

DANIEL SŁYŚ*

MATHEMATICAL MODELLING OF *EcoINFRA2* STORAGE RESERVOIR

The paper presents mathematical model of *EcoINFRA2* storage reservoir functioning which together with hydraulic model, defining boundary conditions of all phases of its functioning, forms the basis for elaborated software instrument enabling the simulation research of design parameters of such type of objects for any function of wastewater inflow. The paper presents the parameters of simulation model and calculation algorithm of its functioning.

DESIGNATIONS

c_H	– coefficient of internal roughness of the channel for the Chezy formula, $m^{0.5} s^{-1}$;
D_o	– diameter of the discharge channel from the reservoir, m;
F_a	– horizontal surface of the accumulation chamber, m^2 ;
f_o	– cross-sectional area of the outflow channel chamber, m^2 ;
F_p	– horizontal surface of the pump chamber, m^2 ;
F_s	– horizontal surface of the control chamber, m^2 ;
F_t	– horizontal surface of the overfall chamber, m^2 ;
g	– terrestrial gravity, m/s^2 ;
h	– momentary height of wastewater filling in the accumulation chamber, m;
h_{max}	– maximum height of wastewater filling in the accumulation chamber, m;
h_o	– level of pumping unit start and operation, m;
$h_{p max}$	– maximum height of wastewater filling in the pump chamber, m;
h_p	– momentary height of wastewater filling in the pump chamber, m;
$h_{s max}$	– maximum height of wastewater filling in the control chamber, m;
h_s	– momentary height of wastewater filling in the control chamber, m;
h_t	– momentary height of wastewater filling in the overfall chamber, m;
l	– length of the discharge channel from the reservoir, m;
QA	– momentary intensity of wastewater inflow into the reservoir, m^3/s ;
QA_{max}	– maximum intensity of wastewater inflow into the reservoir, m^3/s ;
QD	– momentary intensity of wastewater flow into the throttling pipe, m^3/s ;
QD_{max}	– maximum intensity of wastewater flow into the throttling pipe, m^3/s ;

* Rzeszów University of Technology, Department of Infrastructure and Sustainable Development, al. Powstańców Warszawy 6, 35-959 Rzeszów, Poland, phone: (+48 17) 865 1784, fax: (+48 17) 865 1172, e-mail: daniels@prz.edu.pl, <http://www.prz.rzeszow.pl/~kie/>

QP	– momentary intensity of wastewater flow from overfall chamber to the pump chamber, m^3/s ;
QR	– momentary intensity of wastewater flow from pump chamber to the overfall chamber, m^3/s ;
QS	– momentary intensity of wastewater discharge from the accumulation chamber, m^3/s ;
QS_{\max}	– maximum intensity of wastewater discharge from the accumulation chamber, m^3/s ;
QT	– momentary wastewater flow forced by pump-system operation, m^3/s ;
QT_{\max}	– maximum wastewater flow forced by pump-system operation, m^3/s ;
QZ	– momentary intensity of wastewater discharge from the reservoir, m^3/s ;
QZ_{\max}	– maximum intensity of wastewater discharge from the reservoir, m^3/s ;
t	– time, s;
$V_{\text{śc}}$	– volume of wastewater accumulated in the reservoir, m^3 ;
β	– momentary coefficient of wastewater flow reduction through a reservoir, –;
β_{\max}	– maximum coefficient of wastewater flow reduction through a reservoir, –;
μ	– flow rate coefficient at reservoir outflow opening, –;
ζ	– local resistance factor, –;

1. INTRODUCTION

Mathematical modelling of objects' functioning and creating computer simulation models make it possible to investigate significant parameters and phenomena in limited necessity of capital intensive laboratory experiments, to repeat them and obtain the results in a short time. Application of mathematical modelling concerns all scientific and technical fields together with the fields connected with the investigation of sewage systems and their objects.

The research in the field of modelling wastewater accumulation processes in storage reservoirs carried out in the world is concentrated mainly on qualitative aspects of precipitation influence on the efficiency of wastewater treatment in storage reservoir in the light of surface water protection and wastewater treatment plant's work. Very interesting results of simulation research were obtained on the basis of calculation program *COSMOSS* and presented in publications [1]–[2].

In Polish conditions, storage reservoirs within sewage systems most often play the reducing role in order to limit the extreme wastewater flows in sewage systems and their retention.

In recent years, an intensive development of theory and constructions of storage reservoirs may be observed, among them of prior significance are the multichamber storage reservoirs of pump-gravitational type [3]–[4]. The solutions of such schemes of wastewater accumulation enable the efficient usage of building surface by theoretically unlimited filling height in the chambers of reservoirs. This important feature of hydraulic scheme of pump-gravitational reservoirs significantly broadens the scope of economically and technically grounded solutions of sewage systems working jointly with the objects for wastewater retention.

So there is a necessity to carry out a detailed theoretical and simulation analysis that leads to elaboration of the methods of dimensioning for such types of reservoirs for any

hydrograms of wastewater inflow. Taking this into account, the hydraulic model [5]–[6] and simulation model were constructed for *EcoINFRA2* storage reservoir that is designed for regulation of wastewater flow in storm and combined sewage network.

2. MATHEMATICAL MODELLING OF STORAGE RESERVOIR FUNCTIONING

Mathematical model of storage reservoir embraces all significant phases of its functioning concerning the analysis of investigated parameters' influence on resulting parameters. The model is based on a system of differential equations determining the changes of wastewater filling level in all chambers of the reservoir simultaneously. The elaborated mathematical model is consistent with mass preservation law, which in this case can be formulated as follows:

$$dV_{\text{sc}} = F_t \cdot dh_t + F_s \cdot dh_s + F_p \cdot dh_p + F_a \cdot dh = QA(t) \cdot dt - QZ(t) \cdot dt . \quad (1)$$

Mathematical model of storage reservoir functioning demands the separation of characteristic phases of functioning defined by the boundary hydraulic conditions of their occurrence. Using the hydraulic model of *EcoINFRA2* reservoir presented in publication [6] the mathematical description of all stages of its functioning was elaborated.

At the first stage of reservoir's functioning, i.e., the phase of filling overflow chamber, the process of changes of overflow and steering chambers' filling by wastewater is described by the system of equations:

$$\begin{cases} \frac{dh_t}{dt} = QA(t) \cdot F_t^{-1} \cdot t - QD(h_t, h_s) \cdot F_t^{-1}, \\ \frac{dh_s}{dt} = QD(h_t, h_s) \cdot F_s^{-1} - QZ(t) \cdot F_s^{-1}. \end{cases} \quad (2)$$

At the moment of wastewater overflowing above the edge between the chambers from an overflow chamber to a pump chamber, the system of differential equations describing the changes of filling level in hydraulically connected chambers transforms to the following system of equations:

$$\begin{cases} \frac{dh_t}{dt} = QA(t) \cdot F_t^{-1} \cdot t - QD(h_t, h_s) \cdot F_t^{-1} - QP(h_t, h_p) \cdot F_t^{-1}, \\ \frac{dh_s}{dt} = QD(h_t, h_s) \cdot F_s^{-1} - QZ(t) \cdot F_s^{-1}, \\ \frac{dh_p}{dt} = QP(h_t, h_p) \cdot F_p^{-1}. \end{cases} \quad (3)$$

Further wastewater inflow to a pump chamber brings to the switching of wastewater pump transport system and the beginning of accumulation chamber filling. At this phase of storage reservoir functioning all four chambers take part, which are hydraulically connected to each other and play different roles. The system of equations describing the filling process's changes in the chambers of reservoir at this stage is as follows:

$$\begin{cases} \frac{dh_t}{dt} = QA(t) \cdot F_t^{-1} \cdot t - QD(h_t, h_s) \cdot F_t^{-1} - QP(h_t, h_p) \cdot F_t^{-1}, \\ \frac{dh_s}{dt} = QD(h_t, h_s) \cdot F_s^{-1} - QZ(t) \cdot F_s^{-1}, \\ \frac{dh_p}{dt} = QP(h_t, h_p) \cdot F_p^{-1} - QT(h, h_p) \cdot F_p^{-1}, \\ \frac{dh}{dt} = QT(h, h_p) \cdot F_a^{-1}. \end{cases} \quad (4)$$

The beginning of storage reservoir emptying is initiated first of all in an overflow chamber which is directly connected to the sewage system. The reduced wastewater flow causes wastewater level decrease in this chamber and that in consequence influences the beginning of wastewater back-flow through the edge from pump chamber to an overflow chamber with simultaneous constant wastewater outflow by choking pipe to steering chamber. Momentary wastewater filling in storage reservoir chambers can be defined by the following system of equations:

$$\begin{cases} \frac{dh_t}{dt} = QA(t) \cdot F_t^{-1} \cdot t - QD(h_t, h_s) \cdot F_t^{-1} + QR(h_t, h_p) \cdot F_t^{-1}, \\ \frac{dh_s}{dt} = QD(h_t, h_s) \cdot F_s^{-1} - QZ(t) \cdot F_s^{-1}, \\ \frac{dh_p}{dt} = -QR(h_t, h_p) \cdot F_p^{-1} - QT(h, h_p) \cdot F_p^{-1}, \\ \frac{dh}{dt} = QT(h, h_p) \cdot F_a^{-1}. \end{cases} \quad (5)$$

After the end of wastewater inflow from a pump chamber to an overflow chamber the process of steered wastewater flow from accumulation chamber can be started and lasts up to the moment of complete emptying. In hydraulic processes all the chambers take part, except the pump chamber which is already emptied. The level of filling in the chambers of storage reservoir can be calculated based on the following equations:

$$\begin{cases} \frac{dh_t}{dt} = QA(t) \cdot F_t^{-1} \cdot t - QD(h_t, h_s) \cdot F_t^{-1}, \\ \frac{dh_s}{dt} = QD(h_t, h_s) \cdot F_s^{-1} - QZ(t) \cdot F_s^{-1} + QS(h, h_s) \cdot F_s^{-1}, \\ \frac{dh}{dt} = -QS(h, h_s) \cdot F_a^{-1}. \end{cases} \quad (6)$$

The intensity of wastewater inflow to storage reservoir AQ changes with time and has the decisive importance for wastewater mass balance in reservoir chambers and hydraulic processes. Another important parameter is the reduced intensity of wastewater outflow from a steering chamber QZ , the momentary values of which can be calculated as follows:

$$QZ(t) = \beta \cdot QA(t), \quad (7)$$

$$QZ(t) = 2^{1/2} \cdot g^{1/2} \cdot \mu \cdot f\phi \cdot hs(t)^{1/2}, \quad (8)$$

where

$$\mu = (8 \cdot g \cdot l \cdot c_H^{-2} \cdot Do^{-1} + \sum \zeta + 1)^{1/2}. \quad (9)$$

The intensity of wastewater flow by choking pipe QD changes with time. Such changes are the function of wastewater filling level h_p in overflow chamber of reservoir. Maximum value of flow intensity through the choking pipe QD_{\max} as well as the maximum reduced wastewater outflow QZ_{\max} from steering chamber and maximum intensity of wastewater outflow from accumulation chamber during the emptying process must fulfill the following dependence:

$$QD_{\max}(h_{p \max}) = QZ_{\max}(h_{s \max}) = QS_{\max}(h_{\max}) = \beta_{\max} \cdot QA_{\max}. \quad (10)$$

The value of momentary intensity of wastewater outflow from accumulation chamber QS results from hydraulic characteristics of applied control device; this relationship most often has a non-linear character and is the function of accumulation chamber filling h and steering chamber filling h_s .

The intensity of wastewater flow through the edge between an overflow and pump chamber for the values of wastewater flow into the direction of pump chamber without the consideration of wastewater flow speed over the edge [7] is defined by equation (11) in the case of overflow functioning as unflooded, or by equation (12) if wastewater level in overflow chamber on the steering chamber side influences an overflow capacity:

$$QP = 0.943 \cdot \mu_1 \cdot b \cdot g^{0.5} \cdot (h_p - h_{pr})^{1.5}, \quad (11)$$

$$QP = 0.943 \cdot \mu_2 \cdot b \cdot g^{0.5} \cdot (h_t - h_p)^{1.5} + 1.41 \cdot \mu_3 \cdot b \cdot g^{0.5} \cdot (h_p - h_{pr}) \cdot (h_t - h_p)^{0.5}. \quad (12)$$

In the case of back-flow from a pump chamber to an overflow chamber, the above-mentioned relationships for flooded or unflooded overflow are described as follows [7]:

$$QR = 2.95 \cdot \mu_1 \cdot b \cdot g^{0.5} \cdot (h_p - h_{pr})^{1.5}, \quad (13)$$

$$QR = 2.95 \cdot \mu_2 \cdot b \cdot g^{0.5} \cdot (h_t - h_p)^{1.5} + 4.43 \cdot \mu_3 \cdot b \cdot g^{0.5} \cdot (h_p - h_{pr}) \cdot (h_t - h_p)^{0.5}. \quad (14)$$

The capacity of pump device located in a pump chamber of the reservoir results from its hydraulic characteristic which is individual for every type of device and installation. The demanded capacity of a pump system can be determined from mass balance equation for an overflow chamber:

$$QT_{\max} = QA_{\max} - QD_{\max}. \quad (15)$$

3. PARAMETERS OF SIMULATION MODEL OF RESERVOIR

In the course of the study, the mathematical model of storage reservoir operation was qualitatively characterized while defining a set of parameters that affect the simulation model. The analysis of the mathematical model of storage reservoir operation enabled their classification into suitable groups of parameters:

- The factors studied – this group of parameters includes those undergoing intentional changes during research process realization:

F_a – horizontal surface of the accumulation chamber, m^2 ;

F_p – horizontal surface of the pump chamber, m^2 ;

F_s – horizontal surface of the control chamber, m^2 ;

F_t – horizontal surface of the overfall chamber, m^2 ;

h_{\max} – maximum height of wastewater filling in the accumulation chamber, m;

h_o – level of pumping unit start and operation, m;

$h_{p \max}$ – maximum height of wastewater filling in the pump chamber, m;

$h_{s \max}$ – maximum height of wastewater filling in the control chamber, m;

β – momentary coefficient of wastewater flow reduction through a reservoir, –.

- Resulting factors obtained as a result of experimental object functioning constitute the basis for the assessment of hydraulic processes in simulation model and determine its characteristics:

h – momentary height of wastewater filling in the accumulation chamber, m;

h_p – momentary height of wastewater filling in the pump chamber, m;

h_s – momentary height of wastewater filling in the control chamber, m;

h_t – momentary height of wastewater filling in the overfall chamber, m;

QA – momentary intensity of wastewater inflow into the reservoir, m^3/s ;

QD – momentary intensity of wastewater flow into the throttling pipe, m^3/s ;

QP – momentary intensity of wastewater flow from overfall chamber to the pump chamber, m^3/s ;

QR – momentary intensity of wastewater flow from pump chamber to the overfall chamber, m^3/s ;

QS – momentary intensity of wastewater discharge from the accumulation chamber, m^3/s ;

QT – momentary wastewater flow forced by pump-system operation, m^3/s ;

QZ – momentary intensity of wastewater discharge from the reservoir, m^3/s .

• Constant factors – a group of parameters comprising those which do not change their values during a simulation experiment, or varying parameters the effect of which on the prepared simulation model of storage reservoir is not studied here:

g – terrestrial gravity, m/s^2 ;

l – length of the discharge channel from the reservoir, m ;

μ – flow rate coefficient at reservoir outflow opening, –;

ζ – local resistance factor, –.

4. CALCULATION ALGORITHM

The mathematical model of *EcoINFRA2* storage reservoir is the basis for the formulated simulation model which enables the detailed analysis of hydraulic parameters during the accumulation process in this reservoir.

The instrument developed on the basis of *Scilab* platform software gives the possibility of carrying out simulation experiments for any functions of wastewater flow. The calculation procedure is the next significant stage of computer software development for hydraulic processes' investigations in multichamber storage reservoirs serving hydraulic unloading of sewage systems.

Software instruments enable the analyses of hydraulic processes in storage reservoirs in dynamic systems for every chamber simultaneously in a definite order and sequence of operations determined by the hydraulic model. The transition to the successive stages of reservoir's functioning within sewage system takes place as a result of fulfillment of the boundary hydraulic conditions for particular phases in relation to the following parameters: h_{\max} , h_o , $h_{p \max}$ and $h_{s \max}$, which are analyzed relatively to the momentary values of hydraulic parameters in the scope of reservoir's chambers filling, determined by a calculation program.

A simulation program defines for changing time t the times of particular hydraulic processes t_1, t_2, \dots, t_n for the filling levels in particular chambers: h, h_t, h_s and h_p , and the intensity of wastewater flow QA, QD, QP, QR, QS, QT and QZ .

Calculation algorithm in the scope of storage reservoir's filling processes is presented in figure 1. The way of calculation program operation for the process of reservoir emptying is presented in figure 2.

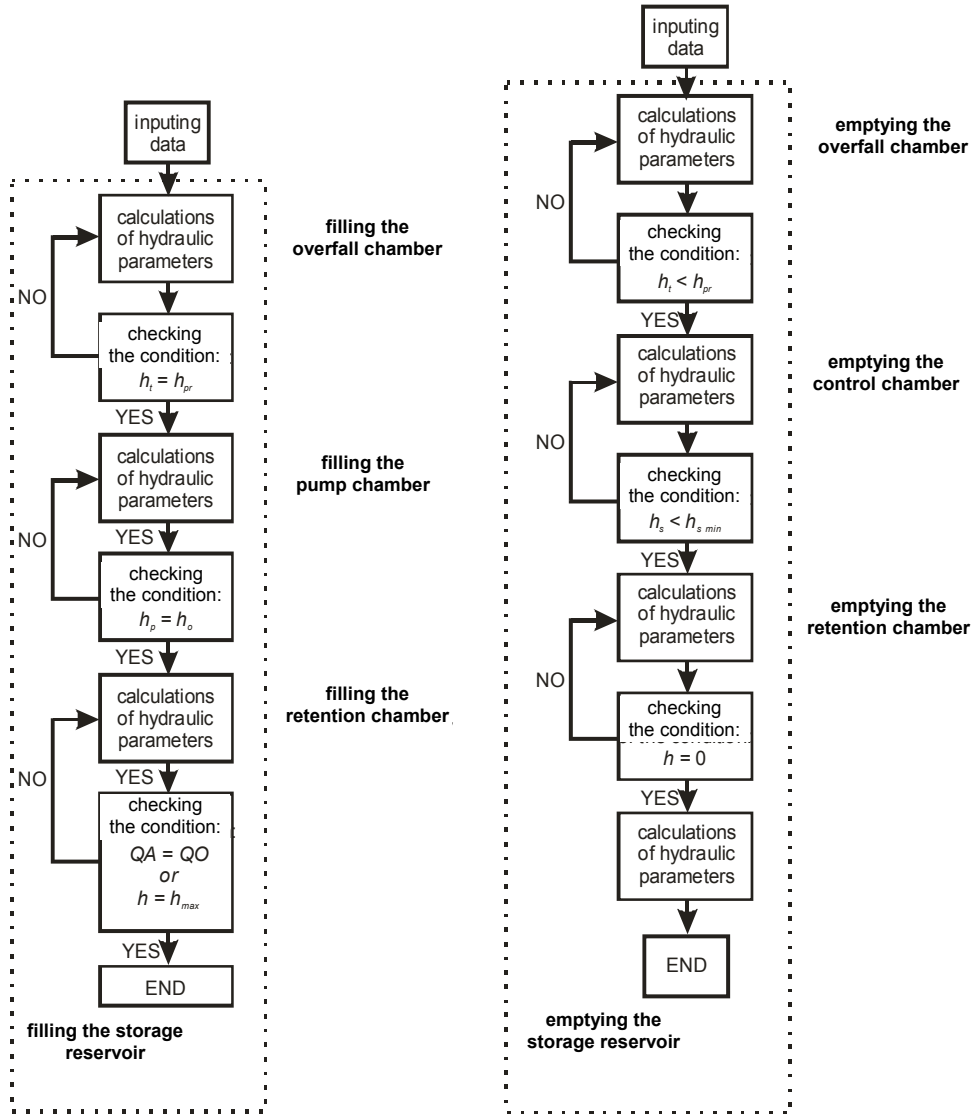


Fig. 1. Calculation algorithm of *EcoINFRA2* storage reservoir simulation model for the stage of its filling

Fig. 2. Calculating algorithm of *EcoINFRA2* storage reservoir simulation model for the stage of its emptying

5. SUMMARY

Sewerage infrastructure is under constant development and modernization together with the development of urban and rural areas. One of the most important technical

problems is wastewater flow regulation in sewage systems in order to reduce the hydraulic overloading and to increase wastewater treatment plant's efficiency. For these purposes different constructions of storage reservoirs with different hydraulic schemes and capacities of wastewater retention are used.

Taking into account the drawbacks of previously applied gravitational reservoirs constructions, which are characterized by limited levels of wastewater retention and great building surfaces, the theoretical bases of pump-gravitational schemes of reservoir chambers are developed [3].

One of such solutions is *EcoINFRA2* storage reservoir within a sewage system, the mathematical and simulation models of which are presented in this paper. The mathematical model of the reservoir is characterized by the features of white-box models. Parameters of mathematical and simulation models have the physical interpretation based on mass conservation law.

The simulation model enables detailed calculations of hydraulic parameters for dynamic system of filling levels changes in all chambers simultaneously. Such complex analysis together with the possibility of reservoir operation investigation for any wastewater inflow hydrograms defines the universal character of the developed simulation model and its usefulness in design practice.

REFERENCES

- [1] CALABRÒ P.S., VIVIANI G., *Simulation of the operation of detention tanks*, Water Research, 2006, 40, 83–90.
- [2] CALABRÒ P.S., *Cosmoss: conceptual simplified model for sewer system simulation*, Urban Water, 2001, 3 (1–2), 33–42.
- [3] DZIOPAK J., SŁYŚ D., *Modelowanie zbiorników klasycznych i grawitacyjno-pompowych w kanalizacji*, Politechnika Rzeszowska, Rzeszów, 2007.
- [4] DZIOPAK J., SŁYŚ D., *Physical and mathematical model of gravitation-pump storage reservoir in sewage system*, Environment Protection Engineering, 2006, Vol. 32, No. 2, 127–137.
- [5] DZIOPAK J., SŁYŚ D., *Hydraulic analysis of EcoINFRA2 storage reservoir functioning*, Environment Protection Engineering, 2008, Vol. 34, No. 1.
- [6] DZIOPAK J., SŁYŚ D., *Zbiornik retencyjny cieczy z regulowanym odpływem*, zgłoszenie patentowe nr P 378387, Urząd Patentowy Rzeczypospolitej Polskiej, Warszawa, 2005.
- [7] DZIOPAK J., *Analiza teoretyczna i modelowanie wielokomorowych zbiorników kanalizacyjnych*, Monografia 125, Politechnika Krakowska, Kraków, 1992.

MODELOWANIE MATEMATYCZNE ZBIORNIKA RETENCJNEGO *ECOINFRA2*

Przedstawiono model matematyczny działania zbiornika retencyjnego typu *EcoINFRA2*, który wraz z modelem hydraulicznym określającym brzegowe warunki hydrauliczne wszystkich faz jego działania stanowi postawę opracowanego narzędzia softwarowego umożliwiającego symulacyjne badania parametrów projektowych tego typu obiektów przy zadanych dowolnych funkcjach opisujących dopływ ścieków. Przedstawiono parametry modelu symulacyjnego oraz algorytm obliczeniowy jego działania.