Improved method for phase wraps reduction in profilometry

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In order to completely eliminate, or greatly reduce the number of phase wraps in 2D wrapped phase map, Gdeisat and co-workers proposed an algorithm, which uses shifting the spectrum towards the origin. But the spectrum can be shifted only by an integer number, meaning that the phase wraps reduction is often not optimal. In addition, Gdeisat's method will take much time to make the Fourier transform, inverse Fourier transform, select and shift the spectral components. In view of the above problems, we proposed an improved method for phase wraps elimination or reduction. First, the wrapped phase map is padded with zeros, the carrier frequency of the projected fringe is determined by high resolution, which can be used as the moving distance of the spectrum. And then realize frequency shift in spatial domain. So it not only can enable the spectrum to be shifted by a rational number when the carrier frequency is not an integer number, but also reduce the execution time. Finally, the experimental results demonstrated that the proposed method is feasible.

Keywords: phase unwrapping, zero-padding, Fourier transform, carrier-frequency, profilometry.

1. Introduction

Phase-based fringe projection technique is an important method in three-dimensional (3D) shape measurement. It has been extensively investigated and widely used in aerospace, biological engineering, military reconnaissance and other fields because of its full field, real-time, non-contact, simple device and higher accuracy [1–4]. The basic procedure is as follows: first, the sinusoidal fringe is projected onto the surface of the tested object, and the deformed fringe is recorded by a camera, and then the phase information of the deformed fringe is demodulated by the four-step phase shift method or the Fourier transform method. Since the phase obtained by the phase demodulation method is wrapped in $(-\pi, \pi)$, phase unwrapping is necessary to obtain the continuous phase [5–7]. In re-

cent decades, a variety of unwrapping algorithms are proposed. Most phase-unwrapping algorithms can be classified into two categories: temporal phase unwrapping and spatial phase unwrapping. The spatial phase unwrapping is a process of integral accumulation. Once in one of these points appears an error, this error will spread to the following points, which will affect the calculation of the phase, and it may lead to the phenomenon of wire drawing [8, 9]. In the actual measurement, discontinuous morphology, noise and shadow may lead to errors. To avoid the impact of these factors on the unwrapping process, some scholars proposed local spatial phase unwrapping and global spatial phase unwrapping method. The local spatial phase unwrapping method includes a branch-cut method [10], quality guided method [11] and so on. The global spatial phase unwrapping method mainly includes the least square method [12] and the minimum norm method [13]. Although these methods have a certain improvement, there are some limitations. Saldner and Huntley [14] proposed the temporal phase unwrapping method, which is made in the temporal domain, where a sequence of maps is acquired while the fringe pitch is changed. Then the phase at each pixel is unwrapped along the time axis to avoid the spread of errors. However, the method needs multiple frames of fringe images which would take much time. Although there are many phase unwrapping algorithms, no algorithm can be applied to address all problems. If the number of phase wraps in the phase diagram can be eliminated or reduced, it will undoubtedly be beneficial to reduce the unwrapping time and improve the noise performance. Qudeisat *et al*. [15] proposed the phase wraps reduction method to completely eliminate, or greatly reduce the number of phase wraps, which would make the phase unwrapping simple and fast. In some cases, all of the phase wraps are eliminated and there is therefore no need to unwrap the resultant phase map. However, according to our own experience with the method, Gdeisat's method has the following disadvantages:

1) This method needs Fourier transform, inverse Fourier transform, select and shift the spectral components, these procedures increase the calculation complexity and the processing time consuming as well.

2) The spectrum can be shifted only by an integer number in Gdeisat's method, meaning that the phase wraps reduction is often not optimal.

Here, in order to improve the calculation efficiency as well as simplify its procedures, we present an improved method for phase wraps elimination or reduction. This method is a good solution to the problems of Gdeisat's method. The capability of the presented method is demonstrated by both theoretical analysis and experiments.

The paper is organized as follows. Section 2 introduces the principle of the system. Section 3 presents the experimental results. Section 4 summarizes this paper.

2. Theory

2.1. Gdeisat's method

Phase-measuring profilometry is one of the simplest methods to convert the phase map to the height of the 3D object surface. The schematic of the profilometry system is as shown in Fig. 1, where P is the exit pupil of the projector, C is the entrance pupil of
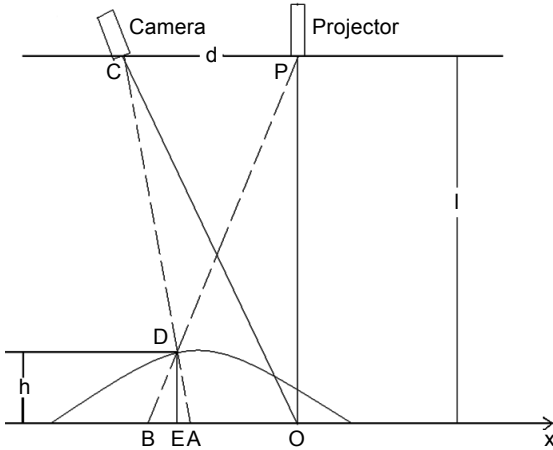
Fig. 1. Optical path of phase measuring profilometry (see text for explanation).

the camera, and D is an arbitrary point on the tested object. Points P and C are assumed to be on the same plane with a distance l to the reference plane. The distance between the projector and the camera is d. The height of the object surface relative to the reference plane can be calculated by [16]

$$h = \frac{l}{1 + 2\pi d/p\Delta\varphi} \tag{1}$$

where p is the pitch of the fringe pattern, $\Delta\varphi$ is the phase difference between points A and B. So when we obtain $\Delta\varphi$, the height value of the object surface can be calculated. However, the phase value obtained by phase demodulation algorithm is usually wrapped. So phase unwrapping is necessary to obtain the continuous phase.

Gdeisat and co-workers proposed an algorithm to completely eliminate, or greatly reduce the number of phase wraps in the phase diagram using spectrum-shifting. The basic process is as follows. First, convert the wrapped phase map into the complex array, and calculate the 2D Fourier transform. Then shift the spectrum towards the origin and make the inverse Fourier transform. Finally, extract the phase using the arctangent function. After this, the number of phase wraps in the new phase map is greatly reduced or eliminated, which would make the phase unwrapping simple, fast, and even without phase unwrapping. In [15], when the maximum phase change in the deformed fringe patterns is less than π radians, by applying Gdeisat's method to the wrapped phase map, all the phase wraps in the image are eliminated. The resultant phase map therefore does not require the application of any phase unwrapping algorithms. We think the condition is same with the initial phase in [4, 17].

2.2. Our method

In [15], Gdeisat's method can eliminate, or greatly reduce the number of phase wraps in the phase map, but it also has some disadvantages, that is, *i*) this method needs twice

Fourier transform, and it needs to search and select the spectral components, which will occupy large amount of processing time; *ii*) because it uses the discrete Fourier transform, the spectrum can be shifted only by an integer number in Gdeisat's method. But the actual frequency may be a fraction, that is, it is a rational number. So when we use Gdeisat's method, the phase wraps reduction is often not optimal. Based on our analysis of the Ref. [15], we proposed an improved method for phase wraps elimination or reduction. This section thoroughly explains the principle of our method.

Let us assume that the wrapped phase extracted using the four-step phase shift algorithm is expressed by,

$$\varphi_{\mathrm{w}}(x, y) = W[2\pi f_x x + 2\pi f_y y + \varphi(x, y)] \tag{2}$$

where f_x and f_y are the spatial carrier frequency along *x*- and *y*-axis, respectively, $\varphi(x, y)$ represents the phase information of the tested object, and $\varphi_{\mathrm{w}}(x, y)$ is the wrapped phase which is wrapped in $(-\pi, \pi)$. The following equation converts the wrapped phase map into the complex array $\varphi_{\mathrm{wc}}(x, y)$:

$$\varphi_{\mathrm{wc}}(x, y) = \exp[j\varphi_{\mathrm{w}}(x, y)] \tag{3}$$

where *j* is equal to $\sqrt{-1}$.

In Gdeisat's method, first, make the Fourier transform to:

$$\Phi_{\mathrm{wc}}(u, v) = F[\varphi_{\mathrm{wc}}(x, y)] \tag{4}$$

where $F[\cdot]$ is the 2D Fourier transform operator, and the terms *u* and *v* are the vertical and horizontal frequencies, respectively. The 2D Fourier transform of the wrapped phase $\Phi_{\mathrm{wc}}(u, v)$ is shifted towards the origin, then the inverse Fourier transform is applied:

$$\varphi_{\mathrm{wcs}}(x, y) = F^{-1}[\Phi_{\mathrm{wc}}(u + u_0, v + v_0)] \tag{5}$$

where $F^{-1}[\cdot]$ is the inverse 2D Fourier transform operator, u_0 and v_0 are the frequency shift values. We can obtain the phase map as follows:

$$\varphi_{\mathrm{ws}}(x, y) = \tan^{-1}\left\{ \frac{\mathrm{Im}[\varphi_{\mathrm{wcs}}(x, y)]}{\mathrm{Re}[\varphi_{\mathrm{wcs}}(x, y)]} \right\}$$

$$= W[2\pi(f_x - u_0)x + 2\pi(f_y - v_0)y + \varphi(x, y)] \tag{6}$$

where $\tan^{-1}$ denotes the four quadrant arctangent operator, Im – the imaginary part and Re – the real part of the complex array $\varphi_{\mathrm{wcs}}(x, y)$.

From Eq. (6), we can see that shifting the spectrum in the frequency domain towards the origin results in decreasing the spatial carrier frequency of projected fringe, thus the number of phase wraps in the phase map is reduced.

Through the analysis, it is easy to see that the spectrum can be shifted only by an integer number with the discrete Fourier transform, while the actual frequency in 3D morphology experiment may not exactly be an integer, but a rational number. Thus when we use Gdeisat's method, the phase wraps reduction is often not optimal. Besides, Gdeisat's method requires the Fourier transform, spectrum search, shifting and the inverse Fourier transform. Therefore, in the actual operation, the process is relatively complex. To solve this problem, we simplify and speed up the implementation process in the spatial domain by the frequency shift property of the 2D Fourier transform [18]. The frequency shift property of the 2D Fourier transform can be written as

$$F(u + u_0, v + v_0) = F\left\{ f(x, y) \exp\left[-j2\pi(u_0 x/m + v_0 y/n) \right] \right\} \tag{7}$$

where $F(u, v)$ is the Fourier transform of $f(x, y)$, and m, n are the length of $f(x, y)$ along x- and y-axis, respectively.

By Eq. (7), we can directly obtain $\varphi_{wcs}(x, y)$ by multiplying the result $\varphi_{wc}(x, y)$ of Eq. (3) by $\exp[-j2\pi(u_0 x/m + v_0 y/n)]$

$$\Phi_{wc}(u + u_0, v + v_0) = F\left\{ \varphi_{wc}(x, y) \exp\left[-j2\pi(u_0 x/m + v_0 y/n) \right] \right\} \tag{8a}$$

$$\varphi_{wcs}(x, y) = F^{-1}\left[\Phi_{wc}(u + u_0, v + v_0) \right]$$

$$= \varphi_{wc}(x, y) \exp\left[-j2\pi(u_0 x/m + v_0 y/n) \right] \tag{8b}$$

By Eq. (8), we can realize the frequency shift in the spatial domain by the frequency shift property of the 2D Fourier transform. It is obvious that our method does not require the Fourier transform, spectrum selection, spectrum shift and the inverse Fourier transform, and u_0, v_0 can be a rational number, so the proposed method overcomes the limitation of the Gdeisat's method on the frequency shift value.

In order to determine the actual frequency of the deformed fringe pattern in 3D profilometry experiment, QI FAN *et al.* [19] proposed a spectrum centroid method for estimating the carrier frequency, yet they still needed the Fourier transform. Although the accuracy is high, the computational burden is large. YONGZHAO DU *et al.* [20] proposed an accurate method to estimate the carrier frequency based on zero padding [21]. According to [20], pad $\varphi_w(x, y)$ in Eq. (2) with zeros,

$$\varphi_{wz}(x, y) = \begin{cases} \varphi_w(x, y), & 0 \le x \le M - 1, \quad 0 \le y \le N - 1 \\ 0, & M \le x \le kM - 1, \quad N \le y \le kN - 1 \end{cases} \tag{9}$$

where M and N are the values of $\varphi_w(x, y)$ along x- and y-axis, respectively, and k is an integer.

Make the Fourier transform to $\varphi_{wz}(x, y)$,

$$\Phi_{wz}\left(\frac{u}{kM}, \frac{v}{kN}\right) = \sum_{m=0}^{kM-1} \sum_{n=0}^{kN-1} \varphi_{wz}(x, y) \exp\left[-j2\pi\left(\frac{u}{kM}x + \frac{v}{kN}y\right)\right] \qquad (10)$$

From Eq. (10), we can see that zero padding in the spatial domain will achieve up-sampling in the frequency domain [20, 21]. In [20], taking $k = 10$, the accuracy is improved by 10 times. However, if we use the method in [20], the fringe pattern will become very large after zero padding, and when we make the Fourier transform to the padded fringe pattern, computation burden will be quite large. So Feng's method requires high performance computer hardware, and the speed is very slow.

Therefore, in order to determine the actual carrier frequency of the deformed fringe pattern, we improve the zero padding method in [20]. The basic idea is that the 2D Fourier transform is replaced by two one-dimensional (1D) Fourier transform, and the horizontal and vertical components of the fringe carrier frequency are estimated by the 1D Fourier transform, respectively. The specific procedure is as follows:

1) Take the middle of the row and the column of $\varphi_w(x, y)$ in Eq. (2), pad them with zeros respectively, thus obtain two 1D vector data.

2) Make the 1D Fourier transform respectively to the two 1D vector data in step 1, estimate their spectrum peak position, and obtain u_0 and v_0.

Only needing two 1D Fourier transforms, even if taking $k = 100$, the procedure with zero padding is still very simple, therefore the speed is still very fast.

The following experiment is used to verify the proposed algorithm.

3. Experimental results

In this section, for evaluating the real performance of our method, we test our method on a series of experiments. Below, we will describe these experiments and practical suggestions for the above procedure.

We develop a fringe projection measurement system, which consists of a DLP projector (Optoma EX762) driven by a computer and a CCD camera (DH-SV401FM). The schematic of fringe-projection profilometry system is as shown in Fig. 1. The distances d and l can be estimated using calibration algorithm [16]. In our experiments, d is about 60 cm, l is about 70 cm, l_0 is about 20 cm. The surface measurement software is programmed by Matlab with i5-4570 CPU at 3.20 GHz.

First, we do an experiment on a face model with smooth shapes, the biggest height is about 3 cm, the phase value of projected fringe is 12π. By calculation with Eq. (5) in [4], φ_{BD} is less than 2π. In theory, the resultant phase map does not require any phase unwrapping after applying Gdeisat's method. Figure 2 shows the first pattern of the sinusoidal fringe captured with the resolution of 688×582. The corresponding wrapped phase is shown in Fig. 3 after the four-step phase shift algorithm is applied.

Take the middle of the row (the 300th row) in Fig. 3, pad it with zeros ($k = 10$), the result is shown in Fig. 4.
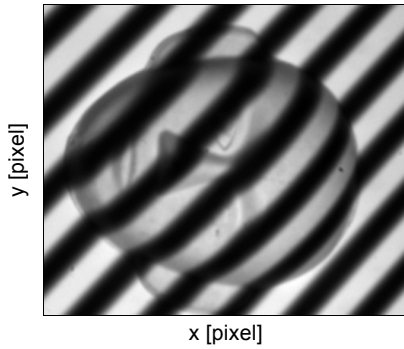
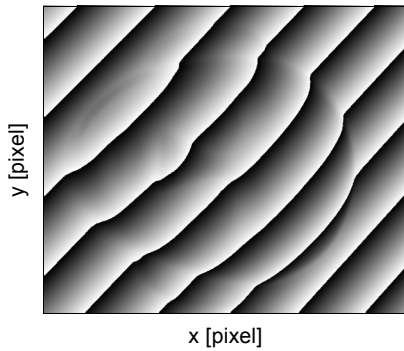Fig. 2. The first pattern of the sinusoidal fringe.
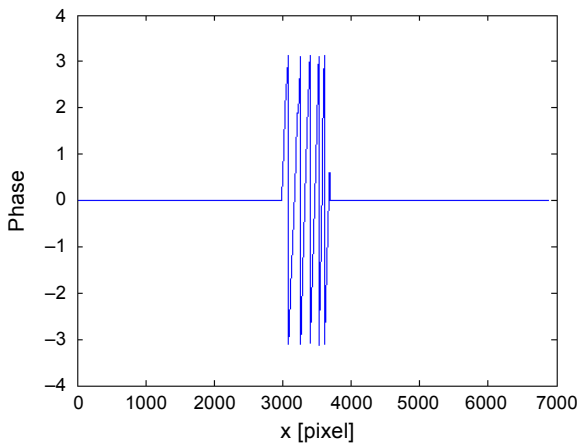

Fig. 3. The wrapped phase map.

Fig. 4. The 300th row of Fig. 3 after zero padding.

Figure 5 shows the spectrum of Fig. 4 by the 1D Fourier transform. We can see that the resolution of frequency domain is improved by ten times due to zero padding. By estimating the peak of the spectrum, determine $v_0 = 4.7$.

With the same method, to take the middle of the column (the 300th column) in Fig. 3, we pad it with zeros ($k = 10$), and we can determine $u_0 = 3.8$.

Then we put u_0 and v_0 into Eq. (8), and the phase map $\varphi_{\mathrm{wcs}}(x, y)$ is shown in Fig. 6. The phase wraps are completely eliminated.
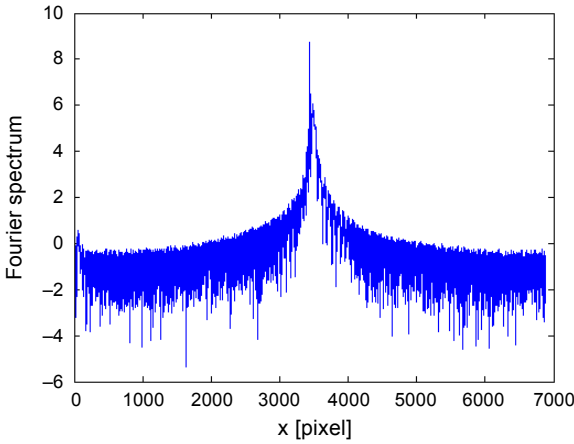
Fig. 5. The spectrum of Fig. 4.

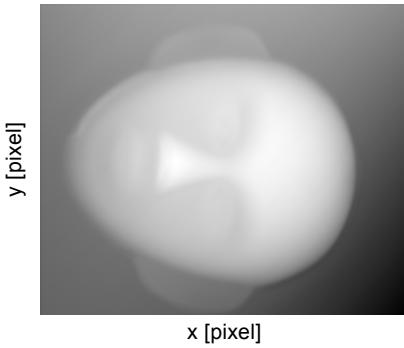

x [pixel]   Fig. 6. The phase map obtained by the proposed method.

For comparison, according to Gdeisat's method, take $u_0 = 4$, $v_0 = 5$, get the phase data as shown in Fig. 7, one can see that there are still phase wraps after applying Gdeisat's method because the actual carrier frequency is not an integer. Therefore it requires phase unwrapping to obtain the continuous phase. From Fig. 6, we can find that with the proposed method, the carrier frequency can be determined more accurately, and the phase wraps are completely eliminated. So the resultant phase map no longer requires any phase unwrapping.
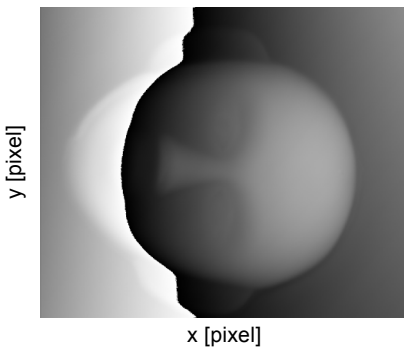

x [pixel]   Fig. 7. The phase map obtained by Gdeisat's method.

In order to compare the time between Gdeisat's method and the proposed method, we conduct a comparative experiment. In the comparisons, all processed fringe patterns have the pixels sizes of 688×582, the computational platform is a laptop with Intel Core i5-4570 CPU at 3.20 GHz and a 4 GB RAM. We use MATLAB 2014a on the same computer to process the same fringe pattern. The proposed method only needs 0.672 s while Gdeisat's method costs 1.390 s. The difference of time consuming between the two methods is obvious, the speed of the proposed algorithm is improved by about 50%. In the proposed method, the calculation procedures such as the Fourier transform, frequency selection and inverse transform are not required, therefore, it can save large amount of processing time.

For a more complete verification of the proposed method, we conduct another experiment on a plastic board with a big hole. To validate that the methods are applied to objects with a hole, we use a black background in the hole area. The biggest height is about 7 cm, the phase value of the projected fringe is 8π. By calculation with Eq. (5) in [4], φ_{BD} is more than 2π. In theory, the resultant phase map is still wrapped after applying Gdeisat's method. Figure 8 shows the first pattern of the sinusoidal fringe. The corresponding wrapped phase is shown in Fig. 9. Then we unwrap the phase map by Herráez's method [22] 10 times, the total processing time is 19.72 s.

By the proposed method, we can determine $v_0 = 3.5$. Because the projected fringe is vertical, we can determine $u_0 = 0$. Put u_0 and v_0 into Eq. (8), the phase map $\varphi_{\mathrm{wcs}}(x, y)$ is shown in Fig. 10.
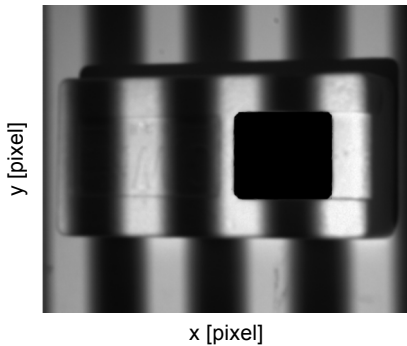

Fig. 8. The first pattern of the sinusoidal fringe.
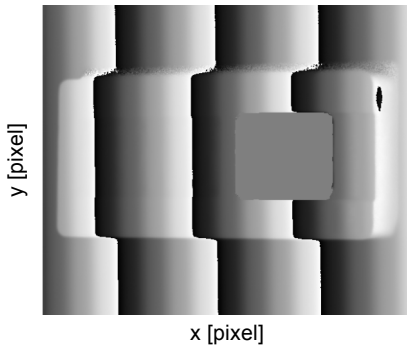

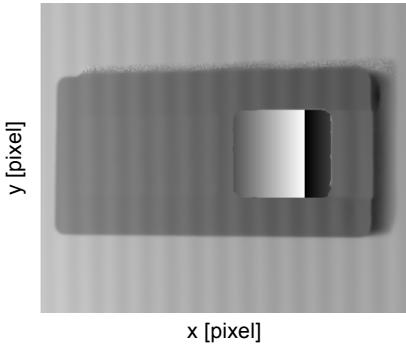Fig. 9. The wrapped phase map.

Fig. 10. The phase map obtained by the proposed method.
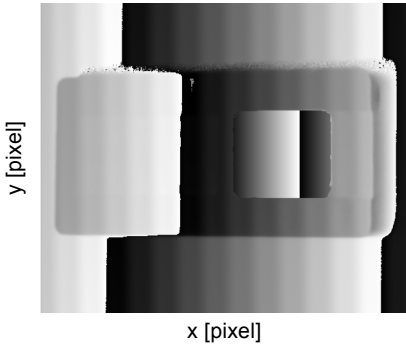

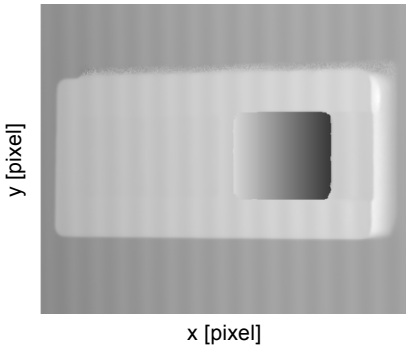Fig. 11. The phase map obtained by Gdeisat's method.

Fig. 12. The unwrapped phase map.

In contrast, according to Gdeisat's method, we take $u_0 = 0$, $v_0 = 4$, to obtain the phase data as shown in Fig. 11. Then we unwrap the wrapped phase in Figs. 10 and 11 by Herráez's method [22] 10 times. The resultant unwrapped phases obtained from Figs. 10 and 11 are the same, which are shown in Fig. 12. The processing times are 17.67 and 16.55 s, respectively.

From Figs. 10 and 11, we can see that, because the tested object is too high, φ_{BD} is more than 2π, and the phase wraps are not eliminated completely after the spectrum shift [15, 16]. But by the proposed method, the phase wraps reduction is optimal, and the proposed method is better than Gdeisat's method.

4. Conclusion

In this paper, we propose an improved method for phase wraps elimination or reduction, which is an extension of Gdeisat's method. Our method overcomes the main disadvantages that Gdeisat's method encounters. The proposed method estimates the actual frequency of the deformed fringe pattern more accurately than the zero padding method, eliminates Fourier transform, inverse Fourier transform and frequency selection in Gdeisat's method, and achieves the frequency shift by a simple multiply operation in the spatial domain by the frequency shift property of 2D Fourier transform. So the proposed method not only improves the operation speed, but also adapts to the situation when the spectrum shift distance is a rational number, which would make the phase wraps reduction optimal.

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