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# Dynamic evolution of mixed circular edge-screw dislocations

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Based on the extended Huygens–Fresnel principle, the analytical expressions for the cross-spectral density function of mixed circular edge-screw dislocations beams propagating through atmospheric turbulence have been derived, and used to study the dynamic evolution of mixed circular edge-screw dislocations in free space and atmospheric turbulence. It is shown that the radius of circular edge dislocations increases with increasing propagation distance, and both the positions of the optical vortex and the center of circular edge dislocations are located at the point (0, 0) when mixed circular edge-screw dislocations propagate in free space. When mixed circular edge-screw dislocations propagate in the atmospheric turbulence, the position of optical vortices varies with increasing propagation distance, the circular edge dislocation evolves into a pair of optical vortices with the opposite topological charge ±1, and the pair of optical vortices will annihilate as soon as the propagation distance becomes large enough.

Keywords: mixed circular edge-screw dislocations, optical vortex, circular edge dislocations, atmospheric turbulence.

## 1. Introduction

Recently, much interest has been exhibited in optical beams carrying phase singularities because of their theoretical importance and attractive potential applications in optical manipulations, atom trapping, optical communication, optical tweezers, semiconductor patterning, high-resolution metrology and micromachining, *etc*. [1–8]. In 1974, NYE and BERRY proposed the concept of optical wavefront dislocation, and summarized the three main phase singularities of monochromatic light waves: screw dislocation, edge dislocation and mixed dislocation [1]. In a screw dislocation a spiral phase ramps around a singularity where the phase of the wave is undefined and whose amplitude vanishes. An edge dislocation is the $\pi$-shift in the wave phase located along a line or circle in the transverse plane, thus edge dislocation can be divided into linear edge dislocation and circular edge dislocation [2]. Mixed dislocation is composed of an edge dislocation and screw dislocation. DIPANKAR *et al.* have investigated the trajectory of an optical vortex in atmospheric turbulence in 2009 [9]. JINHONG LI *et al.* have analyzed the propagation of optical vortices formed by the partially coherent beam array [10]. YAMEI LUO and BAIDA LÜ pointed out that the position of linear edge dislocations is dependent on the amplitude ratio and waist width ratio of beams [11]. The dynamic evolution and classification of optical vortices in atmospheric turbulence have been researched in literatures [12, 13]. The linear edge and circular edge dislocations were also researched in detail in [14–16].

The mixed dislocations have attracted researchers' attention extensively. PETROV reported that an optical vortex induces the splitting of a linear edge dislocation into vortices of topological charges whose positions and number depend on which phase dislocation is shifted from the host beam [17]. HONGWEI YAN and BAIDA LÜ have studied the interaction of the vortex and linear edge dislocation in the presence of an astigmatic lens, and pointed that for the aberration-free case the linear edge dislocation bend and break up into a pair of oppositely charged vortices [18]. DE HE *et al.* have investigated the interaction of the vortex and linear edge dislocation, and found that the linear edge dislocation will be broken, and the vortices may be moved, created and annihilated by varying beam parameters in the free-space propagation [19]. HAITAO CHEN *et al.* have studied analytically and numerically the interaction between a vortex and an edge dislocation nested in a cos-Gaussian beam passing through a tilted lens [20]. KAICHENG ZHU *et al.* have analyzed the intensity distribution and the corresponding phase distribution of the generalized sine-Gaussian beams with a topological charge +1 and an edge dislocation [21]. The interaction of the vortex and linear edge dislocations also has been reported in literature [22–26]. It is interesting to ask: what will happen when the mixed circular edge-screw dislocations are propagating through free space and atmospheric turbulence? The purpose of this paper is to make a detailed study on the dynamic evolution of the mixed circular edge-screw dislocations in free space and atmospheric turbulence. In the second section, the cross-spectral density function of the mixed circular edge-screw dislocations beams propagating through atmospheric turbulence is derived. The dynamic evolution of mixed circular edge-screw dislocations propagating through free space and atmosphere turbulence is studied in the third and forth section, respectively. Finally, the fifth section summarizes the main results of this paper.

## 2. Theoretical model

The initial field of Laguerre–Gaussian (LG) beam at the plane $z = 0$ reads as [27, 28]

$$\mathbf{E}(s, \theta, 0) = \left(\frac{\sqrt{2}\, s}{w_0}\right)^m L_n^m\left(\frac{2s^2}{w_0^2}\right) \exp\left(-\frac{s^2}{w_0^2}\right) \exp(im\theta) \tag{1}$$

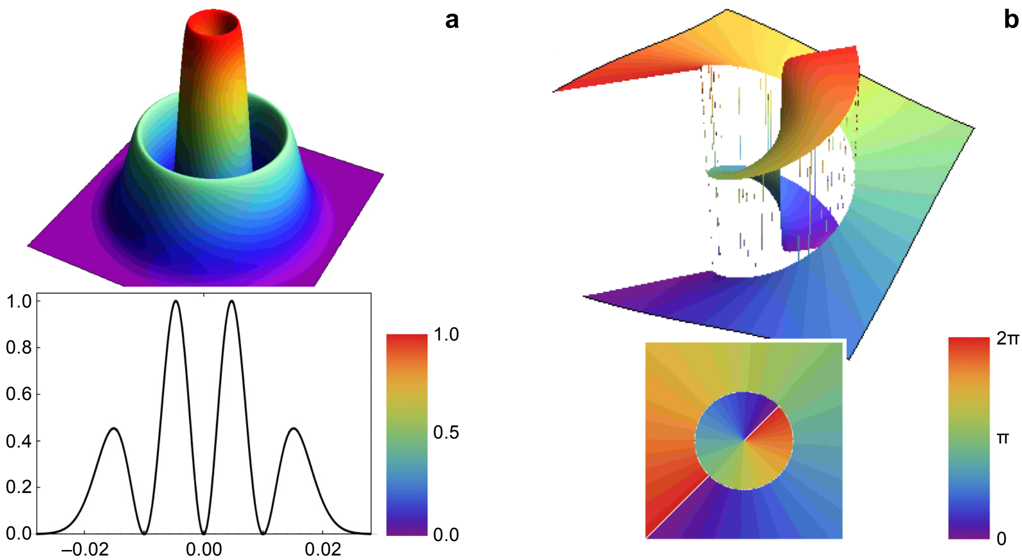


Fig. 1. The normalized intensity distribution (**a**) and phase distribution of mixed circular edge-screw dislocations beams at the source plane (**b**).

where $s$ and $\theta$ are the radial and azimuthal coordinates, $w_0$ denotes the waist width of the Gaussian part, $L_n^m(\cdot)$ denotes Laguerre polynomial. For $m \neq 0$ and $n \neq 0$, LG beams are typical mixed circular edge-screw dislocations beams, namely, Eq. (1) is the initial field of mixed circular edge-screw dislocations beams at the plane $z = 0$. Using Eq. (1), the normalized intensity distribution and phase distribution of mixed circular edge-screw dislocations beams are plotted in Fig. 1. The calculation parameters are $w_0 = 1$ cm, $m = 1$, and $n = 1$. From Figs. 1**a** and 1**b** we can see that the mixed circular edge-screw dislocations beams exist a circular edge dislocation and an optical vortex with topological charge is +1 at the source plane.

Using the relations between Laguerre polynomial and Hermite polynomial [29]

$$\exp(im\theta)\boldsymbol{\rho}^2 L_n^m(\boldsymbol{\rho}^2) = \frac{(-1)^n}{2^{2n+m}n!}\sum_{t=0}^{n}\sum_{r=0}^{m} i^r \binom{n}{t}\binom{m}{r} H_{2t+m-r}(s_x) H_{2n-2t+r}(s_y) \tag{2}$$

the initial field of mixed circular edge-screw dislocations beams at the plane $z = 0$ can be expressed as follows:

$$\mathbf{E}(\mathbf{s}, 0) = \frac{(-1)^n}{2^{2n+m}n!}\sum_{t=0}^{n}\sum_{r=0}^{m} i^r \binom{n}{t}\binom{m}{r} H_{2t+m-r}\left(\frac{\sqrt{2}\, s_x}{w_0}\right) \times H_{2n-2t+r}\left(\frac{\sqrt{2}\, s_y}{w_0}\right)\exp\left(-\frac{\mathbf{s}^2}{w_0^2}\right) \tag{3}$$

The cross-spectral density function of mixed circular edge-screw dislocations beams at the source plane $z = 0$ is expressed as

$$W_0(\mathbf{s}_1, \mathbf{s}_2, 0) = \langle \mathbf{E}(\mathbf{s}_1, 0)^* \bullet \mathbf{E}(\mathbf{s}_2, 0)\rangle$$

$$= \frac{1}{2^{4n+2m}(n!)^2} \sum_{t_1=0}^{n} \sum_{r_1=0}^{m} \sum_{t_2=0}^{n} \sum_{r_2=0}^{m} A_1 A_2 \tag{4}$$

where * denotes the complex conjugate, and

$$A_1 = (-i)^{r_1} i^{r_2} \binom{n}{t_1}\binom{m}{r_1}\binom{n}{t_2}\binom{m}{r_2} H_{2t_1+m-r_1}\left(\frac{\sqrt{2}\, s_{1x}}{w_0}\right) H_{2t_2+m-r_2}\left(\frac{\sqrt{2}\, s_{2x}}{w_0}\right)$$

$$A_2 = H_{2n-2t_1+r_1}\left(\frac{\sqrt{2}\, s_{1y}}{w_0}\right) H_{2n-2t_2+r_2}\left(\frac{\sqrt{2}\, s_{2y}}{w_0}\right) \exp\left(-\frac{\mathbf{s}_1^2 + \mathbf{s}_2^2}{w_0^2}\right)$$

In accordance with the extended Huygens–Fresnel principle [30], the cross-spectral density function of mixed circular edge-screw dislocations beams propagating through atmospheric turbulence is given by

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \left(\frac{k}{2\pi z}\right)^2 \iiiint W_0(\mathbf{s}_1, \mathbf{s}_2, 0) B \, \mathrm{d}\mathbf{s}_1 \mathrm{d}\mathbf{s}_2 \tag{5}$$

where

$$B = \exp\left\{-\frac{ik}{2z}\left[(\mathbf{s}_1 - \boldsymbol{\rho}_1)^2 - (\mathbf{s}_2 - \boldsymbol{\rho}_2)^2\right]\langle \exp\left[\psi(\mathbf{s}_1, \boldsymbol{\rho}_1) + \psi^*(\mathbf{s}_2, \boldsymbol{\rho}_2)\right]\rangle\right\}$$

and $\boldsymbol{\rho}_1 \equiv (\rho_{1x}, \rho_{1y})$, $\boldsymbol{\rho}_2 \equiv (\rho_{2x}, \rho_{2y})$ denote the position vector at the $z$ plane, $k$ is the wave number related to the wavelength $\lambda$ by $k = 2\pi/\lambda$, $\langle \cdot \rangle$ denotes the average over the ensemble of the turbulence medium. It is worth mentioning that a quadratic approximation of the Rytov phase structure function [31] is used in Eq. (5), which can be written as

$$\langle \exp\left[\psi(\mathbf{s}_1, \boldsymbol{\rho}_1) + \psi^*(\mathbf{s}_2, \boldsymbol{\rho}_2)\right]\rangle$$

$$= \exp\left[-\frac{(\mathbf{s}_1 - \mathbf{s}_2)^2 + (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2 + (\mathbf{s}_1 - \mathbf{s}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\rho_0^2}\right] \tag{6}$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ denotes the spatial coherence radius of spherical wave propagation through turbulence and $C_n^2$ denotes the generalized structure constant. The larger $C_n^2$ is, the stronger the atmospheric turbulence is.

Substituting Eqs. (4) and (6) into Eq. (5), we used the following formulas and the Hermite functions [32]:

$$\int \exp\left[-(x-y)^2\right] H_n(ax)\mathrm{d}x = \sqrt{\pi}\,(1-a^2)^{n/2} H_n\left[\frac{ay}{(1-a^2)^{1/2}}\right] \tag{7}$$

$$\int x^n \exp\left[-(x-\beta)^2\right]\mathrm{d}x = (2i)^{-n}\sqrt{\pi}\,H_n(i\beta) \tag{8}$$

$$H_n(x+y) = \frac{1}{2^{n/2}}\sum_{k=0}^{n}\binom{n}{k} H_k(\sqrt{2}\,x) H_{n-k}(\sqrt{2}\,y) \tag{9}$$

$$H_n(x) = \sum_{m=0}^{[n/2]}(-1)^m \frac{n!}{m!(n-2m)!}(2x)^{n-2m} \tag{10}$$

after complex integral operation, we obtain the cross-spectral density function of mixed circular edge-screw dislocations beams propagating through atmospheric turbulence as follows:

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = C_1 \sum_{t_1=0}^{n}\sum_{r_1=0}^{m}\sum_{t_2=0}^{n}\sum_{r_2=0}^{m} C_2 \sum_{c_1=0}^{\left[\frac{2t_1+m-r_1}{2}\right]}\sum_{d_1=0}^{2t_2+m-r_2}\sum_{e_1=0}^{\left[\frac{d_1}{2}\right]} C_3C_4C_5C_6C_7$$
$$\times \sum_{c_2=0}^{\left[\frac{2n-2t_1+r_1}{2}\right]}\sum_{d_2=0}^{2n-2t_2+r_2}\sum_{e_2=0}^{\left[\frac{d_2}{2}\right]} C_8C_9C_{10}C_{11}C_{12} \tag{11}$$

where

$$C_1 = \left(\frac{k}{2\pi z}\right)^2 A_x A_y \exp\left[-\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{\rho_0^2}\right]\exp\left[-\frac{ik}{2z}(\boldsymbol{\rho}_1^2-\boldsymbol{\rho}_2^2)\right]\frac{1}{2^{4n+2m}(n!)^2}$$

$$C_2 = (-i)^{r_1} i^{r_2}\binom{n}{t_1}\binom{m}{r_1}\binom{n}{t_2}\binom{m}{r_2}$$

$$C_3 = \binom{2t_2+m-r_2}{d_1}(-1)^{c_1+e_1}(2i)^{-(2t_2+m-r_1-2c_1+d_1-2e_1)}$$

$$C_4 = \frac{(2t_1+m-r_1)!}{c_1!(2t_1+m-r_1-2c_1)!}\,\frac{d_1!}{e_1!(d_1-2e_1)!}\,\frac{\pi}{\sqrt{D}}\left(1-\frac{2}{w_0^2 D}\right)^{\frac{2t_2+m-r_2}{2}}$$

$$C_5 = 2^{-\frac{2t_2+m-r_2}{2}}\left(\frac{4}{\rho_0^2\sqrt{w_0^2D^2-2D}}\right)^{d_1-2e_1}\left(\frac{2\sqrt{2}}{w_0}\right)^{2t_1+m-r_1-2c_1}$$

$$C_6 = \left(\frac{1}{\sqrt{G}}\right)^{2t_1+m-r_1-2c_1+d_1-2e_1+1} H_{2t_2+m-r_2-d_1}\left[\frac{(\rho_{1x}-\rho_{2x})z-ik\rho_{2x}\rho_0^2}{\rho_0^2 z\sqrt{w_0^2D^2-2D}}\right]$$

$$C_7 = H_{2t_1+m-r_1-2c_1+d_1-2e_1}\left(\frac{iF_x}{2\sqrt{G}}\right)$$

$$C_8 = \binom{2n-2t_2+r_2}{d_2}(-1)^{c_2+e_2}(2i)^{-(2n-2t_1+r_1-2c_2+d_2-2e_2)}$$

$$C_9 = \frac{(2n-2t_1+r_1)!}{c_2!(2n-2t_1+r_1-2c_2)!}\,\frac{d_2!}{e_2!(d_2-2e_2)!}\,\frac{\pi}{\sqrt{D}}\left(1-\frac{2}{w_0^2D}\right)^{\frac{2n-2t_2+r_2}{2}}$$

$$C_{10} = 2^{-\frac{2n-2t_2+r_2}{2}}\left(\frac{4}{\rho_0^2\sqrt{w_0^2D^2-2D}}\right)^{d_2-2e_2}\left(\frac{2\sqrt{2}}{w_0}\right)^{2n-2t_1+r_1-2c_2}$$

$$C_{11} = \left(\frac{1}{\sqrt{G}}\right)^{2n-2t_1+r_1-2c_2+d_2-2e_2+1} H_{2n-2t_2+r_2-d_2}\left[\frac{(\rho_{1y}-\rho_{2y})z-ik\rho_{2y}\rho_0^2}{\rho_0^2 z\sqrt{w_0^2D^2-2D}}\right]$$

$$C_{12} = H_{2n-2t_1+r_1-2c_2+d_2-2e_2}\left(\frac{iF_y}{2\sqrt{G}}\right)$$

and

$$A_x = \exp\left[\frac{1}{4D}\left(\frac{\rho_{1x}-\rho_{2x}}{\rho_0^2}-\frac{ik\rho_{2x}}{z}\right)^2\right]\exp\left(\frac{F_x^2}{4G}\right)$$

$$F_x = \frac{ik\rho_{1x}}{z}-\frac{\rho_{1x}-\rho_{2x}}{\rho_0^2}+\frac{1}{D\rho_0^2}\left(\frac{\rho_{1x}-\rho_{2x}}{\rho_0^2}-\frac{ik\rho_{2x}}{z}\right)$$

$$G = \frac{1}{w_0^2}+\frac{ik}{2z}+\frac{1}{\rho_0^2}-\frac{1}{D\rho_0^4}$$

According to the symmetry, $A_y$ and $F_y$ can be obtained by the replacement of $\rho_{1x}$ and $\rho_{2x}$ in $A_x$ and $F_x$ with $\rho_{1y}$ and $\rho_{2y}$. For $C_n^2 = 0$, Eq. (11) becomes the cross-spectral density function of mixed circular edge-screw dislocations beams in free space.

The spectral degree of coherence is defined as [33]

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \frac{W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)}{\left[I(\boldsymbol{\rho}_1, z) I(\boldsymbol{\rho}_2, z)\right]^{1/2}} \tag{12}$$

where $I(\boldsymbol{\rho}_i, z) = W(\boldsymbol{\rho}_i, \boldsymbol{\rho}_i, z)$, $(i = 1, 2)$ stands for the spectral intensity of the point $(\boldsymbol{\rho}_i, z)$. The position of optical vortices is determined by [34]

$$\mathrm{Re}\left[\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)\right] = 0 \tag{13}$$

$$\mathrm{Im}\left[\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)\right] = 0 \tag{14}$$

where Re and Im denote the real and imaginary parts of $\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$. The sign of optical vortices are determined by the vorticity of phase contours around singularities [35], namely, when varying the phase in counterclockwise direction and clockwise direction, the sign of the topological charge corresponds to plus and minus, respectively, and the phase changes $2m\pi$, the corresponding topological charge is $m$.

## 3. Dynamic evolution of mixed dislocations in free space

Figure 2 gives curves of $\mathrm{Re}(u) = 0$ (solid curves) and $\mathrm{Im}(u) = 0$ (dashed curves) and contour lines of phase of mixed circular edge-screw dislocations beams at the source plane and propagating through free space at the propagation distance $z = 0.3$ and 2 km, the abscissa represents $\rho_{2x}$ direction, ordinate represents $\rho_{2y}$ direction, and their units are cm. The calculation parameters are $\lambda = 632.8$ nm, $w_0 = 1$ cm, $\boldsymbol{\rho}_1 = (7$ cm, $7$ cm$)$, $m = 1$, $n = 1$. Figures 2**a** and 2**d** indicate that there exists a mixed circular edge-screw dislocation at the source plane, which is composed of an optical vortex (marked as A) with topological charge $m = +1$ and a circular edge dislocation (marked as B). The position of the optical vortex A is (0, 0), the radius of circular edge dislocation B is $r_B = 1.02$ cm, the position of the center of circular edge dislocation B is located at (0, 0). Figures 2**b** and 2**e** show that when mixed circular edge-screw dislocations propagate in free space, the positions of the optical vortex A and the center of the circular edge dislocation B do not move, and the radius of the circular edge dislocation B increases to $r_B = 1.18$ cm at $z = 0.3$ km. From Figs. 2**c** and 2**f** we can see that the positions of the optical vortex A and the center of the circular edge dislocation B still do not move, and the radius of the circular edge dislocation B continues to increase to $r_B = 4.20$ cm at $z = 2$ km. Figure 2 indicates that with the increase of the propagation distance, the

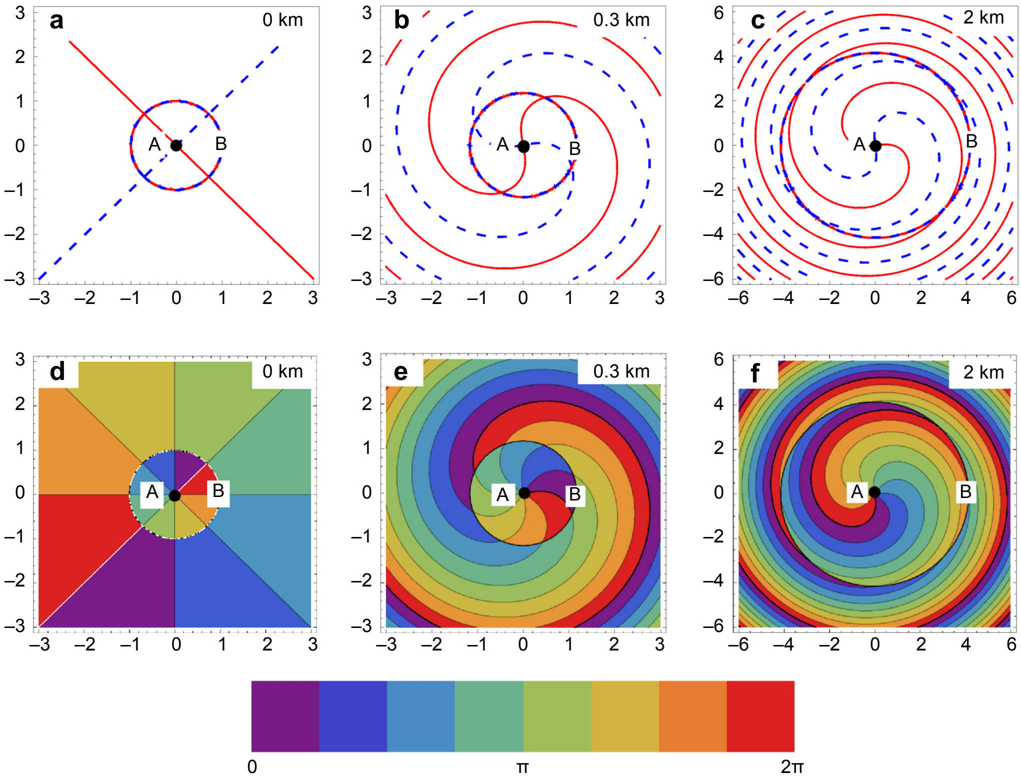


Fig. 2. Curves of Re($u$) = 0 (solid curves) and Im($u$) = 0 (dashed curves) (**a**–**c**) and contour lines of phase of mixed circular edge-screw dislocations beams at the source plane and in free space at different propagation distance $z$ (**d**–**f**); ● – topological charge is +1.

radius of the circular edge dislocation increases gradually. The positions of the optical vortex and the center of the circular edge dislocations do not move when mixed circular edge-screw dislocations beams propagate in free space.

## 4. Dynamic evolution of mixed dislocations in atmospheric turbulence

Figure 3 gives curves of Re($u$) = 0 (solid curves) and Im($u$) = 0 (dashed curves) and contour lines of the phase of mixed circular edge-screw dislocations beams propagating through atmospheric turbulence at the propagation distance $z$ = 0.3, 2 and 5 km. The calculation parameters are $C_n^2 = 10^{-14}$ m$^{-2/3}$, and the others are the same as in Fig. 2. Figures 2**a**, 2**d**, 3**a**, and 3**d** indicate that the position of optical vortex A moves to (0.32 cm, –0.15 cm) from (0, 0), circular edge dislocation B evolves into a pair of optical vortices (marked as $B_+$ and $B_-$) with topological charge +1 and –1, whose positions are located at $B_+$(1.04 cm, –0.67 cm) and $B_-$(–0.65 cm, 1.04 cm) at $z$ = 0.3 km. Figures 3**b** and 3**e** show that with the increment of the propagation distance, the posi-

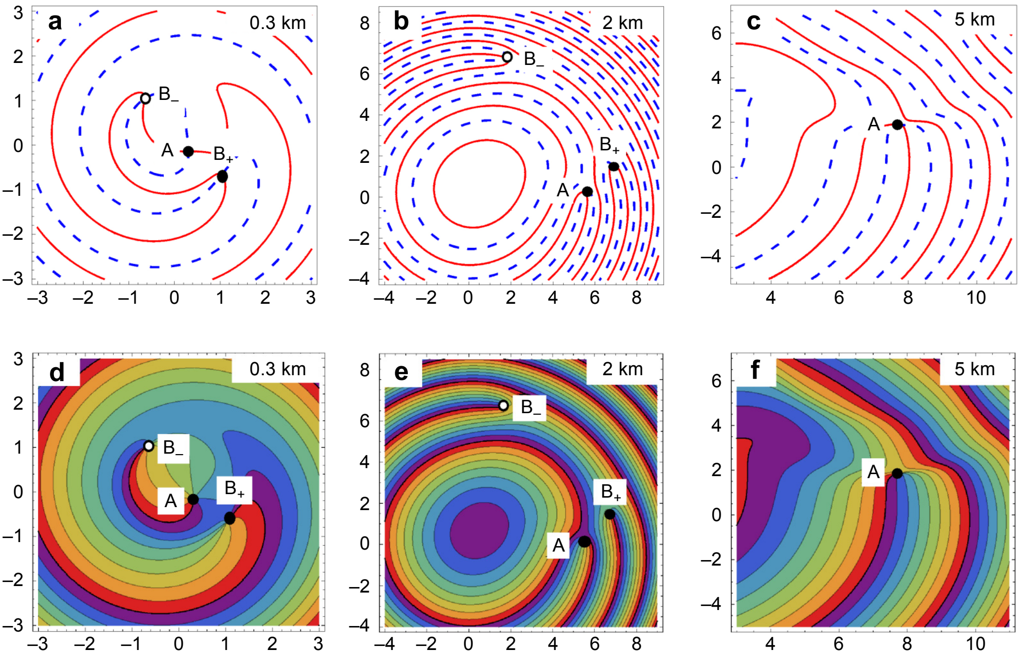


Fig. 3. Curves of Re($u$) = 0 (solid curves) and Im($u$) = 0 (dashed curves) (**a**–**c**) and contour lines of the phase of mixed circular edge-screw dislocations beams propagating through atmospheric turbulence at different propagation distance $z$ (**d**–**f**); ○ – topological charge is –1, ● – topological charge is +1, the color scale is the same as in Fig 2.

tions of optical vortices A, $B_+$, B_ move. Figures 3**c** and 3**f** indicate that when the transmission distance is far enough, the pair of optical vortices $B_+$ and B_ will annihilate, and optical vortex A still exists. Figure 3 indicates that the position of optical vortices varies with the increasing propagation distance, and each circular edge dislocation evolves into a pair of optical vortices with the opposite topological charge when mixed circular edge-screw dislocations beams propagate in atmospheric turbulence. It is assumed that the atmospheric turbulence is a random media, in which the wavefront of the mixed circular edge-screw dislocations beams will be distorted and broken. When the propagation distance is far enough, the pair of optical vortices will annihilate.

## 5. Conclusion

In this paper, by using the extended Huygens–Fresnel principle, the analytical expressions for the cross-spectral density function of mixed circular edge-screw dislocations beams propagating through atmospheric turbulence have been derived, and used to study the dynamic evolution of mixed circular edge-screw dislocations in free space and atmospheric turbulence. For mixed circular edge-screw dislocations beams in free space, with the increment of the propagation distance, the radius of the circular edge dislocations increases gradually, the positions of the optical vortex and the center of the circular edge dislocations do not move. In the atmospheric turbulence, the position of optical vortices varies with increasing propagation distance, each circular edge dislocation evolves into a pair of optical vortices, as soon as the propagation distance becomes large enough, the pair of optical vortices will annihilate. The results obtained in this paper will deepen the understanding of the characteristics of the dynamic evolution of the phase singularities, and will help to explore the potential applications of the new effects of phase singularities.

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