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ON NEW IMMUNIZATION STRATEGIES UNDER RANDOM SHOCKS ON THE TERM STRUCTURE OF INTEREST RATES

We introduce new measures of immunization such as exponential duration referring, in particular, to Fong and Vasiček [7], Nawalkha and Chambers [14], Balbás and Ibáñez [2], and Balbás et al. [3], but under the assumption of multiple shocks in the term structure of interest rates. These shocks are given by a random field. The cases of a single and multiple liabilities are discussed separately.

Keywords: *portfolio, immunization, duration, term structure of interest rates, random field*

1. Introduction

Bondholders are subject to interest risk caused by changes in interest rates. Therefore, researchers have examined the immunization problem for a bond portfolio (see Nawalkha and Chambers [15]). They have proposed multiple-risk measure models (e.g. Fong and Vasiček, [7], Balbás and Ibáñez [2]) or single-risk measure models (e.g. Nawalkha and Chambers [14], Kaluszka and Kondratiuk-Janyska [9]). We propose new strategies for portfolio immunization under multiple shocks in the term structure of interest rates (TSIR for short), where a shock is given by the sum of a polynomial and a random field (see e.g. Kimmel [13]). Under the assumption about shocks in the TSIR, we consider the case of a single liability and develop the approach of portfolio immunization in the case of multiple liabilities introduced in Kon-

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dratiuk-Janyska and Kaluszka [11]. In addition to time-honored duration, we obtain new risk measures, e.g. exponential duration. The remainder of this paper is organized as follows. Section 2 gives the notation. Section 3 presents immunization strategies in the case of a single liability, whereas Section 4 provides results for the case of multiple liabilities.

2. Preliminary notation

Denote by $[0,T]$ the time interval with $t=0$ being the present moment, and let H be the investor's planning horizon, $(0<H<T)$, when the portfolio is rebalanced. The portfolio consists of random inflows $A_t \geq 0$ occurring at fixed times $t \leq T$ and generated by e.g. zero-coupon bonds, coupon bonds, indexed linked bonds, catastrophe bonds, stocks, options or other assets to cover random liabilities $L_t \geq 0$ due at dates $t \leq T$. This is a typical situation e.g. when an insurance company has to discharge its random liabilities and invests money in acquiring an immunized portfolio. Denote the set of available inflows by $\mathcal{A}$. Generally, this is an arbitrary set that might be nonconvex, since we do not assume that the market is complete or that assets are infinitely divisible. Consequently, $N_t = A_t - L_t$ is the net cash flow at time *t*. By $f(t,s)$ we mean an instantaneous forward rate over the time interval $[t,s]$ and therefore investing 1 at time *t* in a zero coupon-bond we get $\exp\left(\int_t^s f(t,u)du\right)$ at time *s*. The set of instantaneous forward rates $\{f(t,s): 0<t\leq s\}$ determines the random term structure of interest rates defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$. At time $t=0$, $s \to f(0,s)$ is deterministic. Hence,

- $a_t = A_t \exp\left(\int_t^H f(0,u)du\right)$ is the time-*H* value of A_t,
- $l_t = L_t \exp\left(\int_t^H f(0,u)du\right)$ is the time-*H* value of L_t,
- $n_t = a_t - l_t$ is the time-*H* value of net worth,
- $A(t) = \sum_{s\leq t} a_s$ is the accumulated value of assets,
- $L(t) = \sum_{s\leq t} l_s$ is the accumulated value of liabilities,

- $N(t) = A(t) - L(t)$,
- $V(0) = \mathrm{E}\sum_t n_t$ is the time-H average value of the portfolio of asset and liability

flows if the forward rates equal the future spot rates.

We assume that

$$f(t,s) = f(0,s) + \sum_{i=1}^{d} \lambda_i s^{i-1} + \varepsilon(t,s), \quad s \ge t > 0, \tag{1}$$

where λ_i are random variables and $\varepsilon(t,s)$ is a random field with mean 0. This means that the TSIR experiences shifts described by a polynomial $\sum_{i=1}^{d} \lambda_i s^{i-1}$ (see Chambers et al. [4], Prisman and Shores [16], Crack and Nawalkha [5]), where the λ_i coefficients describe the shape of a shock: the height is described by λ_1, slope by λ_2, curvature by λ_3 and there may be other higher order term structure shape parameters. Empirical tests show that the coefficients related to the level, slope and curvature of term structure shifts are necessary to guarantee a return close to the target (Soto [17]). Additionally, it has been shown that parallel movements have a significant role in the behaviour of shocks (see e.g. Ilmanen [8]). Therefore, we consider the cases $d = 1$ and $d \ge 1$ separately. Instead of the function s^i, one can use others (see Kaluszka and Kondratiuk-Janyska [10]). We also model the forward rate at time t by a random field $\varepsilon(t,s)$ to catch the noise affecting relevant factors in the sum $\sum_{i=1}^{d} \lambda_i s^{i-1}$. This element ensures that the model is arbitrage-free, in order to be consistent with modern finance theory.

The decision problem of the investor is to design a stream of assets to cover a stream of liabilities. An ideal situation is when $N_t = 0$ for all t. However, in reality, the market is incomplete, which excludes an ideal adjustment of assets to liabilities. Moreover, an investor constructing a portfolio meets two kinds of risks: *reinvestment* and *price*. The first one is connected with the way of reinvesting money paid before an investment horizon. The other appears in pricing assets before their expiry dates. Since the value of a portfolio at time H depends on the reinvestment strategy, we require the following open-loop strategy:

a) If $t < H$, then the value of N_t at time H is equal to $N_t \exp\left(\int_t^H f(t,s)ds\right)$, where $f(t,s)$ satisfies (1). This means that if $N_t = A_t - L_t > 0$ for $0 < t < H$, an investor purchases $(H-t)$-year strip bonds. Otherwise, he sells short $(H-t)$-year strip bonds.

b) If $t > H$, then the value of N_t at H equals

$N_t \exp\left(-\int_H^t f(H,s)ds\right) = N_t \exp\left(\int_t^H f(H,s)ds\right)$, and (1) holds, which means that at time H the portfolio priced according to the TSIR is sold by the investor.

As a consequence, the value of the portfolio at H equals

$$\sum_t N_t \exp\left(\int_t^H f(t \wedge H, s)ds\right) = \sum_t n_t \exp(k(t)),$$

with

$$k(t) = \int_t^H (f(t \wedge H, s) - f(0,s))ds, \tag{2}$$

where $f(t,s)$ is a shock in the instantaneous forward rate satisfying (1) and $a \wedge b = \min(a,b)$. Denote the set of admissible shocks, including arbitrary functions k given in (2), by $\mathcal{K}^d$. From the investor's standpoint, the average time-H value of his portfolio under the above open-loop strategy is given by

$$V(k) = \mathrm{E}\sum_t n_t \exp(k(t)).$$

The classical immunization problem consists in finding a portfolio such that $V(k) \geq V(0)$ for all $k \in \mathcal{K}$, where $\mathcal{K}$ stands for a class of shocks. Our aim is to find a lower bound on $\inf_{k \in \mathcal{K}^d} V(k)$, which is dependent only on the proportions of the portfolio. Next, at $t = 0$ we select a portfolio among the bonds available on the market such that this lower bound is maximal.

3. Single liability

In this section we present the problem of portfolio immunization in the case of a single liability at time H i.e. $l_t = 0$ for $t \neq H$. Assume that $k(\cdot)$ belongs to the following class of shocks

$$\mathcal{K}_1 = \{k(\cdot) \in \mathcal{K}^1 : \lambda_1 \ \textit{is unknown, arbitrary number}\}.$$

Recall that $A(T) = \sum_t a_t$ and denote the duration of an asset flow by

$$D = \frac{\mathbf{E}\sum_t ta_t}{\mathbf{E}\sum_t a_t}.$$

Additionally, throughout the paper

$$R(t) := \ln \mathbf{E} \exp\left(\int_t^H \varepsilon(t \wedge H, s) ds \right).$$

Proposition 3.1. If the sequence (a_t) is independent of the random field $(\varepsilon(t,s))$ and $D = H$, then

$$\inf_{k \in \mathcal{K}_1} V(k) \geq \mathbf{E}A(T) \exp\left(\sum_t R(t) \frac{\mathbf{E}a_t}{\mathbf{E}A(T)} \right) - \mathrm{E}l_H . \tag{3}$$

Proof: Applying Jensen's inequality, we obtain for all $k \in \mathcal{K}_1$

$$V(k) = \sum_t \mathbf{E}a_t \mathbf{E} \exp\left(\lambda_1 (H - t) + \int_t^H \varepsilon(t \wedge H, s) ds \right) - \mathbf{E}l_H$$

$$= \mathbf{E}A(T) \sum_t \frac{\mathbf{E}a_t}{\mathbf{E}A(T)} \exp(\lambda_1 (H - t) + R(t)) - \mathbf{E}l_H$$

$$\geq \mathbf{E}A(T) \exp\left(\lambda_1 (H - D) + \sum_t R(t) \frac{\mathbf{E}a_t}{\mathbf{E}A(T)} \right) - \mathbf{E}l_H ,$$

which completes the proof. □

As a corollary of Proposition 3.1, we obtain the following immunization strategy:

$$\max_{(a_t)} \sum_t R(t) \mathbf{E}a_t \text{ subject to } D = H.$$

In the literature one can find different measures in place of $\sum_t R(t) \mathbf{E}a_t$, in particular, the M-Squared measure of Fong and Vasiček [7]

$$\sum_t (t - H)^2 \mathbf{E}a_t$$

or the M-Absolute measure of Nawalkha and Chambers [14]

$$\sum_t |t-H| \mathbf{E}a_t \, .$$

What differentiates our approach from others is that our risk measure is derived using a random field ε, not under the assumption of shock movement within a given band width. The example given below presents an explicit formula for $R(\cdot)$.

Example. Assume that ε is a Brownian sheet i.e. is a Gaussian random field with expected value (0) and covariance function

$$\mathbf{Cov}(\varepsilon(t,x),\varepsilon(s,y)) = (t \wedge s)(x \wedge y) \, .$$

Then $\int_t^H \varepsilon(t \wedge H, s) ds$ has a Gaussian distribution, which implies after simple algebra that

$$R(t) = \frac{1}{2}\mathbf{Var}\left(\int_t^H \varepsilon(t \wedge H, s) ds\right) = \frac{t \wedge H}{6}(H-t)^2(\max(t,H) + 2(t \wedge H)).$$

In particular, when there is no risk in reinvestment, i.e. $t \geq H$ for all t, the immunization strategy relies on solving the following problem:

$$\max_{(a_t)}\left(\sum_t (t-H)^2 \, \mathbf{E}a_t + \frac{1}{3H}\sum_t (t-H)^3 \, \mathbf{E}a_t\right) \text{ subject to } D = H \, .$$

For more examples of random fields we refer the reader to Adler and Taylor [1], Goldstein [6], Kennedy [12], VanMarcke [18] and the papers referred there.

Remark 1. From the proof of Proposition 3.1, we obtain that for all $k \in \mathcal{K}_1$

$$V(k) = \sum_t \mathbf{E}a_t \mathbf{E}\exp(\lambda_1(H-t) + r(t)) - \mathbf{E}l_H \, ,$$

where $r(t) = \int_t^H \varepsilon(t \wedge H, s) ds$. Define a new random variable τ given by the distribution $\mathbf{P}(\tau = t) = \frac{\mathbf{E}a_t}{\mathbf{E}A(T)}$. Hence,

$$V(k) = \mathbf{E}A(T)\mathbf{E}\exp(\lambda_1(H-\tau) + r(\tau)) - \mathbf{E}l_H \, .$$

From Jensen's inequality we obtain

$$V(k) \geq \mathbf{E}A(T) - \mathbf{E}l_H \, ,$$

since $\mathbf{E}\tau = D$ and

$$\mathbf{E}r(\tau) = \sum_t \frac{\mathbf{E}a_t}{\mathbf{E}A(T)} \mathbf{E}r(t) = \sum_t \frac{\mathbf{E}a_t}{\mathbf{E}A(T)} \mathbf{E}\left(\int_t^H \varepsilon(t \wedge H, s) ds \right)$$
$$= \sum_t \frac{\mathbf{E}a_t}{\mathbf{E}A(T)} \int_t^H \mathbf{E}\varepsilon(t \wedge H, s) ds = 0.$$

Under the assumption $\mathbf{E}(A(T)) = \mathbf{E}l_H$, it follows that each portfolio of duration equal to the investment horizon is immunized in the sense of expected value. Usually, there exists an infinite number of portfolios such that $D = H$ and Proposition 3.1 allows us to choose the best one among them.

In order to get a better estimate of the changes in a portfolio's value, we introduce a new risk measure, which we will call the exponential duration

$$D_{\exp} = \frac{\sum_t t e^{R(t)} \mathbf{E}a_t}{\sum_t e^{R(t)} \mathbf{E}a_t}.$$

When $\varepsilon \equiv 0$ then $D_{\exp} = D$, which means that if the variance of ε is small, then the exponential duration is close to the classical one. However, a better approximation is given by

$$D_{\exp} \approx \frac{D + \frac{1}{\mathbf{E}A(T)} \sum_t tR(t) \mathbf{E}a_t}{1 + \frac{1}{\mathbf{E}A(T)} \sum_t R(t) \mathbf{E}a_t}.$$

Proposition 3.2. If the sequence (a_t) is independent of the random field $(\varepsilon(t,s))$ and $D_{\exp} = H$, then

$$\inf_{k \in \mathcal{K}_1} V(k) \geq \sum_t e^{R(t)} \mathbf{E}a_t - \mathbf{E}l_H . \tag{4}$$

Proof: As in the proof of Proposition 3.1, we obtain

$$V(k) = \sum_t \mathbf{E}a_t \exp(R(t)) \exp(\lambda_1 (H - t)) - \mathbf{E}l_H$$
$$\geq A^*(T) \exp\left(\sum_t \frac{\mathbf{E}a_t e^{R(t)}}{A^*(T)} \lambda_1 (H - t) \right) - \mathbf{E}l_H$$
$$= A^*(T) \exp(\lambda_1 (H - D_{\exp})) - \mathbf{E}l_H ,$$

where $A^*(T) = \sum_t \mathbf{E}a_t e^{R(t)}$, which is our claim. □

It is easy to see that the right hand side of inequality (4) is greater than or equal to the right hand side of inequality (3) using Jensen's inequality. This means that in the class of portfolios satisfying the condition $D_{\exp} = H$ we get a better estimate of the changes in a portfolio's value.

We now derive the lower bound on the portfolio's expected value at time-H, which takes into account a shock in the slope, convexity and other shape parameters of the TSIR. Consider the case where $d \geq 1$ is a fixed number and introduce the class

$$\mathcal{K}_2 = \{k(\cdot) \in \mathcal{K}^d : |\lambda_i| \leq ic_i \ \text{ for } \ i = 1,...,d\},$$

where the c_i are given.

Proposition 3.3. If the sequence (a_t) is independent of the random field $(\varepsilon(t,s))$, then

$$\inf_{k \in \mathcal{K}_2} V(k) \geq \mathbf{E}A(T) \exp\left(-\frac{1}{\mathbf{E}A(T)} \sum_{i=1}^{d} c_i \sum_t \mathbf{E}a_t \left| H^i - t^i \right| + \frac{1}{\mathbf{E}A(T)} \sum_t R(t) \mathbf{E}a_t \right) - \mathbf{E}l_H . \quad (5)$$

Proof: Since the proof is easy, we omit it. □

Inequality (5) suggests the following immunization problem:

$$\textit{find a portfolio which maximizes} \quad \sum_t R(t)\mathbf{E}a_t - \sum_{i=1}^{d} c_i \sum_t \mathbf{E}a_t \left| H^i - t^i \right|. \quad (6)$$

4. Multiple liabilities

Assume that multiple shocks in the term structure of interest rates are contained in $\mathcal{K}^d$ and $(\lambda_1,...,\lambda_d)$ is a random vector with a multidimensional Gaussian distribution with expected value 0 and covariance matrix given by $\sigma_{ij} = \text{cov}(\lambda_i, \lambda_j)$. Then the average time-$H$ value of a portfolio under the assumption of independence between n_t, λ_i and ε is given by

$$V(k) = \sum_t \mathbf{E}n_t \mathbf{E} \exp\left(\sum_{i=1}^{d} \lambda_i (H^i - t^i) + R(t) \right)$$

$$= \sum_t \mathbf{E}n_t \exp\left(\frac{1}{2} \sum_{i,j=1}^{d} (H^i - t^i)(H^j - t^j)\sigma_{ij} + R(t) \right).$$

Now the problem of immunization consists in finding such a feasible sequence (n_t) to obtain the maximum of

$$\sum_t \mathbf{E} n_t \exp\left(\frac{1}{2} \sum_{i,j=1}^{d} (H^i - t^i)(H^j - t^j)\sigma_{ij} + R(t) \right),$$

which is analogous to the risk measure given in (6).

If $\lambda_1,...,\lambda_d$ are independent random variables and λ_i has a uniform distribution on the interval $(-c_i, c_i)$ for $i = 1,...,d$, then an investor should choose a portfolio (n_t), which maximizes

$$\sum_t \mathbf{E} n_t \prod_{i=1}^{d} \frac{\sin h((H^i - t^i)c_i)}{(H^i - t^i)c_i} \exp(R(t)),$$

where $\sin h(x) = \dfrac{e^x - e^{-x}}{2}$.

If $\lambda_1,...,\lambda_d$ are independent random variables and $e^{\lambda_i} \sim \Gamma(\alpha_i, \beta_i)$, where $\alpha_i > \max(0, t^i - H^i)$ for $i = 1,...,d$, then the portfolio should be constructed in such a way that

$$\sum_t \mathbf{E} n_t \prod_{i=1}^{d} \Gamma(\alpha_i + H^i - t^i) \exp\left(\sum_{i=1}^{d} (H^i - t^i) \ln \beta_i + R(t) \right) \to \max_{(n_t)},$$

where $\Gamma(x)$ is the Euler gamma function.

Note that the above cases are easy to solve if the set of admissible sequences (n_t) is finite or convex. Obviously, the results presented are only a few examples of the possible distributions of the vector $(\lambda_1,...,\lambda_d)$ for which we get explicit formulas for the optimization problem (see e.g. the case when $(\lambda_1,...,\lambda_d)$ has a multidimensional *t*-Student distribution).

Different results are obtained when we do not possess any information about the distribution of $\lambda_1,...,\lambda_d$.

Proposition 4.1. Assume that $\mathbf{E}\sum_t n_t = 0$ and the sequence (n_t) is independent of the random field $(\varepsilon(t,s))$, then a lower bound on the portfolio's expected value at time-*H* is given by

$$\inf_{k \in \mathcal{K}_3} V(k) \geq -\int_0^T \left| \sum_{s \leq t} \mathbf{E} n_s \right| c(t) dt, \tag{7}$$

where:

$c(t) \geq 0$ is a known function,

$\mathcal{K}_3 = \left\{ k(\cdot) \in \mathcal{K}^d : \left| \frac{\partial}{\partial t} \exp\left(R(t) + \sum_{i=1}^{d} \lambda_i (H^i - t^i) \right) \right| \leq c(t) \text{ for all } t \right\}$ and d is a fixed number.

Proof: Recall that $N(t) = \sum_{s \leq t} \mathbf{E} n_s$. Then

$$V(k) = \mathbf{E} \sum_t n_t e^{R(t)} \exp\left(\sum_{i=1}^{d} \lambda_i (H^i - t^i) \right) = \int_0^T \exp\left(R(t) \sum_{i=1}^{d} \lambda_i (H^i - t^i) \right) dN(t)$$

$$= -\int_0^T N(t) \frac{\partial}{\partial t} \exp\left(R(t) + \sum_{i=1}^{d} \lambda_i (H^i - t^i) \right) dt \geq -\int_0^T |N(t)| c(t) dt \; .$$

as desired. □

As a corollary of Proposition 4.1 we get the following immunization strategy:

$$\min_{(n_t)} \int_0^T \left| \sum_{s \leq t} \mathbf{E} n_s \right| c(t) dt \, , \tag{8}$$

which is a generalization of the Nawalkha and Chambers problem [14] for a sequence of liabilities.

Proposition 4.1 is analogous to Proposition 3.1 in Kondratiuk-Janyska and Kaluszka [11]. Analogous theorems to the remaining results in Kondratiuk-Janyska and Kaluszka [11] may also be easily formulated.

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Nowe strategie uodpornienia przy losowych zaburzeniach struktury terminowej stóp procentowych

Przedstawiamy nowe strategie immunizacji portfela przy założeniu wielokrotnych zaburzeń struktury terminowej stóp procentowych, gdzie zaburzenie jest opisane za pomocą sumy pewnego wielomianu i pola losowego [13]. Sformułowano twierdzenia dla ogólnej postaci pola losowego, a w przykładzie analizuje się przypadek płachty Browna. Przy kilku rodzajach zaburzeń struktury terminowej stóp procentowych rozważono zarówno przypadek jednego, jak i wielu zobowiązań [11]. Ponieważ rozważano różne postaci zaburzeń, otrzymano różne dolne oszacowania wartości strumienia pieniężnego (jako różnica aktywów i pasywów) w chwili H (horyzont inwestycyjny), gdy pojawią się zaburzenia. W konsekwencji strategie uodpornienia zawierają nowe miary ryzyka jak np. wykładniczy czas trwania.

Słowa kluczowe: *portfel, uodpornienie, czas trwania, struktura terminowa stóp procentowych, pole losowe*