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GROUPED RISK PARITY PORTFOLIOS FOR STOCKS FROM THE WSE

PORTFELE GRUPOWEGO PARYTETU RYZYKA DLA SPÓŁEK Z GPW W WARSZAWIE

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Summary: The risk parity portfolios are characterized by equally weighted contributions of all assets. Very often, the idea of risk parity is considered as a special type of the diversification strategy. This approach became very popular among investors after the last economic crisis, when many portfolios perceived as well-diversified suddenly became undiversified portfolios. Usually, risk parity is calculated for individual stocks. In this article, the method of estimating risk parity portfolios for grouped stocks is discussed. The presented model is applied to selected stocks belonging to different groups (sectors, size of companies) and quoted on the Warsaw Stock Exchange. The main goal of the empirical research is the analysis of the risk parity portfolios calculated for the groups of stocks and also for individual stocks. Additionally, the risk parity portfolios are compared with the naive portfolios and minimum variance portfolios. All portfolios are compared according to the risk, rate of return, Sharpe ratio and the future profits.

Keywords: risk parity portfolio, grouped risk parity, equally risk contribution portfolio, diversification, well-diversified portfolio.

Streszczenie: Portfele parytetu ryzyka określane są jako portfele, których całkowite ryzyko dzielone jest równo na wszystkie jego składniki. Obecnie idea parytetu ryzyka rozważana jest jako szczególny przypadek strategii dywersyfikacyjnej. Metoda parytetu ryzyka stała się popularna wśród inwestorów po ostatnim kryzysie ekonomicznym, kiedy to dobrze zdywersyfikowane portfele, nagle okazały się portfelami niezdywersyfikowanymi. Zazwyczaj parytet ryzyka konstruowany jest dla indywidualnych akcji. W artykule omówiono metodę wyznaczania grupowego parytetu ryzyka. Model ten zastosowano dla wybranych spółek z GPW w Warszawie. Głównym celem badań empirycznych była analiza portfeli parytetowych wyznaczanych zarówno dla danych grupowych, jak i dla indywidualnych spółek. Portfele parytetowe porównane zostały z portfelami naiwnymi i portfelami minimalnej wariancji. Portfele oceniono pod względem ryzyka, stóp zwrotu, współczynnika Sharpe'a oraz przyszłych zysków.

Słowa kluczowe: portfele parytetu ryzyka, grupowy parytet ryzyka, portfele o równym udziale ryzyka, dywersyfikacja, portfel dobrze zdywersyfikowany.

1. Introduction

Usually portfolios are constructed so as to maintain a balance between the rate of return and risk. A good portfolio is the one that brings maximum profits with minimum possible risk. However, this approach, in which both the portfolio return and risk are considered, very often leads to the construction of inefficient portfolios. Better results can be achieved if, during the construction of portfolios, we focus only on the distribution of risk. This approach gives good effects even in the case of rapid changes in the financial markets. The approach in which portfolios are constructed only according to risk contribution refers to risk parity portfolios also called equally weighted risk portfolios. An important characteristic of the risk parity portfolios is that by using this approach we avoid the dominant role of one or more stocks in the portfolio. In addition, we receive portfolios which have a maximum level of diversification [Qian 2005, 2006; Braga 2015].

The studies related to the risk parity conducted so far mainly concerned the methods of construction of such portfolios [Chaves et al. 2011, 2012; Lohre et al. 2012; Maillard et al. 2010; Meucci 2009]. In numerous studies, risk parity portfolios were compared with portfolios such as: minimum variance portfolios, mean-variance portfolios, naive portfolios or the most diversified portfolios [Chaves et al. 2011, 2012; Braga 2015]. In the Polish literature, the results of the research on the method of selection of companies for the portfolios of equal risk contribution were presented by Gluzicka [2015a]. The case of multi-period risk parity portfolios [Gluzicka 2015b] and the parity risk for selected linear risk measures [Gluzicka 2016] were also analysed.

Previous studies were related to risk parity of a single financial instrument. The subject of this article is risk parity calculated for the groups of stocks. In the first part of the article, the definitions of measures used in the construction of parity risk for a single company are presented. Also, the method of construction of this type of portfolios is described. In the next section, the risk parity for the groups of stocks is defined and the optimization model to construct these portfolios is presented. This model is applied to a selected group of stocks listed on the Warsaw Stock Exchange. The results of these studies are presented in the last part of the article.

2. Risk parity for individual stocks

Risk parity portfolios, called also portfolios with equal risk contribution, can be treated as a compromise between well-diversified portfolios and equally weighted portfolios (naive portfolios). For some researchers, the risk parity portfolio is a special type of diversification strategy. In many cases, diversification is achieved on the level of allocation of capital and an example of such a method is an equally weighted portfolio or mean-variance portfolio, whereas the risk parity portfolio is an example of the diversification in the sense of risk contribution.

According to the first definition, the risk parity portfolios were the portfolios of shares proportional to the inverse of the standard deviation of a given stock. This type of portfolio is called naive risk parity. The naive risk parity can be calculated only in one case – when all pairs of rates of return have the same correlation coefficient. But, on the real investment markets, this situation never happens [Qian 2005, 2006].

Another example of parity portfolios are 60% of equity and 40% of bonds portfolios (60/40 portfolio). Usually application of this approach guarantees portfolio of high rate of return. However, volatility of this portfolio can be dominated by the risk of equity. For this reason, 60/40 portfolios are not portfolios with equal risk contribution [Qian 2005, 2006; Bai et al. 2016].

The simple definition of the risk parity portfolio says that this is a portfolio with the total risk equally divided on all components of this portfolio. Let's assume that: x_i is the share of the i-th stock in portfolio, r_i – the rate of return of the i-th stock in portfolio, N – the number of stocks in portfolio. Usually the risk parity is defined when the risk of portfolio is measured by standard deviation:

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}}, \tag{1}$$

where σ_{ij} – covariance between i-th and j-th stocks, $\sigma_{ii} = \sigma_i^2$ – variance of the i-th stock and σ_p – standard deviation of portfolio.

Formally the risk parity is defined by two following measures: Marginal Risk Contribution and the Total Risk Contribution. The first one – the Marginal Risk Contribution for the i-th stock (MRC_i) is described by the formula (Maillard et al. 2010; Chaves et al. 2011, 2012):

$$MRC_i = \frac{\partial \sigma_p}{\partial x_i} = \frac{\sum_{j=1}^{N} x_j \sigma_{ij}}{\sigma_p}. \tag{2}$$

The second measure – the Total Risk Contribution (TRC_i) is equal to the product of the share of i-th stock in portfolio and the Marginal Risk Contribution for this stock:

$$TRC_i = x_i \frac{\partial \sigma_p}{\partial x_i} = \frac{\sum_{j=1}^{N} x_i x_j \sigma_{ij}}{\sigma_p}. \tag{3}$$

These two measures can be used to construct optimal portfolios. If the Marginal Risk Contribution is the same for all N components of a portfolio:

$$\frac{\partial \sigma_p}{\partial x_i} = \frac{\partial \sigma_p}{\partial x_j} \quad \text{for } i, j = 1, 2, \ldots, N \tag{4}$$

then we receive the minimum variance portfolio. But in the situation when the Total Risk Contribution is the same for all N stocks in a portfolio:

$$x_i \frac{\partial \sigma_p}{\partial x_i} = x_j \frac{\partial \sigma_p}{\partial x_j} \quad \text{for } i, j = 1, 2, \ldots, N \tag{5}$$

we receive the risk parity portfolio.

The research conducted so far indicates that portfolios with equal risk contribution have higher Sharpe ratio than the portfolios of minimum variance [Chaves et al. 2011]. Furthermore, risk parity portfolios have a lower risk than the naive risk parity portfolios and the equally weighted portfolios [Braga 2015].

The risk parity portfolios can be constructed by using different methods. Usually the parity portfolios are selected according to the following optimization model [Maillard et al. 2010]:

$$\begin{gathered} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(x_i \frac{\partial \sigma_p}{\partial x_i} - x_j \frac{\partial \sigma_p}{\partial x_j} \right)^2 \rightarrow min \\ \sum_{i=1}^{N} x_i = 1 \\ 0 \leq x_i \leq 1 \ \text{ for } i = 1, 2, \ldots, N. \end{gathered} \tag{6}$$

The abovementioned problem can be solved by using the methods of sequential quadratic programming. To receive the solution of such problems we can use among others the Matlab software.

Some examples of the other methods of selection the risk parity portfolios include: Jacobi algorithm, Newton algorithm [Chaves et al. 2011, 2012; Lohre et al. 2012], cyclical coordinate descent algorithm, least square approach [Bai et al. 2016].

It should be noticed that when we construct the risk parity portfolios for N components, all N components have the non-zero shares. The contribution of risk for all components is approximately equal.

3. Grouped Risk Parity Portfolios

Usually risk parity is defined for individual stocks. But in some situation, it is better to analyse risk parity for groups of stocks. This approach is called the grouped risk parity. It is a good solution for example when we analyse a lot of stocks or when the investor establishes the upper limit of the number of stocks in the portfolio. The

grouped risk parity is also useful in situation when we are considering the transaction costs [Bai et al. 2016].

As a group, we can take the stocks that belong to one sector (for example banking sector, energy sector, IT sector, construction sector). The stocks with a specified market value can be also set as the group of stocks.

The necessary condition for the grouped risk parity is the following: the total risk of the portfolio should be equally divided into all groups of stocks in it. Formally, this condition can be put as:

$$\sum_{i \in G_s} x_i \frac{\partial \sigma_p}{\partial x_i} = \sum_{j \in G_t} x_j \frac{\partial \sigma_p}{\partial x_j} \quad \text{for all groups of stocks } G_s \text{ and } G_t . \tag{7}$$

Similarly, as for the risk parity for individual stocks, the grouped risk parity portfolio can be constructed by using the optimization model:

$$\begin{gathered} \sum_{s=1}^{k-1} \sum_{t=s+1}^{k} \left(\sum_{i \in G_s} x_i \frac{\partial \sigma_p}{\partial x_i} - \sum_{j \in G_t} x_j \frac{\partial \sigma_p}{\partial x_j} \right)^2 \rightarrow min \\ \sum_{i=1}^{N} x_i = 1 \\ 0 \le x_i \le 1 \text{ for } i = 1, 2, \ldots, N, \end{gathered} \tag{8}$$

where k is the number of groups of stocks, G_s – is the s-th group of stocks. This model is also an example of the sequential quadratic programming problem. The grouped risk parity portfolios are constructed under two assumptions:

1) the capital is invested in all the considered groups of stocks,
2) each stock can belong only to one group.

The similar problem of grouped risk parity portfolios can be also solved with the least-square approach proposed by Bai et. al. (2016).

4. Grouped risk parity – empirical research for the WSE

The model for the grouped risk parity portfolio has been applied to different groups of stocks from the Warsaw Stock Exchange. The grouped risk parity portfolios were compared with naive portfolios (equally weighted), minimum variance portfolios and with the risk parity portfolios for individual stocks. For all groups the conclusions were similar, so the results obtained for one group only are presented.

Below the results for portfolios constructed for the weekly rates of return for selected stocks from the 02nd of January 2014 till 30th of June 2016 are presented. Of all stocks listed on the Warsaw Stock Exchange 40 stocks which were listed without breaks throughout the analysed period were randomly selected. These stocks

represented different sectors: banks (11), energy (9), capital market (3), hotels and restaurants (4), media (8) and telecommunications (5).

For this data three cases were considered. In the first case, portfolios were constructed for 20 stocks with the highest average rate of return in 2013. The second case – portfolios were constructed for 20 stocks with the lowest risk (standard deviation) in 2013. The third group consisted of 17 indexes respectively representing big, medium and small companies.

Identification of big, medium and small companies was conducted on the base of the market value of each stock. The market value of stock is calculated as a product of the number of shares in trading and the closing price for the day. The market value of each analysed company was calculated five times: 30.06.2014, 30.12.2014, 30.06.2015, 30.12.2015, 30.06.2016. For each day stocks were sorted by the market value and divided into three groups: big company if the market value was at least equal to the 70^{th} percentile, medium company if the market value was between the 70^{th} and 30^{th} percentile, small company if the market value was equal at most the 30^{th} percentile. In the next step, we selected only those stocks which always have occurred in the same group. There was 17 such stocks – 7 large, 5 medium and 5 small companies.

For each of three groups of data, the following portfolios were constructed: the risk parity portfolio for groups of stocks (GRPP), the risk parity portfolio for individual stock (RPP), equal weighted portfolio (EWP), minimum variance portfolio (MVP). The risk parity for groups in the first two cases was constructed for the six groups (all sectors). In the third case portfolios were constructed for three groups (big, medium and small companies).

Portfolios were compared in terms of the level of risk (standard deviation), rate of return and Sharpe ratio. Received results for all portfolios were presented in Tables 1-3.

First of all, we received the confirmation of properties characteristic for the risk parity portfolios. These portfolios are usually less risky than the naive portfolios. What is more, the risk parity portfolios have a higher Sharpe ratio than the naive portfolios and the minimum variance portfolios. Both relationships are true for portfolios constructed for each of the three groups of data. Moreover, these dependencies are true for risk parity for individual stocks and also for the grouped risk parity.

Table 1. Characteristics of portfolios from the first group (stocks with the highest rate of return)

	Portfolio			
	GRPP	RPP	EWP	MVP
Risk	1.6873E-05	1.8443E-05	1.9996E-05	1.3948E-05
Rate of return	1.0001	1.0001	1.0001	1.0002
Sharpe ratio	51.45%	47.44%	42.99%	48.73%

Source: own study.

Table 2. Characteristics of portfolios from the second group (stocks with the lowest risk)

	Portfolio			
	GRPP	RPP	EWP	MVP
Risk	1.6787E-05	1.8786E-05	2.0117E-05	1.4347E-05
Rate of return	1.0002	1.0001	1.0001	1.00002
Sharpe ratio	59.06%	47.32%	42.74%	11.07%

Source: own study.

Table 3. Characteristics of portfolios from the third group (size of companies)

	Portfolio			
	GRPP	RPP	EWP	MVP
Risk	2.5145E-05	2.0842E-05	0.00011	1.5335E-05
Rate of return	0.9990	0.9991	0.9990	0.9993
Sharpe ratio	9.56%	6.81%	2.05%	9.49%

Source: own study.

The comparison of both types of risk parity portfolios indicates that the risk parity portfolios for groups are a little less risky and have a higher rate of return and Sharpe ratio than the risk parity portfolios for individual stocks. These results were received in the first and second group. In the third group, the risk parity portfolio was a little bit less risky than the grouped risk parity portfolio. This can be explained by the fact that in this group most rates of return were very low.

It should be noticed that in the risk parity portfolios for individual stocks, all components have non-zero shares, while in the risk parity portfolios for groups we can obtain the shares equal zero. This means that these portfolios have a lower level of diversification (in the sense of the number of components in the portfolio). The shares equal to zero were obtained for at least two stocks.

Additionally, for all portfolios the future profits were calculated. It was assumed that 100 000 PLN was invested in each selected portfolio. Future profits were calculated at the value of portfolio, if it would be sold in subsequent days of the third quarter of 2016. Tables 4-6 present the ratio of the value of portfolios from the day of sells compared to 100 000 PLN (value of the portfolio on the day of purchase). In the tables for every day the most profitable portfolio was indicated.

Similarly, as in the previous comparison, the risk parity portfolios for groups are better than the risk parity portfolios for individual stocks. Most often the risk parity portfolios for groups have a little higher value than the value of parity portfolios for individual stocks. Usually the grouped risk parity portfolios gave also higher profits than the naive portfolios. However, in the third group in most cases the best future profits we received for the minimum variance portfolio.

Table 4. The future profits of portfolios from the first group

Date	GRPP	RPP	EWP	MVP
04/07/16	**0.9897**	0.9895	0.9884	0.9895
11/07/16	1.0046	1.0023	**1.0049**	0.9992
18/07/16	1.0003	0.9992	**1.0011**	0.9984
25/07/16	0.9996	0.9969	**1.0002**	0.9949
01/08/16	**0.9889**	0.9875	0.9881	0.9885
08/08/16	**1.0043**	1.0024	1.0042	1.0027
16/08/16	**0.9941**	0.9926	0.9939	0.9922
22/08/16	0.9941	0.9921	**0.9947**	0.9911
29/08/16	**0.9941**	0.9928	0.9940	0.9938
05/09/16	**1.0051**	1.0020	1.0050	0.9991
12/09/16	0.9810	0.9815	0.9813	**0.9840**
19/09/16	**0.9962**	0.9952	0.9960	0.9934
26/09/16	**0.9918**	0.9897	0.9916	0.9916

Source: own study.

Table 5. The future profits of portfolios from the second group

Date	GRPP	RPP	EWP	MVP
04/07/16	**0.9891**	0.9886	0.9864	0.9884
11/07/16	1.0084	**1.0091**	1.0086	0.9983
18/07/16	1.0065	**1.0067**	1.0062	1.0004
25/07/16	1.0011	**1.0024**	1.0016	0.9946
01/08/16	**0.9934**	0.9927	0.9925	0.9853
08/08/16	**1.0117**	1.0097	1.0091	1.0014
16/08/16	0.9984	0.9982	**0.9974**	0.9910
22/08/16	0.9964	0.9966	**0.9965**	0.9898
29/08/16	**1.0006**	0.9992	0.9989	0.9941
05/09/16	1.0141	**1.0160**	1.0158	1.0006
12/09/16	**0.9859**	0.9810	0.9798	0.9820
19/09/16	**1.0029**	1.0016	1.0021	0.9932
26/09/16	**0.9960**	0.9952	0.9944	0.9894

Source: own study.

Table 6. The future profits of portfolios from the third group

Date	GRPP	RPP	EWP	MVP
04/07/16	**1.0069**	1.0028	0.9761	0.9994
11/07/16	1.0068	**1.0073**	1.0064	1.0058
18/07/16	1.0049	1.0054	1.0001	**1.0094**
25/07/16	**0.9981**	0.9955	0.9970	0.9943
01/08/16	**0.9881**	0.9863	0.9877	**0.9881**
08/08/16	1.0023	1.0009	1.0018	**1.0036**
16/08/16	**1.0001**	0.9965	0.9996	0.9948
22/08/16	0.9932	0.9963	0.9920	**0.9976**
29/08/16	**0.9918**	0.9905	0.9911	0.9901
05/09/16	1.0068	1.0076	1.0071	**1.0097**
12/09/16	0.9846	0.9838	0.9830	**0.9906**
19/09/16	0.9997	0.9985	0.9975	**1.0031**
26/09/16	0.9966	0.9960	0.9960	**0.9982**

Source: own study.

To generalize the received properties for the grouped risk parity portfolios, the simulation research was conducted. For all three groups, the rate of return of stocks were randomly generated, then four portfolios were constructed for these rates of return: GRPP, RPP, MVP and EWP. For these portfolios the value of risk (standard deviation), rate of return and Sharpe ratio were calculated. The experiment was repeated a hundred times (in every group one hundred portfolios were generated). The results were similar to the presented example. The values of risk, rates of return and Sharpe ratio for GRPP and RPP portfolios were presented on the Figures 1-3. This is the case for the first regarding group of stocks.

GRPP portfolios proved to be less risky than their corresponding RPP portfolios. Also, according the Sharpe ratio, the GRPP portfolios were better. In contrast, the values of rates of return for both type of portfolios were very similar. GRPP portfolios seem to be a little bit more profitable.

Statistical tests were performed to confirm the results. The tests showed significant differences for the risk and Sharpe ratio. For the rates of return there was no basis for rejecting the hypothesis about equal distribution. The tests were conducted for a confidence level of $\alpha = 0.05$.

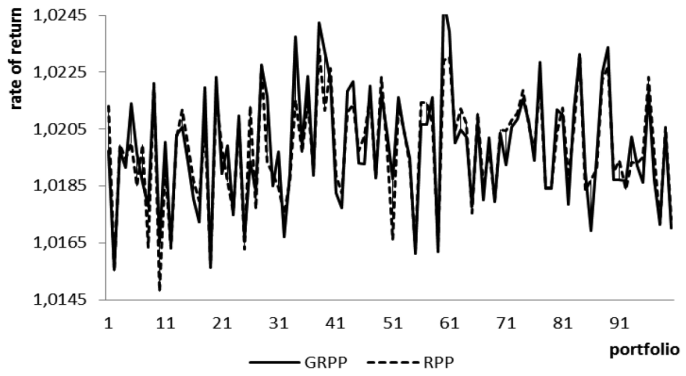

Fig. 1. Rates of return for GRPP and RPP portfolios

Source: own study.

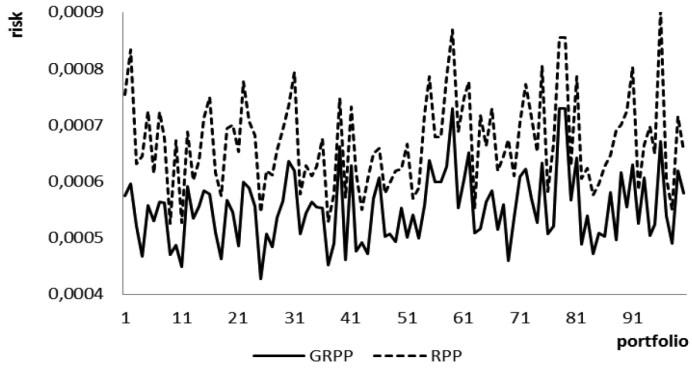

Fig. 2. Values of risk for GRPP and RPP portfolios

Source: own study.

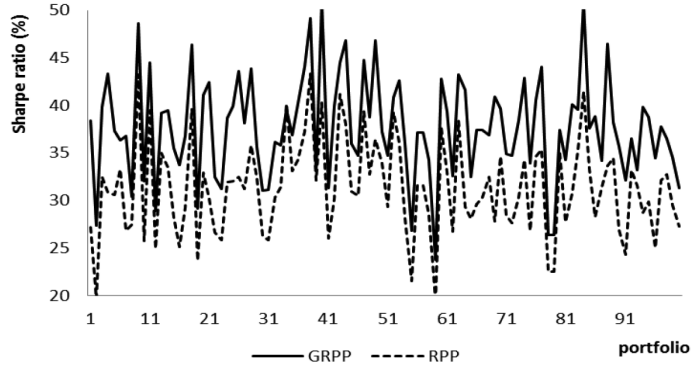

Fig. 3. Values of Sharpe ratio for GRPP and RPP portfolios

Source: own study.

5. Conclusion

In the article, the model for the grouped risk parity portfolios was presented. This model was applied to selected data from the Stock Exchange in Warsaw. This short empirical research indicated that parity portfolios for groups of stocks gave better results than the parity portfolios for the individual stocks. Portfolios with equal risk for groups were characterized by higher Sharpe ratio and the lower risk than the corresponding portfolios for individual risk. The GRPP portfolios have also better rates of return, however these differences are not significant. In most cases these portfolios are better even according to the future profits. The received results prompt further research concerning the risk parity for groups of stocks. This research will concern measures of risk other than the standard deviation, as well as the multi-period case.

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