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Generalized Stokes parameters of random electromagnetic quasi-homogeneous beams on propagation

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The expressions for the elements of the on-axis and transverse generalized Stokes parameters of random electromagnetic quasi-homogeneous beams on propagation are presented and are used to study the polarization properties of the beams. Some typical numerical calculations of the on-axis and transverse Stokes parameters, polarization properties of the beams on propagation are illustrated. The results show that for different sources all the on-axis parameters are identical in the near field and keep fixed values in the far field. But the transverse parameters are affected greatly by the properties of the source even in the near field. We have also found that the spatial profiles of the transverse parameters remain unchanged upon propagation.

Keywords: electromagnetic, quasi-homogeneous, generalized Stokes parameters, polarization.

1. Introduction

As is well-known, the polarization properties of an electromagnetic beam at a point in space can be determined by the use of the Stokes parameters since 1852 [1]. The Stokes parameters have been generalized from one-point quantities to two-point counterparts [2]. The spectral interference law that governs the behavior of the four Stokes parameters in Young's two-pinhole experiment with a random electromagnetic beam was derived in [3]. The changes in the probability density functions of the instantaneous Stokes parameters of a quasi-monochromatic electromagnetic beam propagating in free space have been explored [4]. A physical interpretation for the two-point Stokes parameters and how the interpretation related to a set of simple measurements with Young's interferometer were described in [5]. The experimental determinations of the generalized Stokes parameters have been reported [6, 7]. The generalized Stokes parameters have attracted much attention because they contain information about both the polarization and the coherence properties of the beams [1–10]. Since the so-called quasi-homogeneous (QH) sources [11] are important models for partially coherent sources which are found in nature or developed in laboratory, we will discuss the changes in the on-axis and transverse spectral Stokes parameters of electromagnetic light waves from the QH uniformly polarized beams on propagation. The on-axis and transverse polarization properties of the beams are also discussed by the use of the four Stokes parameters. And some interesting results are obtained since the transverse properties of the scattered field of the QH sources have not been investigated.

2. Theoretical analysis

According to the unified theory of coherence and polarization [12], the correlation properties of a QH source which are located at the source plane ($z = 0$) may be characterized by its cross-spectral density function:

$$W_{ij}^{(Q)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) = \langle E_i^{(Q)*}(\boldsymbol{\rho}_1', \omega) E_j^{(Q)}(\boldsymbol{\rho}_2', \omega)\rangle = \\ = \sqrt{S_i^{(Q)}(\boldsymbol{\rho}_1', \omega)}\sqrt{S_j^{(Q)}(\boldsymbol{\rho}_2', \omega)}\,\mu_{ij}^{(Q)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) \qquad (i, j = x, y) \quad (1)$$

Here $S_i^{(Q)}(\boldsymbol{\rho}', \omega) = W_{ii}^{(Q)}(\boldsymbol{\rho}', \boldsymbol{\rho}', \omega)$ is the spectral density of one component of the electric field, $\mu_{ij}^{(Q)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega)$ is the correlation coefficient between two components of the electric field in the source plane. The brackets denote the average taken over an ensemble of realizations $E^{(Q)}(\boldsymbol{\rho}', \omega)$ of the source distribution, and the asterisk denotes complex conjugation. The superscript Q denotes quantities pertaining to the source. $\boldsymbol{\rho}_1'$, $\boldsymbol{\rho}_2'$ are the position vectors of two points in the source plane. The conditions for a QH source to generate an electromagnetic beam were derived from [13]. By assuming that the source is uniformly polarized, it was verified that the far field of the beam generates and supports two reciprocity relations. For a QH electromagnetic source its spectral degree of coherence depends on $\boldsymbol{\rho}_1'$ and $\boldsymbol{\rho}_2'$ only through the difference $\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1'$. And its spectral density varies so slowly with this position that $S_i^{(Q)}(\boldsymbol{\rho}', \omega)$ is essentially constant. Moreover, the two components $S_x^{(Q)}(\boldsymbol{\rho}', \omega)$ and $S_y^{(Q)}(\boldsymbol{\rho}', \omega)$ are proportional to each other since the uniform polarization of the source, *i.e.*,

$$S_y^{(Q)}(\boldsymbol{\rho}', \omega) = \alpha S_x^{(Q)}(\boldsymbol{\rho}', \omega) \quad (2)$$

where α is a quantity depending only on frequency. So the cross-spectral density matrix $\mathbf{W}^{(Q)}$ of the source can be expressed as:

$$\mathbf{W}^{(Q)} = \mathbf{W}_{ij}^{(Q)}(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) = \alpha_{ij} S^{(Q)}\left(\frac{\boldsymbol{\rho}_1' + \boldsymbol{\rho}_2'}{2}, \omega\right)\mu_{ij}^{(Q)}(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', \omega) \quad (3)$$

where $S^{(Q)}(\boldsymbol{\rho}', \omega) = S_x^{(Q)}(\boldsymbol{\rho}', \omega) + S_y^{(Q)}(\boldsymbol{\rho}', \omega)$ is the spectral density of the electromagnetic field of the source, and

$$\alpha_{ij} = \begin{cases} \dfrac{1}{1+\alpha} & \text{when} \quad i = j = x \\ \dfrac{\alpha}{1+\alpha} & \text{when} \quad i = j = y \\ \dfrac{\sqrt{\alpha}}{1+\alpha} & \text{when} \quad i \neq j \end{cases} \tag{4}$$

Suppose now that the beam propagates in free space from the plane $z = 0$, which we call the source plane, into the half-space $z > 0$. Then, within the accuracy of the paraxial approximation, it follows that [14]

$$\langle E_i^*(\mathbf{r}_1, \omega) E_j(\mathbf{r}_2, \omega)\rangle = $$

$$= \iint_{z=0} \langle E_i^{(Q)*}(\boldsymbol{\rho}'_1, \omega) E_j^{(Q)}(\boldsymbol{\rho}'_2, \omega)\rangle \times K(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_1, \boldsymbol{\rho}_2 - \boldsymbol{\rho}'_2, z, \omega) \mathrm{d}^2\rho'_1 \mathrm{d}^2\rho'_2 \tag{5}$$

where

$$K(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_1, \boldsymbol{\rho}_2 - \boldsymbol{\rho}'_2, z, \omega) = G^*(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_1, z, \omega) \times G(\boldsymbol{\rho}_2 - \boldsymbol{\rho}'_2, z, \omega) \tag{6}$$

$$G(\boldsymbol{\rho} - \boldsymbol{\rho}', z, \omega) = \frac{ik}{2\pi z} \exp\left[-\frac{ik(\boldsymbol{\rho} - \boldsymbol{\rho}')^2}{2z}\right] \tag{7}$$

where $k = \omega/c$ is the wave number, c is the speed of light in vacuum. And $\mathbf{r} \equiv (\boldsymbol{\rho}, z)$ is a point of the electric field.

It is convenient to make changes of variables:

$$\boldsymbol{\rho}'_+ = \frac{\boldsymbol{\rho}'_1 + \boldsymbol{\rho}'_2}{2}, \qquad \boldsymbol{\rho}'_- = \boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_1 \tag{8a}$$

$$\boldsymbol{\rho}_+ = \frac{\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2}{2}, \qquad \boldsymbol{\rho}_- = \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1 \tag{8b}$$

By substituting Eq. (6) into Eq. (5) and using Eq. (8), we obtain the cross-spectral density matrix:

$$\mathbf{W} = W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega) = $$

$$= \left(\frac{k}{2\pi z}\right)^2 \iint_{z=0} \alpha_{ij} S^{(Q)}(\boldsymbol{\rho}'_+, \omega) \mu_{ij}^{(Q)}(\boldsymbol{\rho}'_-, \omega) \exp\left[-ik\frac{(\boldsymbol{\rho}'_+ + \boldsymbol{\rho}_+)(\boldsymbol{\rho}'_- + \boldsymbol{\rho}_-)}{z}\right] \mathrm{d}^2\rho'_+ \mathrm{d}^2\rho'_- \tag{9}$$

The generalized (or two-point) Stokes parameters were defined by the formulae [2]:

$$S_0(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E_x^*(\mathbf{r}_1, \omega) E_x(\mathbf{r}_2, \omega)\rangle + \langle E_y^*(\mathbf{r}_1, \omega) E_y(\mathbf{r}_2, \omega)\rangle \tag{10a}$$

$$S_1(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E_x^*(\mathbf{r}_1, \omega)E_x(\mathbf{r}_2, \omega)\rangle - \langle E_y^*(\mathbf{r}_1, \omega)E_y(\mathbf{r}_2, \omega)\rangle \tag{10b}$$

$$S_2(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E_x^*(\mathbf{r}_1, \omega)E_y(\mathbf{r}_2, \omega)\rangle + \langle E_y^*(\mathbf{r}_1, \omega)E_x(\mathbf{r}_2, \omega)\rangle \tag{10c}$$

$$S_3(\mathbf{r}_1, \mathbf{r}_2, \omega) = i\Big[\langle E_y^*(\mathbf{r}_1, \omega)E_x(\mathbf{r}_2, \omega)\rangle - \langle E_x^*(\mathbf{r}_1, \omega)E_y(\mathbf{r}_2, \omega)\rangle\Big] \tag{10d}$$

The spectral degree of polarization of the beam and the state of polarization of its polarized portion can be determined by $S(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega) \equiv S(\mathbf{r}, \mathbf{r}, \omega)$ with $\mathbf{r} \equiv (\boldsymbol{\rho}, z)$. The spectral degree of polarization $P(\boldsymbol{\rho}, z, \omega)$ can be calculated as follows:

$$P(\boldsymbol{\rho}, z, \omega) = \frac{\sqrt{S_1^2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega) + S_2^2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega) + S_3^2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)}}{S_0(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)} \tag{11}$$

The ellipticity angle ε is defined as $\varepsilon = \mathrm{atan}(A_{\mathrm{minor}}/A_{\mathrm{major}})$, where A_{minor} and A_{major} are the semi-axis sizes of the polarization ellipse. The orientation angle θ of the polarization ellipse and the ellipticity angle ε can be determined by:

$$\theta = \frac{1}{2}\mathrm{atan}\left[\frac{S_2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)}{S_1(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)}\right] \tag{12}$$

$$\varepsilon = \frac{1}{2}\mathrm{asin}\left[\frac{S_3(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)}{\sqrt{S_1^2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega) + S_2^2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega) + S_3^2(\boldsymbol{\rho}, \boldsymbol{\rho}, z, \omega)}}\right] \tag{13}$$

3. Numerical calculations and discussions

Let us assume that the spectral density $S^{(Q)}(\boldsymbol{\rho}', \omega)$ and the correlation coefficients $\mu_{ij}^{(Q)}(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', \omega)$ have the Gaussian form:

$$S^{(Q)}(\boldsymbol{\rho}', \omega) = A\exp\left(-\frac{\boldsymbol{\rho}'}{2\sigma^2}\right) \tag{14}$$

$$\mu_{ij}^{(Q)}(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', \omega) = B_{ij}\exp\left[\frac{(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1')^2}{2\delta_{ij}^2}\right], \quad (i = x, y; j = x, y) \tag{15}$$

where the coefficients A and B_{ij} are independent of the position but may depend on the frequency. By substituting Eqs. (14) and (15) into Eq. (9) the cross-spectral density matrix is derived:

$$\mathbf{W} = W_{ij}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z, \omega) =$$
$$= \frac{\alpha_{ij} A B_{ij}}{N_{ij}^2(z)} \exp\left[-\frac{(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)^2}{2\sigma^2 N_{ij}^2(z)}\right] \exp\left[-\frac{(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2M_{ij}^2 N_{ij}^2(z)}\right] \exp\left[-\frac{ik(\boldsymbol{\rho}_2^2-\boldsymbol{\rho}_1^2)}{2R_{ij}^2(z)}\right] \tag{16}$$

where

$$\frac{1}{M_{ij}^2} = \frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2} \tag{17a}$$

$$N_{ij}^2(z) = 1 + \frac{z^2}{k^2\sigma^2}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}\right) \tag{17b}$$

$$R_{ij}(z) = z\left[1 + \frac{k^2\sigma^2}{z^2}\left(\frac{1}{4\sigma^2} + \frac{1}{\delta_{ij}^2}\right)^{-1}\right] \tag{17c}$$

With the choice of $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$ the spectral Stokes parameters of the beams that radiated from the QH uniformly polarized electromagnetic sources can be determined by Eq. (10) while the degree of polarization and the state of the polarization of the beam will be determined by Eqs. (11)–(13).

By substituting Eqs. (16), (17) into Eq. (10) the on-axis spectral Stokes parameters are obtained. The behaviors of the on-axis spectral Stokes parameters $S_m(0, 0, z, \omega)$ ($m = 0, 1, 2, 3$) of the electromagnetic QH beams versus the propagation distance z for different values of correlation lengths are showed in Fig. 1. It is seen that all the on-axis spectral Stokes parameters change monotonously with the growing propagation distance z. All the Stokes parameters remain positive during the propagation and tend to be zero in the far field. We can also see that larger δ_{xx} (or δ_{yy}) and δ_{xy} (or δ_{yx}) result in lager S_0, S_2 and S_3.

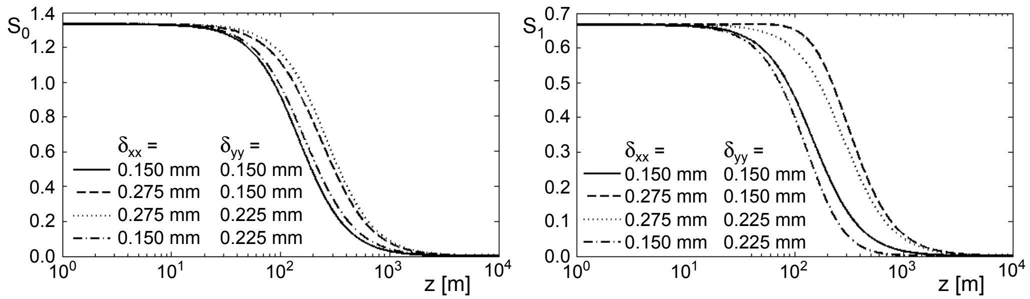


Fig. 1. To be continued on the next page.

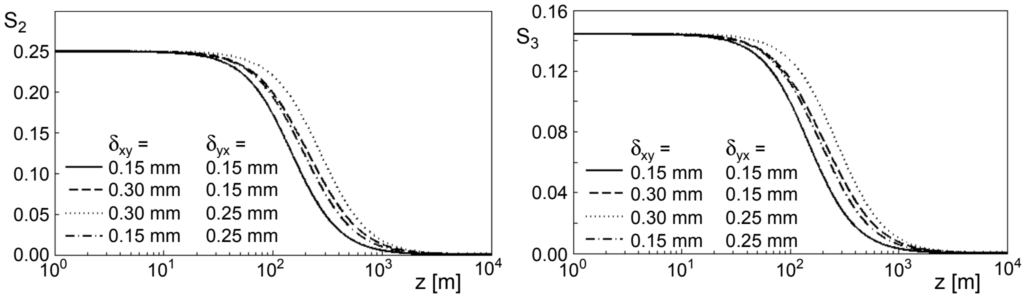
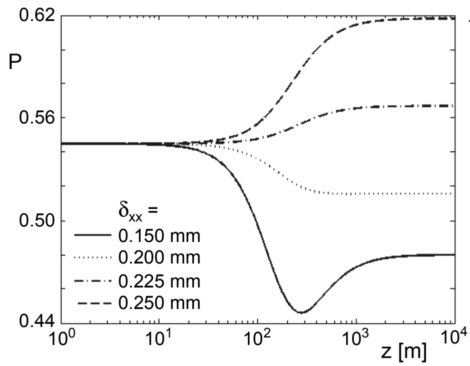
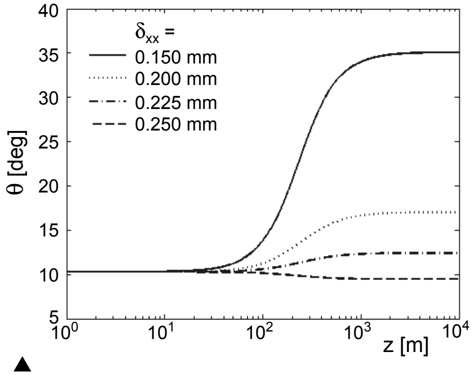


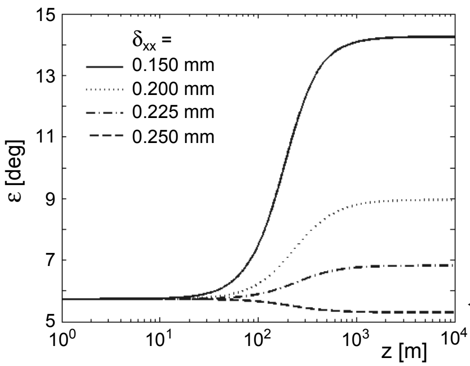
Fig. 1. The changes in the on-axis spectral Stokes parameters of the electromagnetic QH beams on propagation. The parameters are chosen as: $\alpha = 0.5$, $A = 1.5$, $B_{xx} = B_{yy} = 1$, $B_{xy} = 0.25\exp(i\pi/6)$, $B_{yx} = 0.25\exp(-i\pi/6)$, $\sigma = 1$ cm.



◀ Fig. 2



▲ Fig. 3



◀ Fig. 4

Fig. 2. The change in the on-axis spectral degree of polarization of the electromagnetic QH beams on propagation. The parameters are chosen as: $\alpha = 0.5$, $A = 1.5$, $B_{xx} = B_{yy} = 1$, $B_{xy} = 0.25\exp(i\pi/6)$, $B_{yx} = 0.25\exp(-i\pi/6)$, $\sigma = 1$ cm, $\delta_{yy} = 0.225$ mm, $\delta_{xy} = \delta_{yx} = 0.25$ mm.

Fig. 3. The change in the orientation angle θ along the z-axis of the electromagnetic QH beams on propagation. The source parameters are chosen the same as those in Fig. 2.

Fig. 4. The change in the ellipticity angle ε along the z-axis of the electromagnetic QH beams on propagation. The source parameters are chosen the same as those in Fig. 2.

By the use of the on-axis spectral Stokes parameters in Fig. 1 and Eqs. (11)–(13) the on-axis degree of polarization P, orientation angle θ and ellipticity angle ε are derived and depicted in Figs. 2–4, respectively. From Figures 2–4 one can see that the difference between δ_{xy} and the smallest of δ_{xx} and δ_{yy} plays a crucial part in determining the magnitude and variance of both ellipsometric quantities and the degree of polarization. Specifically, for $\delta_{xx} = 0.15$, the degree of polarization changes non-monotonously with increasing z. All the polarization properties will not tend to be zero but keep fixed values in the far field.

By using Equations (10), (16) and (17) the transverse spectral Stokes parameters $S_m(\rho, \rho, z, \omega)$ ($m = 0, 1, 2, 3$) are obtained. The influence of the source correlation

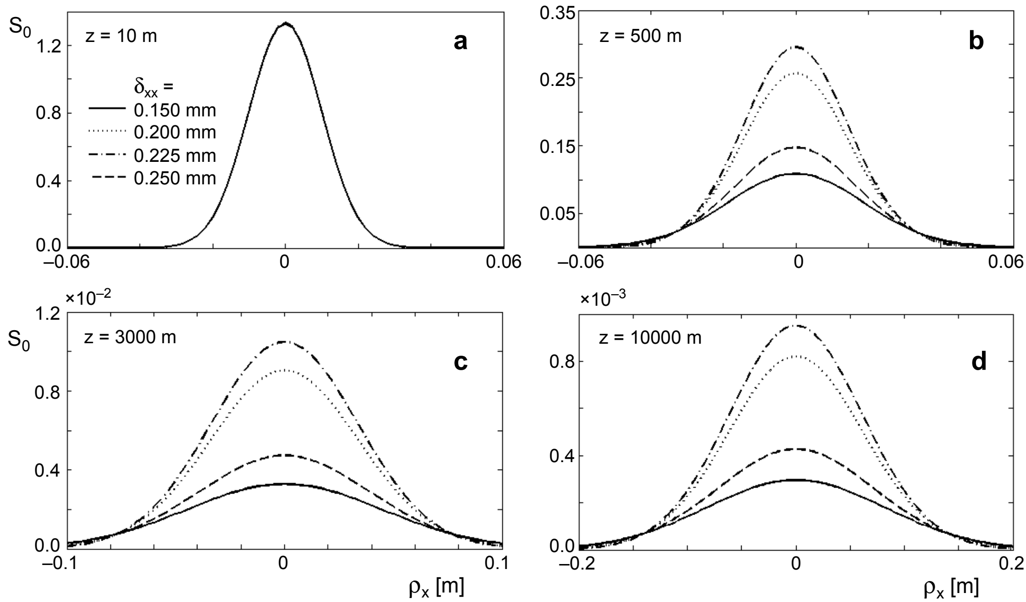


Fig. 5. The evolution of the transverse spectral Stokes parameters S_0 of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

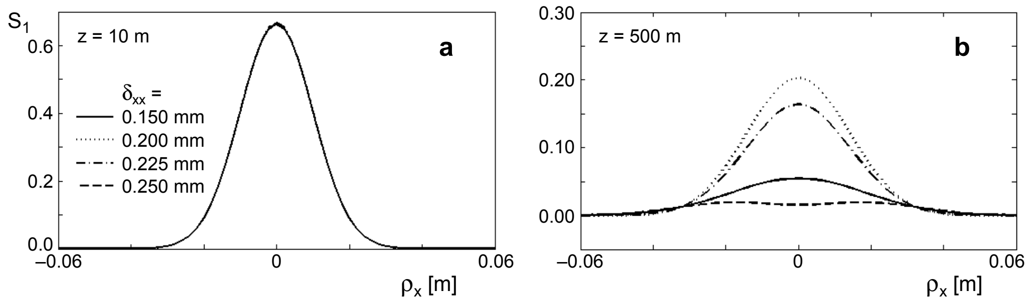


Fig. 6. To be continued on the next page.

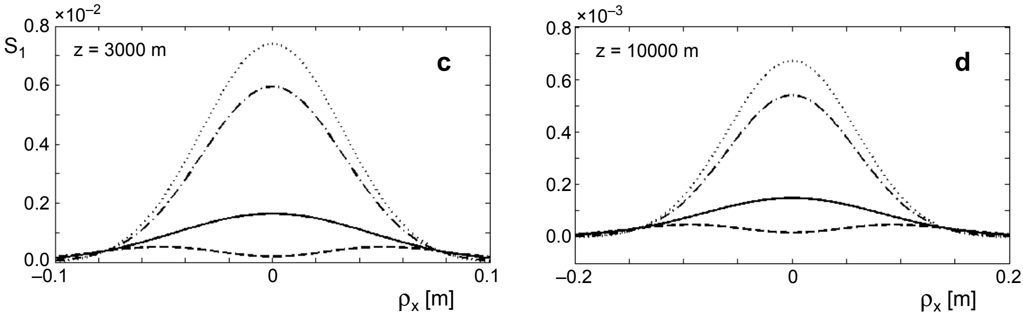


Fig. 6. The evolution of the transverse spectral Stokes parameters S_1 of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

coefficients and the propagation distance on the transverse spectral Stokes parameters is shown in Figs. 5–8. Figures 5**a**–8**a** indicate that the properties of the source have only a small or no effect on all the generalized Stokes parameters over short distances. From Figures 5, 7 and 8 we can see that the spatial profiles of transverse spectral Stokes parameters remain unchanged upon propagation, although the magnitude and the contrast decrease apparently. Parameter δ_{xx}, on the other hand, influences the magnitude rather than the shape of the transverse spectral Stokes parameters. Furthermore, it is clearly seen that the changes in the spectral Stokes parameters not only "propagate"

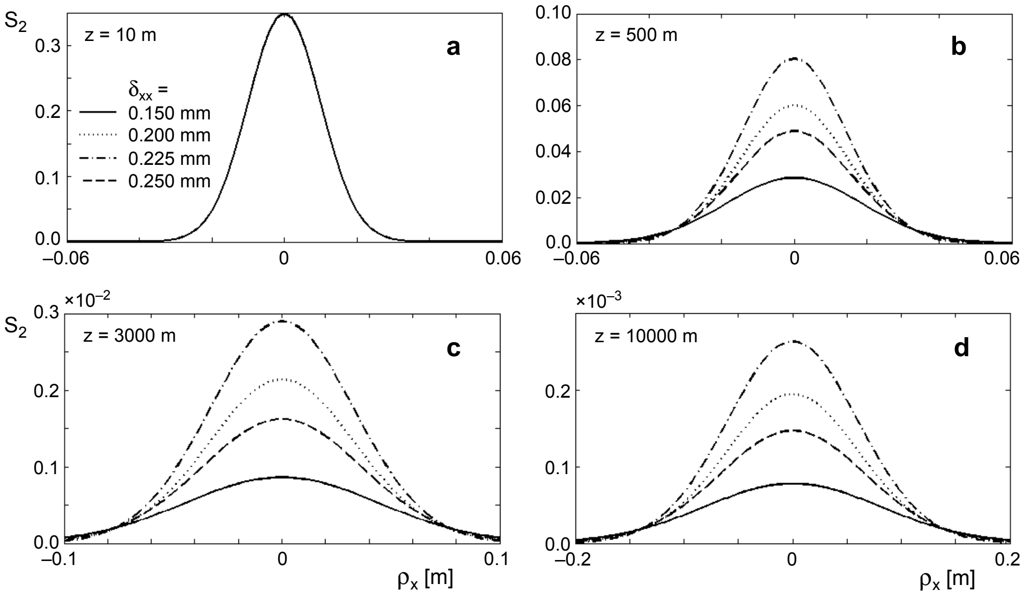


Fig. 7. The evolution of the transverse spectral Stokes parameters S_2 of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

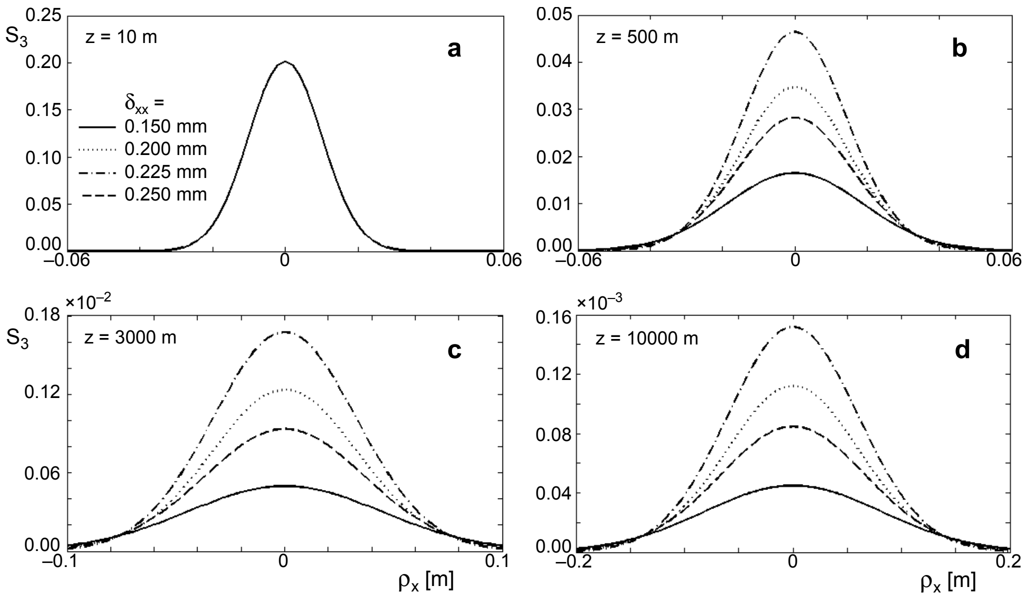


Fig. 8. The evolution of the transverse spectral Stokes parameters S_3 of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

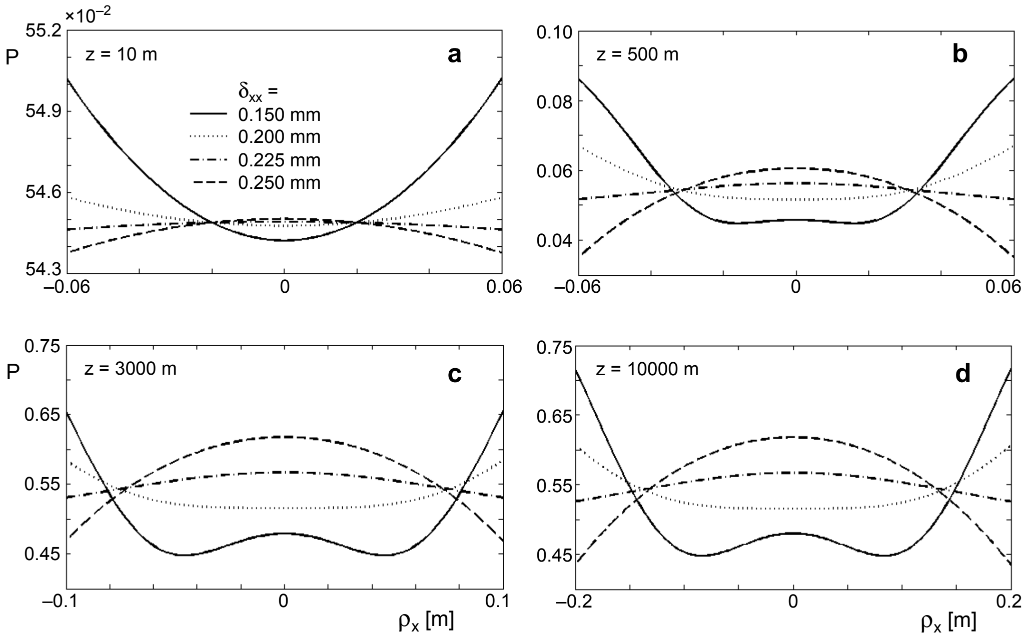


Fig. 9. The evolution of the spectral degree of polarization of the *x*-axis of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

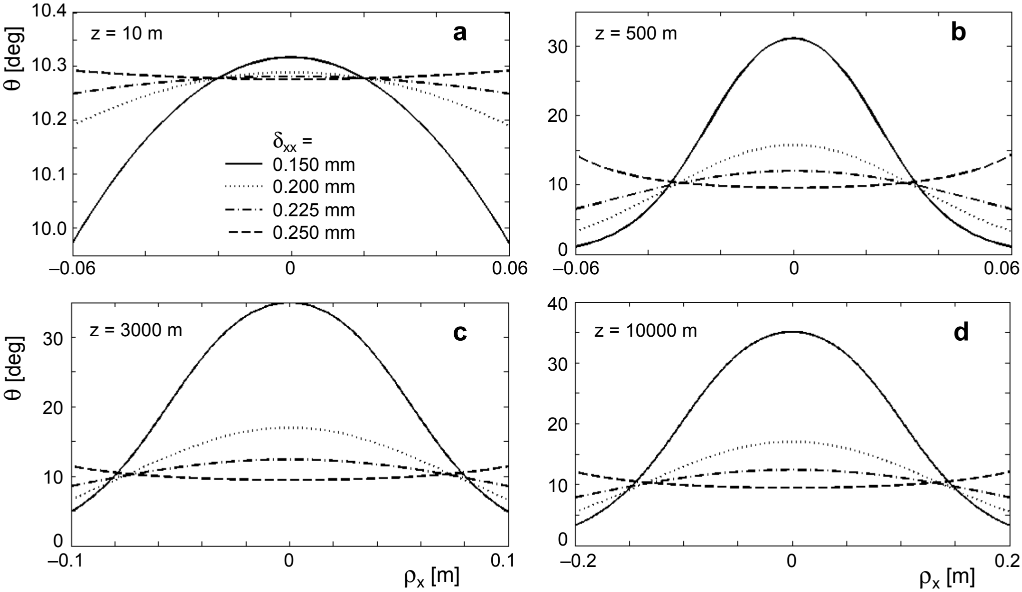


Fig. 10. The evolution of the orientation angle θ of the x-axis of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

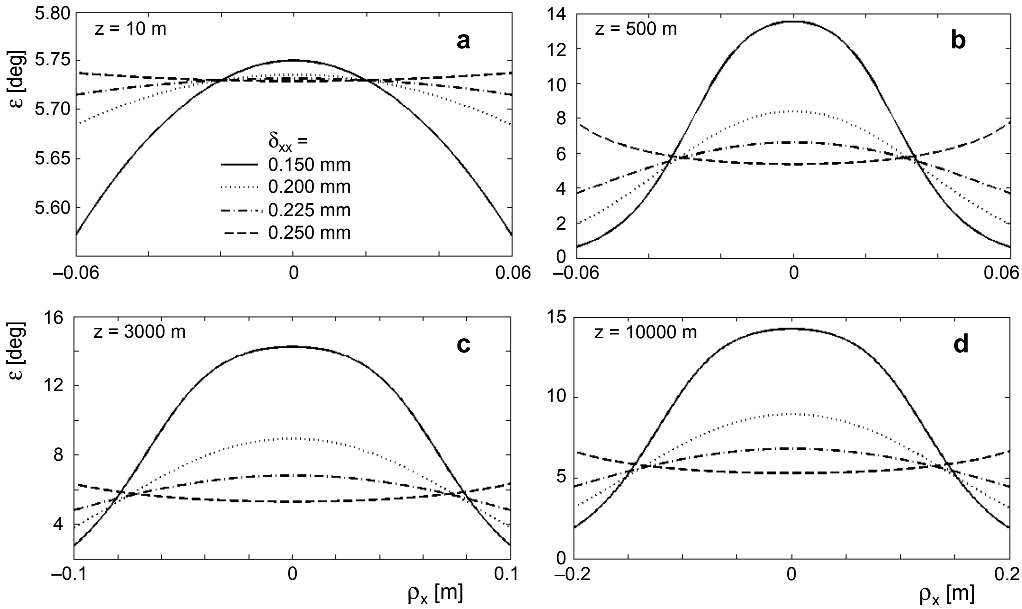


Fig. 11. The evolution of the ellipticity angle ε of the x-axis of the electromagnetic QH beams for different values of δ_{xx} on propagation. The source parameters are chosen the same as those in Fig. 2; $z = 10$ m (**a**), $z = 500$ m (**b**), $z = 3000$ m (**c**), $z = 10000$ m (**d**).

with distance z from the source but also “spread” with radial distance r from the axis of the beam.

By the use of the transverse spectral Stokes parameters in Figures 5–8 and Eqs. (11)–(13) the transverse polarization properties of the beams are obtained. The changes in the polarization properties along the x-axis and the evolution of the transverse polarization properties on propagation are showed in Figs. 9–11. Figures 9**a**–11**a** show that one can easily distinguish different sources by the use of the transverse polarization properties over short distances from the source, since the intensities due to different sources are identical over short distances (S_0 represents the spectral density). And one also cannot distinguish different sources by the use of the on-axis parameters because from Figs. 1–4 it is seen that the parameters are the same within short distances. This will be useful for the inversing problems from QH sources. From Figure 9 we can also see that the transverse degree of polarization is greatly affected by the properties of the QH beam. Figures 9–11 show that the spatial profiles of the transverse polarization properties remain unchanged upon propagation. Apart from the degree of polarization, the transverse orientation angle and ellipticity angle have their maximum or minimum value at the point of $\rho = 0$, *i.e.*, the point on the z-axis.

4. Conclusions

In conclusion, the expressions for the elements of the generalized Stokes parameters of random electromagnetic QH beams on propagation are presented. The degree of polarization, orientation angle and ellipticity angle can be determined by the Stokes parameters. With the help of numerical calculations we studied the changes in the on-axis and transverse Stokes parameters of the electromagnetic QH beams on propagation. The on-axis and transverse polarization properties of the beams are also discussed. For different sources all the on-axis parameters are identical in the near field and keep fixed values in the far field. The transverse degree of polarization and the state of polarization are affected greatly by the properties of the source even in the near field. So one can distinguish different sources by the transverse polarization properties easily even in the near field. And the spatial profiles of the transverse parameters remain unchanged upon propagation. This will be useful for inversing problems from the QH beams and for the properties of the QH beams.

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