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COST ALLOCATION IN PARTITION FUNCTION FORM GAMES

A cooperative game in partition function form is proposed for a cost allocation problem. The game describes a real situation in which a payoff of any coalition does not only depend on the players in the coalition but also on the coalition structure of the other players. Solution concepts like the stable set and the core are analyzed. Relations of the concepts in the case of the game in partition function form and of an appropriately formulated game in characteristic function form are shown.

Keywords: *partition function form game, core, stable set, cost allocation*

1. Introduction

The paper deals with a problem of cost allocation where groups of actors can collectively obtain a bundle of goods and share the costs. Actors interested in obtaining the goods can create coalitions to implement the project jointly. The cost allocation problem deals with allocation of the cost of the project among the actors as well as among the goods. A family of cooperative games in characteristic function form has been proposed by Kruś, Bronisz [11] to describe the problem. A pricing mechanism has been introduced in formulation of the games. Different solution concepts have been presented together with algorithms which allow to derive the solutions. In the paper, similarly as in other papers dealing with the cost allocation problems by: Littlechild, Thompson [15], Young, Okada, Hashimoto [25], Seo, Sakawa [22], Legros [14], utilizing cooperative games in characteristic function form, it was assumed that the payoff of each coalition depends on the players who create it. The cost allocation problems are attacked using different

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techniques in many papers, among others by Fernández, Hinojosa, Puerto [5], Matsubayashi, Umezawa, Masuda, Nishino [19], Krajewska, Kopfer [9], Cruijssen, Cools, Dullaert [4].

There are also practical situations where the payoff of any coalition depends not only on the actors/players creating it but also on the coalition structure of the other players (more generally on partition of the players). It is typical in the case of firms sharing a given market of goods or services. If several firms decide to create a coalition, its gain depends also on other firms, whether the other firms will act independently or create other coalition.

A research study dealing with possible decision support in such situations is currently carried on. It is assumed that actors/players try to obtain some goods and are ready to cover required costs. To reach the goods they can act independently or create coalitions. The costs which have to be covered depend not only on the coalition itself but also on the coalition structure of the other players. The problem deals with cost allocation among the players, but also among the goods. The research includes: – formulation of a cooperative game in partition function form (PFF game) motivated by the cost allocation problem, – formulation and analysis of solution concepts, – looking for a cooperative game in characteristic function form (CFF game) related to the PFF game, such that solutions like stable sets and cores of the games coincide, – utilization of solution concepts like cores and nucleoli derived for the CFF game in decision support analogously as it has been done by Kruś, Bronisz [11].

This paper presents preliminary results of the research study. A cooperative game in partition function form is proposed to model the above decision situation. Solution concepts to the game are proposed, based on the introduced domination relation. Properties of the solution concepts have been shown in five theorems, presenting also relations to a specially formulated CFF game. The paper refers to the original paper by Thrall and Lucas [23] and the similar notation is used. The concepts proposed are also similar to those presented by Thrall and Lucas [23] but they are formulated for a weaker domination relation which seems to be more relevant. Since the time several concepts of cores have been proposed including γ-core by Chander, Tulkens [3], r-core (theory) by Huang, Sjöström [6], recursive cores by Kóczy [7]. A survey of the concepts, referring also to α-core by Auman, Peleg [1], has been given by Kóczy [7].

2. Cooperative game in partition function form

Let $N = \{1, ..., n\}$ be a finite set of players; its subsets are coalitions.

For any $S \subseteq N$ let $P_S = \{P_1, ..., P_n\}$ be a partition of S, i.e.

$$\bigcup_{i=1}^{r} P_i = S, \forall j\, P_j \neq 0, \forall k\, P_j \cap P_k = \varnothing \ \text{ if } \ k \neq j, \tag{1}$$

and let Π_S denote the set of all partitions of S. For simplicity we will denote by P a partition of N, and by Π the set Π_N. Let P_I be the partition consisting of individual players, i.e. $P_I = \{\{1\}, \{2\}, ..., \{n\}\}$.

For each coalition $P_k \subseteq N$ and partition $P \in N$, such that $P_k \in P$ there are given functions describing the cost required to obtain the goods desired by the coalition. We assume, that the functions depend on the total amount of goods and do not depend on a division of them among players in P_k, but may depend on the partition of the other players. Using the functions the benefit of the players acting together in coalition P_k can be derived in comparison to their individual actions.

Definition 2.1. A cooperative game in partition function form (PFF game) is defined by a pair (N, F), where N is the set of players and F is a function which assigns r-dimensional real vector $F_P = (F_P(P_1), ..., F_P(P_r))$ to each partition $P \in \Pi$, $P = \{P_1, P_2, ..., P_r\}$.

$F_P(P_k)$ for $k = 1, 2, ..., r$ describes the benefit the players can obtain acting together in coalition P_k in comparison to their individual actions. Let us remark that $F_P(P_k)$ depends on the partition P, that means it depends on possible coalition structure of other players.

For each $S \subseteq N$ and each partition $P_S \in \Pi_S$ we define the following functions:

$$v(S) = \min_{\{P \in \Pi : S \in P\}} F_P(S), \ v(\varnothing) = 0, \tag{2}$$

$$u(P_S) = \min_{\{P \in \Pi : P_S \subset P\}} \sum_{T \in P_S} F_P(T), \tag{3}$$

$$\overline{v}(S) = \max_{\{P_S \in \Pi_S\}} u(P_S), \ v(\varnothing) = 0. \tag{4}$$

Intuitively, $v(S)$ denotes the guaranteed worth of a coalition S independent on the behavior of the players, $u(P_S)$ denotes the amount which is guaranteed for the players arranged in P_S independent on the behavior of the others players, $\overline{v}(S)$ denotes maximal amount which is guaranteed for the players in S independent on the behavior of the other players.

Example. To illustrate the functions introduced above let us consider a game with four players. Let the functions $F_P(P_k)$ be as follows: for each different $i, j, k, m \in N$,

$F_P(\{i\}) = a$ for any partition P such that $\{i\} \in P$; $F_P(\{i, j\}) = b$ for $P = \{\{i, j\}, \{k\}, \{m\}\}$;

$F_P(\{i, j\}) = c$ for $P = \{\{i, j\}, \{k, m\}\}$; $F_P(\{i, j, k\}) = d$ for $P = \{\{i, j, k\}, \{m\}\}$,
$F_P(N) = e$ for $P = \{N\}$.

In such a case the function $v(S)$ for any nonempty coalition $S \subseteq N$ takes the values:

$$v(\{i\}) = a,\ v(\{i, j\}) = \min (b, c),\ v(\{i, j, k\}) = d,\ v(N) = e$$

and the values of the function $\overline{v}(S)$ are as follows:

$\overline{v}(\{i\}) = a$, $\overline{v}(\{i, j\}) = \max(\min(b,c),2a)$, $\overline{v}(\{i, j,k) = \max(d, b + a, 3a)$,
$\overline{v}(N) = \max(e, d + a, 2c, b + 2a, 4a)$.

It is easy to verify that for any PFF game (N, F) the following inequalities hold:

$$\overline{v}(\{i\}) = v(\{i\}) \text{ for each } i \in N, \tag{5}$$

$$v(S) \leq \overline{v}(S) \text{ for each } S \subset N, \tag{6}$$

$$u(P_S) + u(P_T) \leq u(P_S \cup P_T) \text{ for each } P_S \in \Pi_S,\ P_T \in \Pi_T, \tag{7}$$

where $S, T \subset N$ such that $S \cap T = \varnothing$,

$$\overline{v}(S) + \overline{v}(T) \leq \overline{v}(S \cup T) \text{ for each } S, T \subset N \text{ such that } S \cap T = \varnothing. \tag{8}$$

3. Solution concepts

Definition 3.1. A vector $x = (x_1, ..., x_n)$ is called an **imputation** if

$$x_i \geq \overline{v}(\{i\}) \text{ for each } i \in N, \tag{9}$$

$$\sum_{i \in N} x_i = \sum_{S \in P} F_P(S) \text{ for some } P \in \Pi. \tag{10}$$

Conditions (9) and (10) are called individual rationality and realizability, respectively. The individual rationality means that nobody will agree to obtain payoff lower than his payoff when he acts independently. The realizability means the there exists a partition that can realize the payoffs. Let R denote the set of all imputations, and let R^P denote the set of all imputations realized by partition $P \in \Pi$.

Definition 3.2. Let S be a nonempty subset of N and let $x, y \in R$. Then x **dominates** y **via** S (denoted:
$x\ Dom_S\ y$) if

$$x_i > y_i \text{ for each } i \in S, \tag{11}$$

and there exists $P_S \in \Pi_S$ such that

$$\sum_{i \in N} x_i \le u(P_S)\,, \tag{12}$$

$$\sum_{i \in N} x_i = \sum_{T \in P} F_P(T) \text{ for some } P \in \Pi \text{ such that } P_S \subset P. \tag{13}$$

Condition (11) says that each player in S prefers his payoff in x to that in y. Condition (12) states that the players in S can form such partition $P_S \in \Pi_S$ that they can assure realization of payoffs x_i, $i \in S$. Condition (13) states that the payoff x is realizable by some partition P.

We say that x **dominates** y (denoted by x *Dom* y) if $x\ Dom_S\ y$ for some $S \subset N$. It is easy to show that relation *Dom* is neither transitive nor antisymmetric.

Let X be subset of R. Then $Dom_S X = \{y \in R: x\ Dom_S y \text{ for some } x \in X\}$,
$Dom\, X = \{y \in R: x\ Dom\, y \text{ for some } x \in X\}$.

Definition 3.3. A set of imputations K is a **stable set** if

$$K \cap Dom\ K = \varnothing, \tag{14}$$

$$K \cup Dom\ K = R. \tag{15}$$

Definition 3.4. A set of imputations C is a **core** if

$$C = R \setminus Dom\ R. \tag{16}$$

Condition (14) says that if x and y are in K then neither dominates the other, condition (15) states that if z is not in K then there exists x in K which dominates z**.** The above formal definition is based on the idea that instead of one imputation which every coalition is satisfied with, there is a set of imputations, so that if we take any imputation outside the set, there is an imputation inside the set, which is more beneficial for some coalition and the coalition has an incentive to obtain it. Not everyone might be satisfied with this new imputation, and some subset of players might force a change to another imputation outside the set. But the new imputation is again dominated by an imputation inside the set. Thus the bargaining process resolves around the set. Therefore, the whole set can be considered as a possible solution. All the imputations in the set are as important as one another. So there is no domination among the imputations in the set. The relation (14) is called as the internal stability condition, and the relation (15) as the external stability condition.

The core is the set of nondominated imputations in R, i.e. for any partition P there is no coalition $S \in P$ that gives its members payoffs better than payoffs in the core. Clearly, the core is contained in every stable set.

The definitions of the stable set and the core are described in a similar way to those proposed by Thrall and Lucas [23] and by Lucas [16] but they are based on a weaker domination relation. Thrall and Lucas assumed that a given coalition $S \in P$

can not be subdivided. In our approach we assume that if subdividing coalition S gives better result for the coalition then it is possible to realize it.

For each partition $P \in \Pi$ in a game (N, F) let

$$\| P \| = \sum_{S \in P} F_P(S). \tag{17}$$

Definition 3.5. Any imputation x has the property of **group rationality** if

$$\sum_{i \in N} x_i = \max_{\{P \in \Pi\}} \| P \|. \tag{18}$$

Let $R^{\max}$ denote the set of all imputations satisfying the property of group rationality in the game (N, F).

If the players choose an imputation $x \in R^{\max}$ as the payoff at the end of the game it means that they divide maximal possible gain in the game. It will be shown that imputations belonging to the concepts presented above fulfill this property.

Theorem 3.1. The core C of a game (N, F) is a subset of the core proposed by Thrall and Lucas [23]. Moreover, each imputation $x \in C$ has the property of group rationality.

Proof: Let $x \in C$ and $x \notin R^{\max}$. Then $\sum_{i \in N} x_i = \max_{\{P \in \Pi\}} \| P \| - M$, where $M > 0$. If $y \in R^{\max}$ is defined by $y_i = x_i + M/n$ for each $i \in N$ then $y \, Dom_N \, x$. Contradiction.

If $x \, dom_S \, y$ for some $S \subset N$ in the sense proposed by Thrall and Lucas (1963) then $x \, Dom_S \, y$. Therefore $C = R \setminus Dom\, R \subset \mathrm{R} \setminus dom\, R$.

It can happen that the core C is empty though the core proposed by Thrall and Lucas [23] is nonempty.

On the basis of the definition of function $\overline{v}$ (see equation 4) i.e. by the condition $\overline{v}(\varnothing) = 0$ and superadditivity condition (8) we have that the pair $(N, \overline{v})$ is a well defined cooperative game in characteristic function form with side payments. The following theorem shows relation between cores defined for games in partition function form and games in characteristic function form.

Theorem 3.2. The core of the PFF game (N, F) is equal to the core of the cooperative game in characteristic function form $(N, \overline{v})$, i.e. it satisfies the following conditions:

$$\sum_{i \in S} x_i \geq \overline{v}(S) \text{ for each } S \subset N, \tag{19}$$

$$\sum_{i \in N} x_i = \overline{v}(N). \tag{20}$$

Proof: Let CR denote the core of the game $(N, \overline{v})$. Let $x \in C$ and $x \notin CR$. From (5) it follows that $x_i = \overline{v}(\{i\})$, theorem 3.1 states that $\sum_{i \in N} x_i = \overline{v}(N)$, so x is an imputation in the game $(N,\bar{v})$. Because $x \notin CR$ then there exists an imputation y and a coalition S in the game $(N,\bar{v})$, such that $y_i > x_i$ for each $i \in S$ and $\sum_{i\in S} y_i \le \bar{v}(S)$. Let P_S be a partition of S such that $\bar{v}(S) = u(P_S)$. Consider a partition of N such that $P = P_S \cup \{i_1\}, \{i_2\}, ..., \{i_{n-s}\}\}$ where s denotes the number of players in S, $i_j \in N \setminus S$, $j = 1, 2, ..., n - s$ and an imputation z of the game (N, F) defined by $z_i = y_i$ for each $i \in S$ and $z_i = F_P(\{i\}) + M/(n - s)$ for each $i \in N \setminus S$ where $M = \sum_{T\in P_S} F_P(T) - \sum_{i\in S} y_i \ge 0$. Because $z\ Dom_S\ x$ then $x \notin C$. Contradiction.

Let $x \in CR$ and $x \notin C$. x is an imputation in the game (N, F). Because $x \notin C$ then there exists a coalition $S \subset N$, a partition $P_S \in \prod_S$ and an imputation y in the game (N, F) such $y_i > x_i$ for each $i \in S$, $\sum_{i\in S} y_i \le u(P_S)$ and $\sum_{i\in N} y_i = \sum_{T\in P} F_P(T)$ for some $P \in \prod$, $P_S \subset P$.

Consider an imputation z in the game $(N,\bar{v})$ defined by $z_i = y_i$ for each $i \in S$ and $z_i = y_i + M/(n - s)$ for each $i \in N \setminus S$ where $M = \bar{v}(N) - \sum_{T\in P_S} F_P(T) \ge 0$.

It follows that z dominates x via S in the game $(N,\bar{v})$, so $x \notin CR$. Contradiction.

This proves the theorem.

Theorem 3.3. If K is any stable set of the game (N, F) then each imputation $x \in K$ has the property of group rationality.

Proof: Let $x \in R \setminus R^{\max}$ and y be an imputation defined as in the proof of theorem 3.1. We have that $y\ Dom_N\ x$. If $y \in K$ then $x \in Dom\ K$. If $y \notin K$ then $y \in Dom_S\ K$ for some $S \subset N$, so there exists $z \in K$ such that $z \in Dom_S\ y$. But it is easy to verify that if $z\ Dom_S\ y$ and $y\ Dom_S\ y$ and $y \in Dom_N\ x$ then $z\ Dom_S\ x$. Therefore $x \in Dom\ K$. This proves the theorem.

From theorem 3.1 and 3.3 it follows that when discussing the core and the stable sets, without loss of generality we can restrict our considerations to the imputations satisfying the property of group rationality. It can happen that for a game (N, F) there is no stable set, there is one stable set or there are many stable sets. We can prove the following result:

Theorem 3.4. For an n-person game with the partition $\{N\}$ such that $\|\{N\}\| > \|P\|$ for each $P \in \prod$, $P \ne \{N\}$, there exists a unique stable set $K = R^{\{N\}} = R^{\max}$.

Proof: Let $x, y \in R^{\max}$ and let $x\ Dom\ y$. Domination may be realized only by the partition $\{N\}$, so we have $x_i > y_i$ for each $i \in N$. It follows that $\sum_{i\in N} x_i > \sum_{i\in N} y_i$. Contradiction.

Let $x \in R \setminus R^{\max}$ and let y be an imputation defined as in the proof of theorem 3.1. We have that $y\ Dom_N\ x$, so $x \in Dom\ R^{\max}$. This proves that $R^{\max}$ is a solution. It is unique solution by theorem 3.3.

For n-person games in which the outcome to the partition $\{N\}$ is greater than the sum of the outcomes for any other partition, we have no trouble in finding solution. Moreover, the unique solution is the same as that by Thrall and Lucas [23].

The following theorem shows relation between stable sets defined for games in partition function form and games in characteristic function form.

Theorem 3.5. If $\hat{P}=\{\hat{P}_1,...,\hat{P}_r\}$ is a partition of N such that $\|\hat{P}\|>\|P\|$ for each $P\in\prod$, $P\neq\hat{P}$ then the game (N, F) has the same stable sets as the game in characteristic function form $(N,\bar{v})$ defined by

$$\bar{v}(S)=\bar{v}(T)+\sum_{i\in S\setminus T}\bar{v}(\{i\}) \text{ for each } S\subset N,\ \bar{v}(\varnothing)=0\,, \tag{21}$$

where: $T\subset S,\quad T=\bigcup_{i=1}^{r}\{\hat{P}_i:\hat{P}_i\subset S\}$.

Proof: From (8) it is easy to verify that the game $(N,\bar{v})$ is well defined. x is an imputation in the game $(N,\bar{v})$ if $x_i\geq\bar{v}(\{i\})=v(\{i\})$ and $\sum_{i\in N}x_i=\bar{v}(N)=\|\hat{P}\|$. In such a case the set of imputations in the game $(N,\bar{v})$ is equal to the set $R^{\max}$. Moreover, from theorem 3.3, imputations which are not in $R^{\max}$ play no role in the game (N, F) so we can only consider the set $R^{\max}$.

Let $x, y\in R^{\max}$ and let y dominates x in the game $(N,\bar{v})$. Then there exists a coalition $S\subset N$ such that $y_i>x_i$ for each $i\in S$ and $\sum_{i\in S}y_i\leq\bar{v}(S)$. It follows that $y_i>x_i$ for each $i\in T\subset S$ and $\sum_{i\in T}y_i\leq\bar{v}(T)+\sum_{i\in S\setminus T}(\bar{v}(\{i\})-y_i)\leq\bar{v}(T)$, so y *Dom* x.

Let $x, y\in R^{\max}$ and let y *Dom* x. Then there exists a coalition $S=T$ such that $y_i>x_i$ for each $i\in S$ and $\sum_{i\in S}y_i\leq\bar{v}(S)$ so y dominates x in the game $(N,\bar{v})$. It proves the theorem.

4. Final remarks

In the paper a cooperative game in partition function form has been proposed for the cost allocation problem. The game describes real situations in which payoff of any coalition does not only depend on the players in the coalition but also on the coalition structure of the other players. The theory of such games has been developed. In particular solution concepts like core and stable sets have been proposed on the basis of introduced domination relations. Properties of the concepts have been analyzed. The concepts are similar to those presented by Thrall and Lucas [23] but they have been formulated for weaker domination relation which seems to be more relevant in real situations. The ideas developed are close to those presented by Kóczy [7], [8]. However further theoretical studies are required and are planned, including analysis of relations among the different solution concepts, their existence and others. It has been shown that the core of the game in partition function form is equal to the core of an appropriately formulated game in characteristic function form. This theoretical result is very important for construction of decision support systems. General ideas of decision support and construction of computer based systems can be found in the papers by Wierzbicki, Kruś, Makowski [24], Kruś [10], [12], [13]. On the basis of the results presented in this paper, a respective CFF game can be formulated, the core can be derived and proposed to the players as the set describing frames of their negotiations. Different nucleoli can be calculated analogously as it is presented by Kruś, Bronisz [11], by solving a sequence of linear programming problems and presented to the players as mediation proposals. In the decision support several different solution concepts can be used. Especially the concepts of the recursive (pessimistic, optimistic) cores proposed by Kóczy [7], [8] seem to be very attractive, as they provide additional information important to the players. Further research on the decision support problems are also planned taking into account the ideas of sequential coalition formation and application of the recursive cores.

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Alokacja kosztów w grach w postaci funkcji partycji

W artykule rozpatrywana jest klasa gier kooperacyjnych w postaci funkcji partycji opisujących problem alokacji kosztów. Problem ten dotyczy sytuacji, w której grupa aktorów – graczy może wspólnie pozyskać pewien zestaw dóbr, realizując odpowiednie projekty rozwojowe i dzieląc między siebie związane z tym koszty. Gracze mogą w celu pozyskania wymaganych dóbr tworzyć różne koalicje i realizować różne projekty. Proponowana klasa gier kooperacyjnych w postaci funkcji partycji umożliwia analizę tego problemu. Gry takie opisują rzeczywiste sytuacje, w których wypłaty każdej koalicji zależą nie tylko od graczy, którzy ją tworzą, ale także od struktury koalicji tworzonych przez pozostałych graczy. W pracy rozwijana jest teoria takich gier. Proponuje się koncepcje takich rozwiązań jak rdzeń gry i zbiory stabilne na podstawie wprowadzonych relacji dominacji. Analizuje się własności tych koncepcji rozwiązań. Podaje się twierdzenia pokazujące, że w określonych przypadkach rozważane koncepcje rozwiązań gier w postaci funkcji partycji mogą być wyznaczone jako odpowiednie rozwiązania gier w postaci funkcji charakterystycznej.

Słowa kluczowe: *gra w postaci funkcji partycji, rdzeń, zbiór stabilny, alokacja kosztów*