Optica Applicata, Vol. XLIII, No. 2, 2013
DOI: 10.5277/oa130204

Malus law – interferometric interpretation

WŁADYSŁAW A. WOŹNIAK*, PIOTR KURZYNOWSKI, MARZENA ZDUNEK

Institute of Physics, Wrocław University of Technology,
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

*Corresponding author: wladyslaw.wozniak@pwr.wroc.pl

The aim of the present work is the description of a novel interferometric approach to the commonly known Malus law. In this approach we have described an analyzer as an element realizing the interference of two waves being the components of the linearly polarized wave emerging from the polarizer. We have proposed a decomposition of the polarization state of the light incident on the analyzer into two different bases. The choice of a first base – linearly polarized – allows interpreting Malus law as an interference of two linearly polarized waves with the same polarization state, different amplitudes and the same phases. The second decomposition, based on circularly polarized vectors, leads to the description in which Malus law can be interpreted as an interference of two waves with the same amplitudes but different phases. This allows the introduction of the concept of the geometric phase into Malus law as well as the visualization of this phase on the Poincaré sphere.

Keywords: Malus law, interferometry, geometric phase.

1. Introduction

Like all wave phenomena, light carries information was encoded not only in amplitude but also in its phase. This seems of high interest for most scientists. Simply speaking, this phase (or rather phase difference) is defined as the system's property, which can be seen by way of interference, as specific changes in an interfering waves intensity [1]. However, the phase difference between two (or more) interfering light beams may arise due to two reasons. Firstly, some optical elements can introduce their "own" phase shift as, for example, retardation plates used in anisotropic media optics – dynamical phase. Secondly, the mutual orientation of optical elements can introduce another kind of phase, called geometrical, which was firstly discovered by PANCHARATNAM [2] and investigated by BERRY [3] and many other authors [4–7]. These investigations consisted of both theoretical and practical [8, 9] experiments confirming the existence and importance of the geometric phase. Many interesting measurements and observations of the geometrical phase have been published so far also with the reference to quantum mechanics [10–14]. Classical Pancharatnam's idea as well as Berry's concept were strictly connected with the evolution of light polarization state (or quantum mechanical spin ½ particles) on a cyclic trajectory, *i.e.*, for the light of the same initial and final polarization states. Nevertheless, one can also observe and calculate the geometric phase in non-cyclic polarization changes [15].

There is a great deal of various descriptions of experiments in which geometric phase phenomena have been shown. Some of them used the language of quantum mechanics; some used the polarization formalism. The first description of the geometric phase phenomenon using the concept of birefringent medium's eigenwaves was made by COURTIAL [16]. The authors of the present paper have modified Courtial's idea [17] and extended it for the dichroic media [18]. The modification and extension were based on using both the fast and slow eigenwaves of the birefringent wave plate to describe the geometric phase changes in two specific polarization experiments. This also helps us drawing the specific constructions of the triangles on the Poincaré sphere, which simplified the geometric phase calculation. Some of these calculations were confirmed experimentally in our laboratory, some were verified using the results published in the literature. This convinced us about the correctness of the presented descriptions and also encouraged to try to describe another well-known classic polarization experiment in the geometric phase formalism (and visualize it using the triangle constructions on the Poincaré sphere). The experiment mentioned shows the changes in the intensity of the light passing through two polarizers (one of them usually called an analyzer) with mutual angular orientation. The results of this experiment are well-known to every follower of optics as Malus law [19, 20].

2. Malus law experiment – description

In the simplest version, for two ideal linear polarizers, this law states that the intensity I_{out} of the light after passing through the setup (polarizer + analyzer) is proportional to the square cosine functions of the difference between the first polarizer's (α_P) and analyzer's (α_A) eigenvectors' azimuth angles [21]:

$$I_{\text{out}} \propto \cos^2(\alpha_A - \alpha_P) \tag{1}$$

In the classic approach, such a change of the outcoming light intensity is explained by the analyzers attenuation. An ideal linear analyzer is a medium with linearly polarized eigenwaves from which one is propagated while the second fully attenuated. Therefore, the output light intensity results from the projection of the vector representing the polarization state of the light incident on the analyzer (and outcoming from the polarizer) onto the first eigenvector of the analyzer. On the other hand, the square cosine dependence of the output light beam intensity in Eq. (1) seems a typical element of all interferometric formulas where the cosine function argument (here: $\alpha_A - \alpha_P$) serves as the phase difference between two interfering waves. The question arises: which waves should be chosen to explain Malus law in terms of the phase difference? The choice of analyzer's eigenwaves as a possible base for postulated "interferometric interpretation" seems to be at a first sight an unsatisfactory; as it was mentioned, for an ideal analyzer only one of its eigenwaves is propagated while the second one is

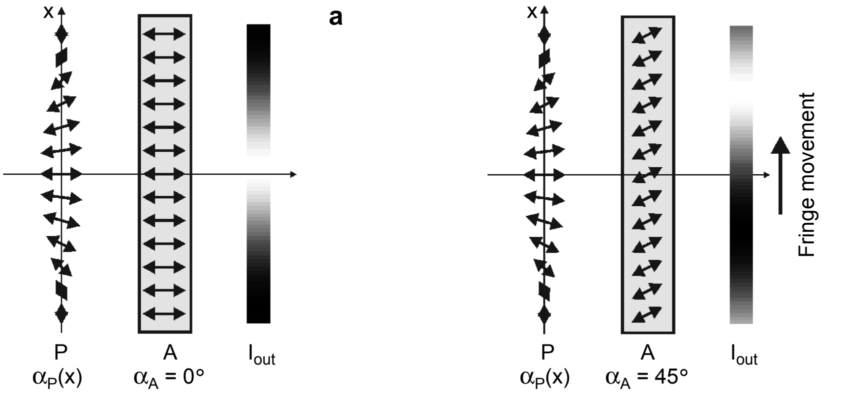

Fig. 1. Malus law experiment with spatial polarizer: fringe movement caused by rotation of the analyzer similar to that observed in interferometric experiments; analyzer with the azimuth angle $\alpha_A = 0°$ (**a**) and analyzer rotated to azimuth angle $\alpha_A = 45°$ (**b**).

fully attenuated and it is impossible to talk about the interference having only one wave emerging from the analyzer. Additionally, the desired phase difference cannot be derived from the dynamical phase of the medium – it is impossible to observe the phase difference between the analyzer's two eigenwaves bearing in mind that only one of these waves is propagated. So the only possible phase difference should be of geometrical character connected with the mutual geometry of the polarizer–analyzer system.

Let us describe the experiment in which the interference nature of Malus law is easy to observe (Fig. 1). We have used the special construction called a spatial polarizer [22] which allows generation of spatially varying polarization state distribution: linearly polarized lights with the continuous change of the azimuth angle. Placing a linear analyzer on such light's way, one can observe the variable light intensity distribution in accordance with Eq. (1) (constant α_A and spatially changeable α_P). Due to the specific construction of our spatial polarizer, this intensity distribution forms a set of parallel fringes. Rotating the analyzer (which means: changing the analyzer's azimuth angle α_A) causes the shift of the fringes set – the same shift, commonly observed in many interferometric experiment, which is always interpreted as a result of changes in the relative phase (phase shift) between two (or more) interfering waves. This might be a good starting point for supposing that the light intensity calculated with Malus law can actually be treated as a result of an interference thus making it possible to find the desired phase shift which we postulate as being geometrical.

3. Malus law experiment – interference of two waves with different amplitudes

Let us describe the above-mentioned Malus law experiment formally using the Jones vector formalism [21]. The Jones vector $\mathbf{E}_P$ of the light emerging from the polarizer can be written as:

$$\mathbf{E}_\mathrm{P} = \Big[\cos(\alpha_P), \sin(\alpha_P)\Big] \tag{2}$$

– linearly polarized, with the azimuth angle α_P, where α_P depends on x-coordinate in the case of our spatial polarizer. Analogically, the Jones vector of the first analyzer's eigenvectors can be written as:

$$\mathbf{A} = \Big[\cos(\alpha_A), \sin(\alpha_A)\Big] \tag{3}$$

According to Malus law the Jones vector $\mathbf{E}_\mathrm{out}$ of the light outcoming from the analyzer should be written as:

$$\mathbf{E}_\mathrm{out} = \cos(\alpha_A - \alpha_P)\mathbf{A} \tag{4}$$

– the polarization state of the light emerging from the analyzer is designated by the analyzer's first eigenvector while the intensity I_out is proportional to the square of the $\mathbf{E}_\mathrm{out}$ module (as in Eq. (1)). To obtain Eq. (4), one should "propagate" the polarization state $\mathbf{E}_\mathrm{P}$ of the light emerging from the polarizer through the analyzer, which can be formally written as the projection of the Jones vector $\mathbf{E}_\mathrm{P}$ onto the analyzer's first "transmission" eigenvector $\mathbf{A}$ (the second eigenvector is fully attenuated):

$$\mathbf{E}_\mathrm{out} = \langle \mathbf{E}_\mathrm{P}, \mathbf{A}\rangle \mathbf{A} \tag{5}$$

where $\langle \mathbf{E}_\mathrm{P}, \mathbf{A}\rangle$ denotes a scalar product of proper vectors. Before we start further calculations, let us look carefully at Eq. (2). This equation means, in fact, that vector $\mathbf{E}_\mathrm{P}$ is represented in a linear base consisting of two versors: $\hat{\mathbf{H}} = [1, 0]$ (x-axis, horizontal) and $\hat{\mathbf{V}} = [0, 1]$ (y-axis, vertical) and can be formally written as:

$$\mathbf{E}_\mathrm{P} = \langle \mathbf{E}_\mathrm{P}, \hat{\mathbf{H}}\rangle \hat{\mathbf{H}} + \langle \mathbf{E}_\mathrm{P}, \hat{\mathbf{V}}\rangle \hat{\mathbf{V}} = \cos(\alpha_P)\hat{\mathbf{H}} + \sin(\alpha_P)\hat{\mathbf{V}} \equiv \mathbf{E}_\mathrm{PH} + \mathbf{E}_\mathrm{PV} \tag{6}$$

where $\mathbf{E}_\mathrm{PH}$ and $\mathbf{E}_\mathrm{PV}$ denote the horizontal and vertical components of $\mathbf{E}_\mathrm{P}$ vector. Inserting Eq. (6) into Eq. (5), one can obtain the following formula:

$$\begin{aligned}\mathbf{E}_\mathrm{out} &= \Big[\cos(\alpha_P)\langle \hat{\mathbf{H}}, \mathbf{A}\rangle + \sin(\alpha_P)\langle \hat{\mathbf{V}}, \mathbf{A}\rangle\Big]\mathbf{A} = \\ &= \Big[\cos(\alpha_P)\cos(\alpha_A) + \sin(\alpha_P)\sin(\alpha_A)\Big]\mathbf{A}\end{aligned} \tag{7}$$

which leads (using simple trigonometric identity) immediately to Eq. (4). However, Eq. (7) shows that the output vector $\mathbf{E}_\mathrm{out}$ can be treated as a sum of two waves with the same linear polarization state:

$$\mathbf{E}_\mathrm{out} = \cos(\alpha_P)\cos(\alpha_A)\mathbf{A} + \sin(\alpha_P)\sin(\alpha_A)\mathbf{A} \equiv \mathbf{E}_\mathrm{outH} + \mathbf{E}_\mathrm{outV} \tag{8}$$

where $\mathbf{E}_\mathrm{outH}$ and $\mathbf{E}_\mathrm{outV}$ denote the horizontal and vertical components of the $\mathbf{E}_\mathrm{out}$ (see the middle part of Eq. (7)). For clarity, the graphical representation of the $\mathbf{E}_\mathrm{P}$ vector's

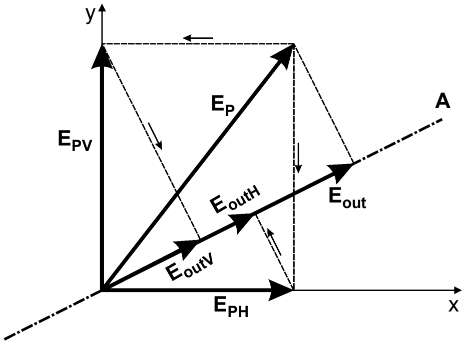

Fig. 2. The graphical representation of the $\mathbf{E}_P$ vector's decomposition into vertical ($\mathbf{E}_{PV}$) and horizontal ($\mathbf{E}_{PH}$) linear components as well as the projection of $\mathbf{E}_{PH}$ and $\mathbf{E}_{PV}$ components into vector **A** to obtain the vector $\mathbf{E}_{out}$ of the light emerging from the analyzer.

decomposition (described by Eq. (6)) as well as the projection of $\mathbf{E}_{PH}$ and $\mathbf{E}_{PV}$ components into vector **A** (described by Eq. (7) and Eq. (8)) is presented in Fig. 2. Finally, we can conclude that the Malus law equation (Eq. (1)) can be obtained as an effect of the interference of two linearly polarized waves ($\mathbf{E}_{outH}$ and $\mathbf{E}_{outV}$) with the same polarization states (represented by vector **A**), different amplitudes (dependent on the quantities α_P and α_A) and the same phases. The phases are, in fact, equal to zero as there are no phase terms in Eq. (8), which means we can assume them as $e^{i \cdot 0}$. This, at least, proved that Malus law can be explained in terms of interferometry. However, it does not provide us with the chance to find the postulated geometrical phase. So the question arises: is it possible to come up with another description of the experiment and explain the behavior of the light using the concept of the geometrical phase? Such an explanation, even though a bit sophisticated, could be interesting; note that opticians for many years had not been aware of the Pancharatnam's phase while conducting and explaining their interferometric experiments. However, the concept of the geometrical phase allowed approaching these experiments from a different angle and link wave optics phenomena to quantum physics. Thus, even if the presented geometric phase approach does not yield any direct practical applications, our new look at Malus law seems worth the hassle.

4. Malus law experiment – interference of two waves with different phases

The prerequisite for the proposed geometrical phase approach to Malus law is the construction of the specific triangles on the Poincaré sphere described previously by the authors [17, 18] for the birefringent medium placed in a polariscopic setup. According to Pancharatnam, the geometric phase can be calculated as a half of the solid angle on the Poincaré sphere formed by the points corresponding to transformation of the light's polarization state passing through the optical elements. According to COURTIAL'S [16] and ours [17, 18] works, the points, representing one or both birefringent medium's eigenvectors, can also be used to form this solid angle. To create the proper construction on the Poincaré sphere, we need at least three (or four, as in our modified construction described earlier [17]) points capable of forming specific spherical triangles. Because there is no birefringent medium in Malus law experiment, the points representing its eigenvectors cannot be used as the base of the postulated construction. What is more, it also applies to the points representing the analyzer's eigenvectors as well as to any other linear vectors – as all the points representing the linear polarization states lie on the Poincaré sphere equator, *i.e.*, the great circle, so they cannot form any triangles at all. Hence, the choice of the states $\hat{\mathbf{H}}$ and $\hat{\mathbf{V}}$ used in the output vector $\mathbf{E}_{\mathrm{out}}$ decomposition (described in Section 3) as the desired points would not render any solution. There is also an additional argument supporting this – the need to take into account the analyzer's attenuation (and therefore different rates of excitations, see Eq. (8)) would result in moving the construction points beyond the Poincaré sphere [23] which is inconsistent with the assumptions of Pancharatnam's constructions.

However, the problem can be possibly solved by using the decomposition of the output vector $\mathbf{E}_{\mathrm{out}}$ similar to that described in Section 3. To introduce the geometric phase as the result of the analyzer's rotation, one can choose another base in which the polarization state of the light incident on the analyzer can be decomposed. The choice of these vectors' base may be arbitrary as the laws of physics, in principle, are not choice-dependent. Nevertheless, this base is intended to ensure the necessary condition: the excitation coefficients of both basis vectors should be identical (and equal to one) for any vector $\mathbf{E}_{\mathrm{P}}$ of the light incident on the analyzer. This condition suggests immediately a possible solution. It is easy to show that all linear polarization states can be represented as a combinations of two circularly polarized states with opposite circularity and the same excitation coefficients. The azimuth angle of the decomposed $\mathbf{E}_{\mathrm{P}}$ linear state will depend on the mutual phase difference between circular states – the quantity we are, after all, looking for. As a consequence, finally we have proposed to choose as the desired base two circularly polarized polarization states: $\hat{\mathbf{R}}$ (right-handed) and $\hat{\mathbf{L}}$ (left-handed). The Jones vectors of these circularly polarized states can be written as:

$$\hat{\mathbf{R}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} \quad \text{and} \quad \hat{\mathbf{L}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix} \tag{9}$$

where i denotes an imaginary unit. Now the vector of the light $\mathbf{E}_{\mathrm{P}}$ emerging from the polarizer and falling on the analyzer can be written as:

$$\mathbf{E}_{\mathrm{P}} = \langle \mathbf{E}_{\mathrm{P}}, \hat{\mathbf{R}} \rangle \hat{\mathbf{R}} + \langle \mathbf{E}_{\mathrm{P}}, \hat{\mathbf{L}} \rangle \hat{\mathbf{L}} = \exp(-i\alpha_P)\hat{\mathbf{R}} + \exp(+i\alpha_P)\hat{\mathbf{L}} \equiv \mathbf{E}_{\mathrm{PR}} + \mathbf{E}_{\mathrm{PL}} \tag{10}$$

where $\mathbf{E}_{\mathrm{PR}}$ and $\mathbf{E}_{\mathrm{PL}}$ denote the right-handed and left-handed components of $\mathbf{E}_{\mathrm{P}}$ vector, respectively. Inserting Eq. (10) into Eq. (5), one can obtain the following formula in this case:

$$\mathbf{E}_{\text{out}} = \left[\exp(+i\alpha_P)\langle\hat{\mathbf{R}}, \mathbf{A}\rangle + \exp(-i\alpha_P)\langle\hat{\mathbf{L}}, \mathbf{A}\rangle\right]\mathbf{A} =$$

$$= \left\{\exp\left[-i(\alpha_A - \alpha_P)\right] + \exp\left[+i(\alpha_A - \alpha_P)\right]\right\}\mathbf{A} \tag{11}$$

which, after simple transformations, leads again to Eq. (4). However, Eq. (11) shows that we can treat the output vector $\mathbf{E}_{\text{out}}$ again as a sum of two waves with the same linear polarization states:

$$\mathbf{E}_{\text{out}} = \left\{\exp\left[-i(\alpha_A - \alpha_P)\right]\right\}\mathbf{A} + \left\{\exp\left[+i(\alpha_A - \alpha_P)\right]\right\}\mathbf{A} \equiv \mathbf{E}_{\text{outR}} + \mathbf{E}_{\text{outL}} \tag{12}$$

where $\mathbf{E}_{\text{outR}}$ and $\mathbf{E}_{\text{outL}}$ denote the right-handed and left-handed components of the $\mathbf{E}_{\text{out}}$, respectively. Thus we can conclude that the Malus law equation can be obtained again as an effect of the interference of two linearly polarized waves with the same polarization states (represented by vector $\mathbf{A}$) – only this time the component waves have the same amplitudes but different phases (dependent on α_P and α_A difference). These phases (or rather their difference) can be interpreted as a geometric phase; it depends only on mutual orientation of polarizer's and analyzer's azimuth angles (α_P and α_A). Again, for clarity, the graphical representation of the $\mathbf{E}_{\text{P}}$ vector's decomposition (described by Eq. (10)) as well as the projection of $\mathbf{E}_{\text{PR}}$ and $\mathbf{E}_{\text{PL}}$ components into vector $\mathbf{A}$ (described by Eqs. (11) and (12)) are presented in Fig. 3. Unfortunately, this graphical visualization is not as clear as in the previous case (see Fig. 2) due to the difficulties in presenting the circle vectors $\mathbf{E}_{\text{PR}}$ and $\mathbf{E}_{\text{PL}}$ (in fact, we can draw only their amplitudes and there is no simple way to present their mutual phases).

To complete our considerations, we should describe the proper construction of the characteristic spherical triangles on the Poincaré sphere illustrating the Pancharatnam

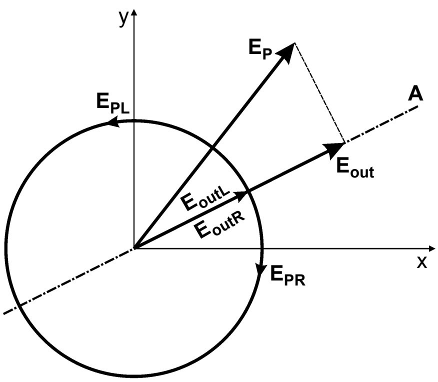

Fig. 3. The graphical representation of the $\mathbf{E}_{\text{P}}$ vector's decomposition into right-handed ($\mathbf{E}_{\text{PR}}$) and left-handed ($\mathbf{E}_{\text{PL}}$) circular components as well as the projection of $\mathbf{E}_{\text{PR}}$ and $\mathbf{E}_{\text{PL}}$ components into vector $\mathbf{A}$ to obtain the vector $\mathbf{E}_{\text{out}}$ of the light emerging from the analyzer.

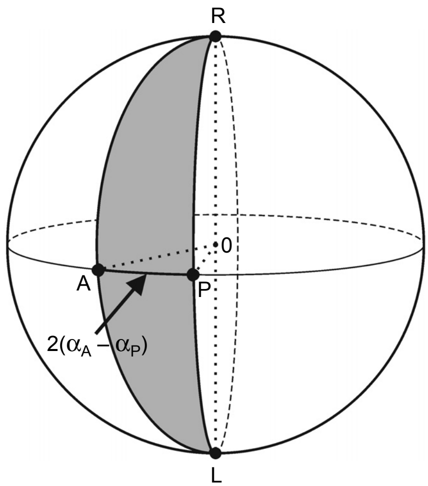

Fig. 4. Construction of the lune (two triangles) on the Poincaré sphere, which explains the geometric phase calculating method for Malus law experiment (see description in the text).

(geometric) phase. The extremely simple proper design in this case is presented in Fig. 4. The four points: P (representing the polarization state $\mathbf{E}_P$ of the light emerging from the polarizer), R and L (representing the "intermediate" polarization states $\hat{\mathbf{R}}$ and $\hat{\mathbf{L}}$) and finally A (representing the polarization state $\mathbf{A}$ of the first analyzer's eigenvector) form two triangles with common base forming a shape of the lune. The final element of our investigations should prove that the half of the area of the proposed figure (namely, half of the solid angle based on this figure) is equal to the postulated geometric phase. In fact, this becomes obvious if we take into account the well-known formula for the lune (section of the sphere) area – being proportional to the angular distance between the two points, P and A, and determining the lune's latitudinal width. For the normalized sphere, this area simply equals $2(\alpha_A - \alpha_P)$ – the doubled relative difference of the polarizer's and analyzer's azimuth angles.

5. Conclusions

The aim of the presented work is the description of a novel approach to the commonly known Malus law. The traditional approach assumes that the observed intensity changes of the light passing through a system of linear polarizer and analyzer with a variable angular orientation can be interpreted as an effect of the analyzer's eigenwaves attenuation. Our new description assumes that we can explain the above-mentioned changes in light intensity using the interferometry concepts. This approach means that we resigned from the eigenwave's description of the analyzer's behavior and instead proposed a decomposition of the incident light polarization state into two different new bases. The first base choice, linearly polarized, allows interpreting Malus law as an interference of two linearly polarized waves with the same polarization state, different amplitudes and the same phases (in fact, equal to zero). The second decomposition, using circularly polarized vectors as a base, leads to the description in which Malus law can be interpreted as an interference of two waves with the same amplitudes but different phases. This allows us to introduce the concept of the geometric phase into Malus law as well as to visualize this phase on the Poincaré sphere. The specific construction of the lune whose area is, following the original Pancharatnam idea, proportional to the desired geometric phase was drawn with the help of the points representing the chosen circular base. The analyzer's first eigenvector position was now used only to close this lune construction and to determine its width.

To sum up, we have described the analyzer as an element realizing the interference of two waves using the geometric phase formalism. Extension of the Courtial's idea described earlier [15, 16] proved useful also in the explanation of Malus law. Note that in previous descriptions of the experiments showing the existence of a geometric phase, the analyzer was treated only as an element "closing" the constructions of the corresponding triangles on the Poincaré sphere. Currently in our interpretation, the analyzer is treated as an element realizing the interference of two waves, which are the components of the wave leaving the polarizer. The explanation of analyzer's behavior in the Malus law experiment by way of introducing, admittedly slightly peculiar, geometric phase can help in the description of more complex systems (*e.g.*, a cascade of phase plates).

References

[1] HARIHARAN P., *The geometric phase*, [In] *Progress in Optics*, [Ed.] E. Wolf, Vol. 48, Elsevier, Amsterdam, 2005, pp. 149–201.

[2] PANCHARATNAM S., *Collected Works*, Oxford University Press, Oxford, 1975.

[3] BERRY M.V., *Anticipation of the geometric phase*, Physics Today **43**(12), 1990, pp. 34–40.

[4] LOVE G.D., *The unbounded nature of geometrical and dynamical phases in polarization optics*, Optics Communications **131**(4–6), 1996, pp. 236–240.

[5] CHYBA T.H., WANG L.J., MANDEL L., SIMON R., *Measurement of the Pancharatnam phase for a light beam*, Optics Letters **13**(7), 1988, pp. 562–564.

[6] BERRY M.V., KLEIN S., *Geometric phases from stacks of crystal plates*, Journal of Modern Optics **43**(1), 1996, pp. 165–180.

[7] HARIHARAN P., MUJUMDAR S., RAMACHANDRAN H., *A simple demonstration of Pancharatnam phase as a geometric phase*, Journal of Modern Optics **46**(9), 1999, pp. 1443–1446.

[8] VON BERGMANN J., VON BERGMANN H., *Foucault pendulum through basic geometry*, American Journal of Physics **75**(10), 2007, pp. 888–892.

[9] GIL S., *A mechanical device to study geometric phases and curvatures*, American Journal of Physics **78**(4), 2010, pp. 384–390.

[10] BHANDARI R., *SU(2) phase jumps and geometrical phases*, Physics Letters A **157**(4–5), 1991, pp. 221–225.

[11] WAGH A.G., RAKHECHA V.C., SUMMHAMMER J., BADUREK G., WEINFURTER H., ALLMAN B.E., KAISER H., HAMACHER K., JACOBSON D.L., WERNER S.A., *Experimental separation of geometric and dynamical phases using neutron interferometry*, Physical Review Letters **78**(5), 1997, pp. 755–759.

[12] GALVEZ E.J., CRAWFORD P.R., SZTUL H.I., PYSHER M.J., HAGLIN P.J., WILLIAMS R.E., *Geometric phase associated with mode transformations of optical beams bearing orbital angular momentum*, Physical Review Letters **90**(20), 2003, article 203901.

[13] BEN-ARYEH Y., *Berry and Pancharatnam topological phases of atomic and optical systems*, Journal of Optics B: Quantum and Semiclassical Optics **6**(4), 2004, pp. R1–R18.

[14] BLIOKH K.Y., *Geometrodynamics of polarized light: Berry phase and spin Hall effect in a gradient--index medium*, Journal of Optics A: Pure and Applied Optics **11**(9), 2009, article 094009.
[15] VISSER T.D., VAN DIJK T., SCHOUTEN H.F., UBACHS W., *The Pancharatnam–Berry phase for non--cyclic polarization changes*, Optics Express **18**(10), 2010, pp. 10796–10804.
[16] COURTIAL J., *Wave plates and the Pancharatnam phase*, Optics Communications **171**(4–6), 1999, pp. 179–183.
[17] KURZYNOWSKI P., WOŹNIAK W.A., SZARYCZ M., *Geometric phase: two triangles on the Poincaré sphere*, Journal of the Optical Society of America A **28**(3), 2011, pp. 475–482.
[18] KURZYNOWSKI P., WOŹNIAK W.A., *Geometric phase for dichroic media*, Journal of the Optical Society of America A **28**(9), 2011, pp. 1949–1953.
[19] GLADDEN S.C., *An experiment on Malus' law for the elementary laboratory*, American Journal of Physics **18**(6), 1950, pp. 395–395.
[20] TEWARSON S.P., *Law of Malus from phenomenological matrix calculus*, American Journal of Physics **36**(9), 1968, 841–842.
[21] BORN M., WOLF E., *Principles of Optics*, Cambridge University Press, Cambridge, 1999, pp. 25–38.
[22] WOŹNIAK W.A., KURZYNOWSKI P., *Compact spatial polariscope for light polarization state analysis*, Optics Express **16**(14), 2008, pp. 10471–10479.
[23] KURZYNOWSKI P., WOŹNIAK W.A., *Phase difference superposition rule for dichroic media*, Optik **103**(2), 1996, pp. 66–68.

Received October 31, 2012