in the on-axis degree of polarization, orientation angle and degree of ellipticity of random electromagnetic ChG beams propagating through non-Kolmogorov atmospheric turbulence. It has been shown that the states of polarization (P, θ, ε) of random electromagnetic ChG beams depend on the structure constant, auto-correlations length, cross-correlations length, cosh-part parameter and the propagation distance z. The states of polarization of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence are different from those in free space.

At a fixed z, the larger the structure constant C_n^2, the smaller the degree of polarization P, the larger the orientation angle θ and degree of ellipticity ε. The P, θ and ε will increase with an increase in cross-correlations length. The on-axis degree of polarization P and degree of ellipticity ε appear to have an oscillatory behavior when $\Omega_0 = 70$ and 90 of random electromagnetic ChG beams in non-Kolmogorov atmospheric turbulence, and there exists a rapid transition of the on-axis orientation angle of the polarization ellipse θ when $\Omega_0 \geq 50$. The results obtained may have beneficial applications to the space optical communications and remote sensing.

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