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THE USE OF VALUE-AT-RISK METHODOLOGY IN THE ASSESSMENT OF INVESTOR'S RISK ATTITUDES ON THE PRECIOUS METALS MARKET

ZASTOSOWANIE METODOLOGII *VALUE-AT-RISK* W OCENIE POSTAW INWESTORA WOBEC RYZYKA NA RYNKU METALI

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Summary: The decision making process is directly associated with risk, no matter what area of interest it is. In the case of financial investments we can define a special type of risk called investment risk. Taking into account the financial time series of assets' characteristics (prices, returns), small deviations of future values comparing to their expected level are not a major threat to the investor's portfolio. In contrast, if the changes in prices/returns are significant, unpredictable and result from unexpected and adverse events, one should pay attention to the proper risk measurement. The topic of this article refers to the new family of risk measures related to investments in assets on the precious metals market. These risk measures are called the GlueVaR risk measures. The name itself suggests that the GlueVaR is related to the commonly used measure of risk – VaR. As will be presented in the article, the family of the GlueVaR risk measures may be expressed as a linear combination of VaR and conditional VaR for fixed tolerance levels. Moreover, the new risk measure allows for assessing risk more personally, taking into account the investor's attitudes towards risk. If portfolio investments are of interest, the GlueVaR risk measures meet the assumption of subadditivity. This property of risk measure is required, as it is strongly related to the diversification problem.

Keywords: risk, precious metals market, subadditivity, VaR, GlueVaR.

Streszczenie: Proces podejmowania decyzji jest bezpośrednio powiązany z zagadnieniem ryzyka, bez względu na obszar, jakiego dotyczy. W przypadku inwestycji finansowych należy wspomnieć o szczególnym rodzaju ryzyka, mianowicie o ryzyku inwestycyjnym. Biorąc pod uwagę finansowe szeregi czasowe (ceny oraz stopy zwrotu), można zauważyć, że niewielkie odchylenia przyszłej realizacji inwestycji od jej wartości oczekiwanej nie stanowią głównej troski wśród inwestorów. Inaczej jest, jeśli owa zmienność jest istotna, nieprzewidywalna oraz wynika z nieoczekiwanych i niepożądanych zdarzeń losowych. W takiej sytuacji należy zwrócić szczególną uwagę na właściwy pomiar ryzyka. Głównym celem prezentowanej pracy jest omówienie nowej rodziny mierników ryzyka na przykładzie inwestycji realizowanych na rynku metali szlachetnych. Nowa rodzina miar ryzyka nosi nazwę GlueVaR i jest bezpośrednio powiązana z popularnymi w teorii i praktyce miarami VaR oraz CVaR. Miara GlueVaR stanowi kombinację liniową wspomnianych mierników dla ustalonego systemu wag oraz poziomów tolerancji. Ważną cechą miar należących do rodziny GlueVaR jest to, iż uwzględniają stosunek inwestora wobec ryzyka (który jest podejściem czysto subiektywnym, indywidualnym). Dodatkowo mierniki GlueVaR należą do rodziny miar koherentnych, spełniając jednocześnie wszystkie aksjomaty dobrej miary ryzyka. Podkreśla się zwłaszcza spełnianie zadość własności subaddytywności, mającej szczególne znaczenie w budowie portfeli inwestycyjnych.

Słowa kluczowe: ryzyko, rynek metali szlachetnych, subaddytywność, VaR, GlueVaR.

1. Introduction

The existence of individuals in the market economy, regardless of how developed and advanced it is, requires from their participants the analysis of the surrounding reality. Such analysis has a relevant impact in the decision-making process. The diverse nature of events which may affect market decisions generates risk that the undertaken investment may produce final results different from the expectation. The term "risk" is usually associated with a situation when the final result of an investment is at a lower level than one may expect. Therefore it is possible to define two concepts of risk: neutral and negative. The result of an investment, no matter what is concerned, is exposed to certain disturbances (factors) which may be classified into two groups:

a) internal factors, specified for a particular investment,

b) external factors, not directly related, but having an influence on the final values of an investment.

One of the internal factors might be the decision of the board of the company regarding further paths of its development. In turn, external factors might be related to the political situation (national, international) or some information from the market. This example precisely determines the sources of the mentioned disturbances. In summary, it may be said that a risky investment produces an outcome which in unknown, uncertain and may differ from the expected one.

In this article the emphasis is placed on the specific type of investment risk, namely such risk which is associated with events which are highly unlikely to occur, but if they do take place, can produce large losses. In the literature such type of risk is called extreme risk [Jajuga 2009], and is defined as Low Frequency, High Severity (LFHS). From a theoretical point of view there are statistical tools for modelling the probability of extreme events, but this is not the topic of this article. In the context of this paper an extreme event is defined as an event for which the probability of occurrence significantly differs from the average level.

2. The measure of risk

The measurement of risk is possible only if the proper risk measure is defined. Let X be the set of all random variables defined for a given probability space (Ω, A, P). A risk measure ρ is a mapping from X to R, which means the mapping from the set of random variables to the line representing the real numbers: $X \to \rho(X) \in R$. Following Artzner et. al. [Artzner et al. 1999] an acceptable risk measure should meet the following axioms:

- positive homogeneity: $\rho(\lambda X) = \lambda\rho(X)$,
- subadditivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$,
- monotonicity: $X \leq Y \Rightarrow \rho(X) \leq \rho(Y)$,
- translation invariance: $\rho(X + \alpha R_{free}) = \rho(X) - \alpha$.

The foregoing properties define the coherent risk measure. From the point of view of investment activity. a particular attention is required by the property of subadditivity. This axiom means that the overall risk of the portfolio is equal or less than the sum of the individual risks of its components. In practice, the most popular risk measure is VaR. For a given time horizon and for a certain fixed probability level α the Value-at-Risk defines the loss that is exceeded over this specified time horizon and the probability $(1 - \alpha)$. From the mathematical point of view VaR at level α is defined as a α −quantile of a random variable.

Despite the popularity of this measure of risk in practical use, VaR is not coherent because does not meet the subadditivity assumption. Value-at-Risk is subadditive for the random variables which are elliptically distributed [McNeil et al. 2005], so in terms of other statistical distributions do not measure the risk. An alternative tool for Value-at-Risk is its conditional version called conditional Value-at-Risk (CVaR) which measures the average level of loss in the most adverse cases. The value of CVaR is usually higher than the value of VaR and the selection of risk measure depends on the investor's attitude towards risk.

3. GlueVaR risk measure

The level of risk which might be accepted by investors depends on their personal preferences and additional factors, which are rarely independent. If the investor's attitude towards risk is of interest, the corresponding risk measure should include this feature. Belles-Sampera et al., in the paper "Beyond Value-at-Risk: GlueVaR distortion Risk Measures" [Belles-Sempera et al. 2014] proposed a tool for measuring risk – the GlueVaR risk measure. The family of GlueVaR risk measures may be expressed in terms of the Choquet integral and distortion function. Without going into details, the distortion function for a GlueVaR risk measures is of the form:

$$\eta_{\gamma_2,\gamma_1}^{h_1,h_2}(u) = \begin{cases} \dfrac{h_1}{1-\gamma_2} u & \text{if} \quad 0 \le u < 1-\gamma_2 \\ h_1 + \dfrac{h_2-h_1}{\gamma_2-\gamma_1}[u-(1-\gamma_1)] & \text{if} \quad 1-\gamma_2 \le u < 1-\gamma_1 \\ 1 & \text{if} \quad 1-\gamma_1 \le u \le 1 \end{cases}$$

where: γ_1 and γ_2 stand for tolerance levels meeting assumption: $\gamma_1 \le \gamma_2$, while parameters h_1 and h_2 are defined as hits of the distortion function, such that $h_1 \in [0,1]$ and $h_2 \in [h_1, 1]$. The expression of the GlueVaR risk measure in terms of the Choquet integral is of the form:

$$GlueVaR_{\gamma_2,\gamma_1}^{h_1,h_2}(X) = \int X \, d\mu = \int X \, d(\eta_{\gamma_2,\gamma_1}^{h_1,h_2} \circ P)$$

where "∘" stands for the function composition.

In might be shown that any GlueVaR risk measure is a linear combination of VaR and conditional VaR at the fixed tolerance levels γ_1 and γ_2. Hence:

$$GlueVaR_{\gamma_1,\gamma_2}^{h_1,h_2}(X) = w_1 CVaR_{\gamma_2}(X) + w_2 CVaR_{\gamma_1}(X) + w_3 VaR_{\gamma_1}(X)$$

where weights are defined as below:

$$\begin{cases} w_1 = h_1 - \dfrac{(h_2-h_1)(1-\gamma_2)}{\gamma_2-\gamma_1} \\ w_2 = \dfrac{h_2-h_1}{\gamma_2-\gamma_1}(1-\gamma_1) \\ w_3 = 1 - w_1 - w_2 = 1 - h_2 \end{cases}$$

It may be pointed out that for fixed tolerance levels and for fixed weights the GlueVaR risk measures reduce to:

- VaR at the level γ_1, if $w_1 = w_2 = 0$,
- CVaR at the level γ_1, if $w_1 = w_3 = 0$,
- CVaR at the level γ_2, if $w_2 = w_3 = 0$.

As presented by Belles-Sempera et al., the pairs (h_1, h_2) representing hits of a distortion function of the GlueVaR and (w_1, w_2) representing weights of CVaR at the levels γ_2 and γ_1 respectively are linearly related to each other. The relationship can be expressed as follows:

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \mathrm{H} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \text{ and } \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \mathrm{H}^{-1} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

where matrices H and H^{-1} are of the form:

$$\mathrm{H} = \begin{bmatrix} 1 & \frac{1-\gamma_2}{1-\gamma_1} \\ 1 & 1 \end{bmatrix} \text{ and } \mathrm{H}^{-1} = \begin{bmatrix} \frac{1-\gamma_1}{\gamma_2-\gamma_1} & \frac{\gamma_2-1}{\gamma_2-\gamma_1} \\ \frac{\gamma_1-1}{\gamma_2-\gamma_1} & \frac{1-\gamma_1}{\gamma_2-\gamma_1} \end{bmatrix}$$

As we can find in the definition of the GlueVaR as a linear combination of VaR and CVaR, the selection of risk measure depends on the weights w_1, w_2 and w_3. These weights may help to define a particular investor in terms of his/her attitude toward risk as [Belles-Sampera et al. 2015]:

- highly conservative, if $w_1 = 1$ and $w_2 = w_3 = 0$;
- conservative, if $w_2 = 1$ and $w_1 = w_3 = 0$;
- less conservative, if $w_3 = 1$ and $w_1 = w_2 = 0$.

Therefore for given confidence levels γ_1 and γ_2 and for certain preferred levels of weights w_1, w_2 and w_3 reflecting investor's attitude toward risk, the appropriate risk measure within the new family of risk measures may be selected.

An interesting feature of the GlueVaR risk measures is that they allow for subadditivity depending on the associated weights. As the GlueVaR may be defined as a linear combination of VaR and CVaR, and as the CVaR is a coherent risk measure, therefore the subadditivity of GlueVaR holds if the weight w_3 corresponding to VaR_{γ_1} is equal to zero. More generally, the GlueVaR is subadditive if $\frac{\gamma_2-1}{\gamma_2-\gamma_1} \leq w_1 \leq 1$ and $w_1 + w_2 \leq 1$.

One of the most interesting features of the GlueVaR risk measure is that it can be expressed using formulas for some popular probability distributions which describe returns. If a normally distributed random variable X is considered, any GlueVaR risk measure can be calculated as:

$$GlueVaR_{\gamma_2,\gamma_1}^{h_1,h_2}(X) = \mu + \sigma q_{\gamma_1}[1-h_2] + \sigma \frac{h_2-h_1}{\gamma_2-\gamma_1}\left[\phi(q_{\gamma_1}) - \phi(q_{\gamma_2})\right] + \frac{h_1}{1-\gamma_2}\phi(q_{\gamma_2})$$

where: $X \sim N(\mu, \sigma)$, q_{γ_1}, q_{γ_2} represent $\gamma_1 -$ quantile and $\gamma_2 -$ quantile of standard normal distribution respectively, and $\phi(\cdot)$ represents density of standard normal distribution. If a random variable X is described by *t*-Student distribution, the expression for GlueVaR risk measure is of the form:

$$GlueVaR_{\gamma_2,\gamma_1}^{h_1,h_2}(X) = \mu + \sigma[\left(\frac{h_1}{1-\gamma_2} - \frac{h_2-h_1}{\gamma_2-\gamma_1}\right) f(t_{\gamma_2}) \left(\frac{k+t_{\gamma_2}^2}{k-1}\right) + \frac{h_2-h_1}{\gamma_2-\gamma_1} f(t_{\gamma_1}) \left(\frac{k+t_{\gamma_1}^2}{k-1}\right) + (1-h_2)t_{\gamma_1}]$$

where: t_{γ_1}, t_{γ_2} represent $\gamma_1 -$ quantile and $\gamma_2 -$ quantile of *t*-Student distribution respectively, k represents degrees of freedom and $f(\cdot)$ represents density function of *t-S*tudent distribution.

4. Empirical results on the precious metals market

The use of the GlueVaR risk measure in risk assessment is presented using the example of the precious metals market. The financial and economic crises observed in the first decade of the 21st century forced investors to search for other possibilities to invest capital which would generate positive returns [Krężołek 2012]. Among commodities the most popular, from the investment point of view, are electricity, fuel, agricultural products, precious stones and metals, etc. In this paper we focus only on the precious metals market. The analysis is based on the daily log-returns of spot closing prices of precious metals quoted on the London Metal Exchange from January 2008 to June 2015. The set of assets includes GOLD, SILVER, PLATINUM and PALLADIUM. The quantile-based risk measures such as VaR, CVaR and GlueVaR have been calculated, for quantile 0.952 and 0.996, using empirical and theoretical distributions: normal, t-Student and α-stable. As an example, Figures 1 and2 present the levels of prices and log-returns for GOLD and PLATINUM.

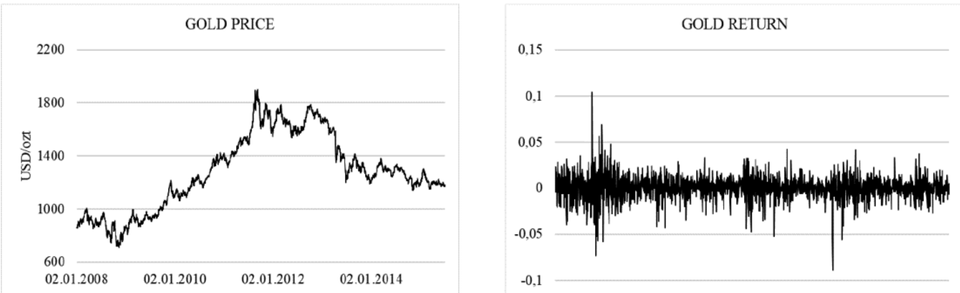

Fig. 1. Spot prices and log-returns of GOLD

Source: own calculations.

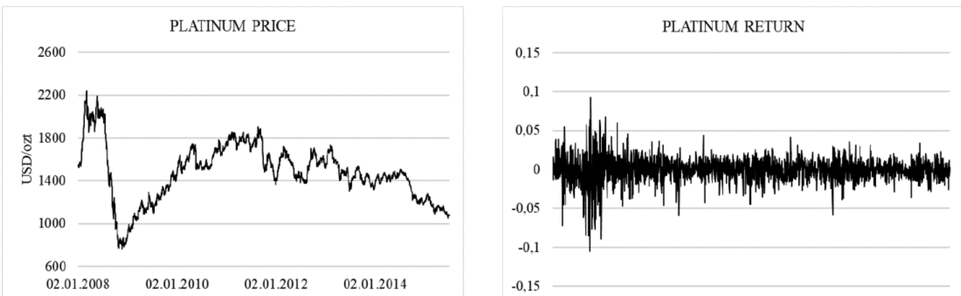

Fig. 2. Spot prices and log-returns of PLATINUM

Source: own calculations.

The level of jumps observed in the volatility of prices of precious metals results in clusters of variance in log-returns. Some selected descriptive statistics are presented in Table 1.

Table 1. Descriptive statistics

Descriptive Statistics	GOLD	SILVER	PLATINUM	PALLADIUM
Mean	0.0002	0.0000	–0.0002	0.0003
Standard deviation	0.0128	0.0228	0.0156	0.0209
Kurtosis	5.9635	6.9337	5.0923	4.8639
Skewness	–0.2429	–0.9875	–0.5409	–0.6037
Min	–0.0888	–0.1693	–0.1045	–0.1656
Max	0.1039	0.1393	0.0925	0.0999
Number of exceedances over + 3 std.dev.	7	4	10	8
Number of exceedances below – 3 std.dev.	17	18	16	15

Source: own calculations.

The average level of returns for precious metals oscillated around zero within the analysed period. The highest level of volatility was observed for SILVER and PALADIUM. Moreover, all the analysed empirical distributions are leptokurtic and left-skewed. These features indicate a higher probability that the hypothesis of normality has to be rejected. The goodness-of-fit tests (Jarque-Bera and Anderson-Darling tests) confirmed this suggestion. The analysis is based on the three theoretical distributions: normal, t-Student and alpha-stable. The estimates of alpha-stable model are presented in Table 2.

Table 2. Maximum likelihood estimates of alpha-stable parameters

Parameters	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	$\hat{\sigma}$
GOLD	1.66503	–0.19187	0.00005	0.00707
SILVER	1.66237	–0.19224	0.00013	0.01238
PLATINUM	1.71036	–0.20893	–0.00026	0.00873
PALLADIUM	1.67730	–0.23803	0.00010	0.01171

Source: own calculations.

All empirical distributions seem to be leptokurtic and asymmetric. Moreover, it has been discovered that there is some disparity in the numbers of observations exceeding three standards deviations from the mean (for both left and right tail of the distribution). Two different confidence levels have been assumed. The confidence level $\gamma_1 = 0.952$ denotes that an extreme event appears twelve times per year and

$\gamma_1 = 0.996$ denotes that an extreme event appears one time per year (one year is understood as a 250 trading days). Table 3 presents seven scenarios for some selected weights w_1, w_2 and w_3.

Table 3. Weights given to VaR and CVaR in the GlueVaR risk measure

Weights/hits	S1	S2	S3	S4	S5	S6	S7
w_1	1.00	0.00	0.00	0.33	0.33	0.67	0.50
w_2	0.00	1.00	0.00	0.33	0.67	0.33	0.50
w_3	0.00	0.00	1.00	0.33	0.00	0.00	0.00
h_1	1.00	0.08	0.00	0.36	0.39	0.69	0.54
h_2	1.00	1.00	0.00	0.67	1.00	1.00	1.00

Source: own calculations.

Three scenarios S1-S3 allow for estimating classical risk measures:

- scenario S1 – CVaR at the level γ_2,
- scenario S2 – CVaR at the level γ_1,
- scenario S3 – VaR at the level γ_1.

The results of estimating GlueVaR risk measures based on scenarios S1-S7 are presented below.

Table 4. GlueVaR risk measures for GOLD

GOLD	S1	S2	S3	S4	S5	S6	S7
Empirical	0.05746	0.02814	0.01977	0.03512	0.03791	0.04769	0.04280
Normal	0.04313	0.02694	**0.02048**	0.03018	0.03234	0.03773	0.03503
t-Student	0.04719	**0.02923**	0.02059	**0.03233**	**0.03521**	0.04120	**0.03821**
Stable	**0.06344**	0.03538	0.02260	0.04047	0.04473	**0.05408**	0.04941

Source: own calculations.

Table 5. GlueVaR risk measures for SILVER

SILVER	S1	S2	S3	S4	S5	S6	S7
Empirical	0.08076	0.04731	0.03445	0.05417	0.05846	0.06961	0.06404
Normal	0.07049	**0.04786**	**0.03769**	**0.05201**	**0.05540**	0.06294	**0.05917**
t-Student	**0.08129**	0.05857	0.03807	0.05931	0.06614	**0.07372**	0.06993
Stable	0.09094	0.05478	0.03960	0.06178	0.06684	0.07889	0.07286

Source: own calculations.

Table 6. GlueVaR risk measures for PLATINUM

PLATINUM	S1	S2	S3	S4	S5	S6	S7
Empirical	0.06566	0.03346	0.02264	0.04059	0.04419	0.05493	0.04956
Normal	0.04696	0.03134	**0.02327**	0.03386	0.03655	0.04175	0.03915
t-Student	0.06283	**0.03411**	0.02346	**0.04013**	**0.04368**	0.05326	0.04847
Stable	**0.06710**	0.03089	0.02153	0.03984	0.04296	**0.05503**	**0.04899**

Source: own calculations.

Table 7. GlueVaR risk measures for PALLADIUM

PALLADIUM	S1	S2	S3	S4	S5	S6	S7
Empirical	0.08386	0.04545	0.03180	0.05370	0.05825	0.07105	0.06465
Normal	0.06059	0.04267	0.03493	0.04606	0.04864	0.05461	0.05163
t-Student	0.07372	0.04336	**0.03388**	0.05032	0.05348	0.06360	0.05854
Stable	**0.08577**	**0.04726**	0.03453	**0.05585**	**0.06010**	**0.07293**	**0.06652**

Source: own calculations.

The values in bold in Tables 5 to 7 indicate the assessments of risk measures closest to the empirical ones. The best results were achieved for heavy-tailed distributions. However, the use of GlueVaR risk measures in risk assessment has to be associated with the decision-maker's attitudes towards risk. It is easy to show that, running some simple transformations, the weight corresponding to Value-at-Risk at the level γ_1 may be expressed using weights w_1 and w_2 as $w_3 = 1 - w_1 - w_2$. The set of acceptable weights of the GlueVaR components are presented in Figure 3.

As discussed in the previous part of this paper, the GlueVaR risk measure is related to the confidence levels and weights given to VaR and CVaR. The confidence levels reflect the probability of occurrence of extreme events and weights reflect how much these events are important for a particular investor. To hold the assumption of subadditivity for the GlueVaR risk measure, the weight corresponding to the non-subadditive risk component of GlueVaR (i.e. VaR) should meet the relation that $w_3 = 0$. Belles-Sampera et al. showed some proof that GlueVaR is subadditive if both weights w_1 and w_2 belong to the area delimited by the triangle S1BS2, especially if they lie on the line segment in a coordinate system described by points: $A = (w_1, w_2) = (\frac{\gamma_2 - 1}{\gamma_2 - \gamma_1}, \frac{1 - \gamma_1}{\gamma_2 - \gamma_1})$ and $S2 = (w_1, w_2) = (1{,}0)$ for fixed values of γ_1 and γ_2 $(0 < \gamma_1 \leq \gamma_2 < 1)$. Moreover, the position of a particular point on this line represents the investor's attitude toward risk. The nearer to point A, the less conservative attitude toward risk. For scenarios S4-S7 the values of GlueVaR risk measure are presented in Figures 4 and 5.

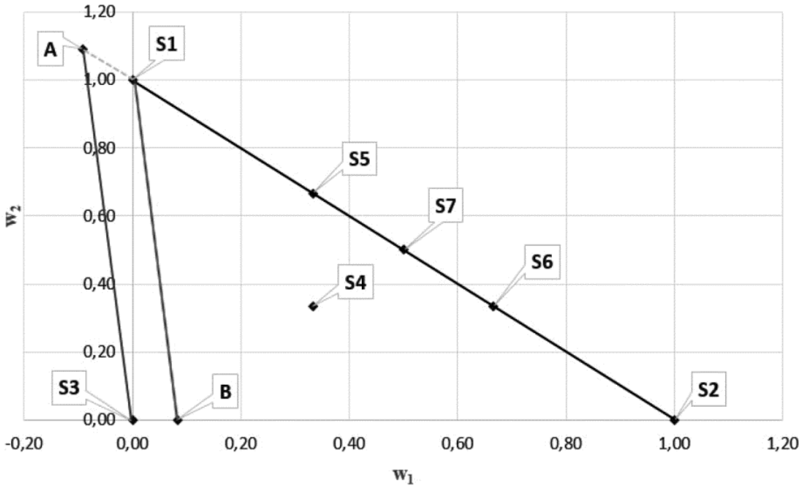

Fig. 3. Investor's attitudes towards risk
Source: own calculations.

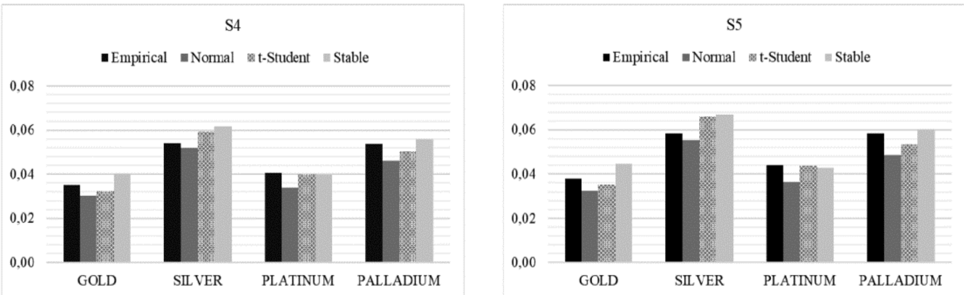

Fig. 4. Investor's attitudes towards risk
Source: own calculations.

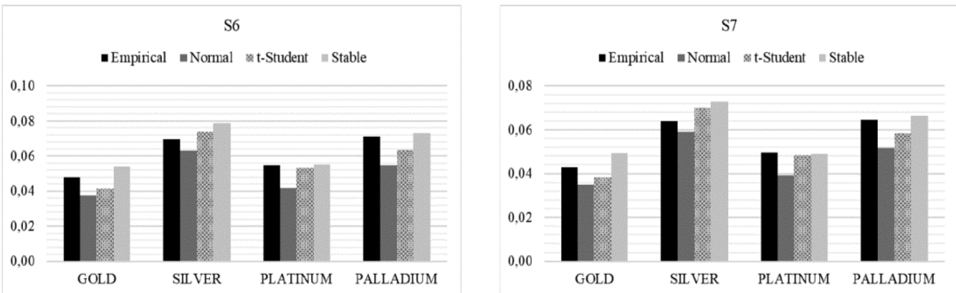

Fig. 5. Investor's attitudes towards risk
Source: own calculations.

Based on the information above, one may conclude that heavy-tailed distributions usually overestimate the real (empirical) level of risk (most of all for alpha-stable distribution), while if the normal approach is maintained, the assessments of risk values are usually underestimated. Taking into account that the analysis is related to extreme risk, it is worth to focus on models dedicated to such phenomena (using the Extreme Value Theory).

5. Conclusions and remarks

The decision-making process requires the acceptance of risk that the undertaken investment may generate results which differ from our expectations. To minimize this risk it is recommended to use the appropriate statistical tool. Taking into account financial investments, there is a lot of risk measures with Value-at-Risk as one of the most popular. However, the risk measure itself should meet assumptions to be defined a good risk measure. In this article the axioms of coherent risk measure have been briefly mentioned. VaR is not coherent, as it does not meet the assumption of subadditivity. This feature is of great interest if the portfolio's investment is considered.

The acceptable level of risk is a personal issue and has to be analysed in that way. A good risk measure has to reflect the investor's attitudes towards risk. In this paper the application of a new family of risk measures has been presented – the GlueVaR risk measures (proposed by Belles-Sampera et al.). The family of GlueVaR risk measures may be easily expressed in terms of VaR and CVaR for given tolerance levels and for given weights. The advantage of GlueVaR compared to the other risk measures is that it takes into account the investor's attitudes towards risk and, most importantly, it meets the assumption of subadditivity.

The application of the GlueVaR risk measures has been presented using the example of investments on the precious metals market. The extreme events have been defined as events occurring one time per financial year and twelve times per financial year. The assessment has been made using normal, $t-$Student and alpha-stable distributions. The results show that if extreme events are of interest, it is better to use heavy-tailed distribution to calculate the GlueVaR risk measure (the results generalize the GlueVaR risk measures calculated for different scenarios).

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