On the spectral efficiency
of incoherent optical MIMO systems

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Spectral efficiency of incoherent optical MIMO systems is investigated. It is proved that under the best conditions (no channel crosstalk at the receiver) incoherent optical MIMO transmission gives potentially a better spectral efficiency than single channel transmission above a certain value of SNR in a single channel system, which varies from 9 dB for 2×2 MIMO to 12.3 dB for 4×4 MIMO. However, in practical optical 2×2 MIMO transmission, this border value of SNR is much higher and is around 25–30 dB.

1. Introduction

The MIMO technique, which is well-known from radio transmission, has also been applied in the optical fiber communications. One example are systems that employ few mode or multicore fibers [1] but at the beginning this technique was used in typical multimode (MM) fibers [2, 3]. There are two versions of the latter technique: using incoherent detection [2] and coherent one (COMIMO) [3]. The coherent technique is complicated and costly so we concentrate on the incoherent transmission. The latter technique makes use of the analogy between multipath propagation known in radio systems, and intermodal dispersion characteristic for MM fibers.

A generic block scheme of the incoherent MIMO system operating over MM fiber is depicted in Fig. 1. For simplicity we assume that the number of receivers is equal to the number of transmitters, and both equal n. Each of the transmitting lasers excites in the MM fiber (slightly) different modes. Furthermore, both the input optical coupler and the output optical splitter have mode selective properties (*i.e.*, to some extent they filter modes/mode groups). Therefore, each connection between i-th ($i = 1, 2, ..., n$) receiver and j-th ($j = 1, 2, ..., n$) transmitter is realized by different modes whose
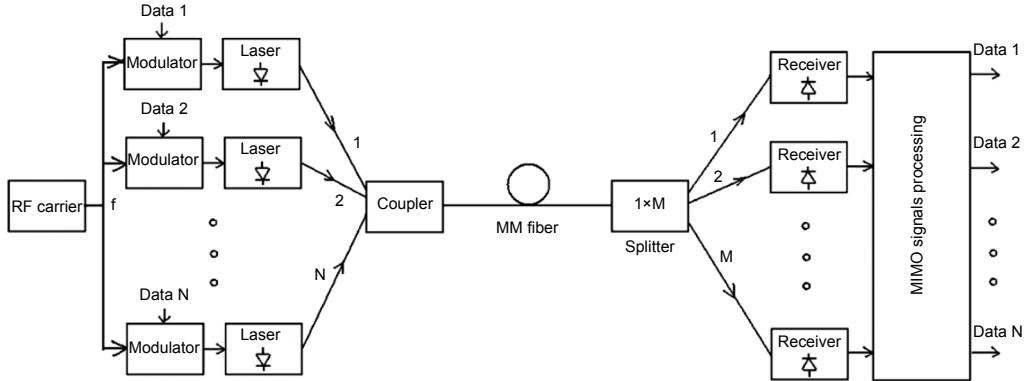
Fig. 1. Block scheme of the incoherent optical MIMO system operating over MM fiber.

powers depended on the (i, j) pair. The relation between two input signals $S = [s_1, s_2]^T$ and corresponding two output signals $R = [r_1, r_2]^T$ may be written as

$$R = H \cdot S \tag{1}$$

Here, $H = [h_{ij}]$ is the transmission matrix containing the transfer coefficients between all receiver/transmitter pairs. The matrix elements are complex numbers as they take into account both the attenuation and the phase shift of the RF carrier between i-th receiver and j-th transmitter. By inverting the matrix $H = [h_{ij}]$ we may recover the original signals (the so-called zero forcing (ZF) algorithm [4]):

$$S = H^{-1} \cdot R \tag{2}$$

Obviously, the matrix H may not be singular and its numerical conditioning directly determines the values of the recovered channels SNR [5]. The matrix H is usually measured at the system initialization stage. Apart from the ZF algorithm there are other methods of transmitted signals recovery [6].

2. Spectral efficiency of incoherent optical MIMO

Let us compare now the maximum theoretical bit rates, which can be obtained in a $n \times n$ incoherent optical MIMO system with that of a single channel optical system. As seen in Fig. 1, the MIMO system consists of n transmitters, n receivers, and two $n \times n$ couplers/splitters. On the other hand, the single channel system excludes the couplers/splitters, *i.e.*, it consists only of the transmitter and the receiver connected with an optical fiber. Let us assume that in both systems the transmitter laser power is the same and equals P_T. Furthermore, let us assume that the noise power at the receiver of each system is the same and equals N. Neglecting the additional coupler losses (*i.e.*, assuming an ideal wide band coupler), the total power launched into the fiber in

both cases (MIMO and single channel) is the same and equals P_T. Assuming again an ideal splitter at the receiver side, the optical power, P_{Rn}, at each optical receiver input is given by

$$P_{Rn} = \frac{P_T \exp(-\alpha L)}{n} \tag{3}$$

where α is the fiber attenuation, L is its length, $n = 1$ for the single channel system, and $n \geq 2$ for the MIMO system. Assuming that the photodetector sensitivity is R, and taking into account that the photodetector current is proportional to the light power (the photodetection process is essentially nonlinear), the electrical signal to the noise ratio after photodetection, $\rho_n = \mathrm{SNR}_n$, is given by

$$\rho_n = \frac{R^2 P_{Rn}^2}{N} = \frac{R^2 P_T^2 \exp(-2\alpha L)}{n^2 N} = \frac{\rho_1}{n^2} \tag{4}$$

where ρ_1 is SNR of the single channel system.

It is necessary to stress that Eq. (4) differs from the expression valid in radio systems [7–10] due to the photoreceiver nonlinearity, and distribution of the incoming optical power among the optical splitter branches. The greatest total throughput (spectral efficiency) of the MIMO system will be obtained in the absence of crosstalk between received channels. That corresponds to the situation when (somehow) the signal transmitted by the first laser is transferred only to the first receiver, the signal from the second laser is sent only to the second receiver, and so on. In this ideal case, we have n independent communication links characterized by the receiver SNR equal to ρ_n, and the total spectral efficiency C is then easily calculated from the Shannon rule:

$$C = n \log_2 \left(1 + \frac{\rho_1}{n^2} \right) \tag{5}$$

The maximum values of C against $\rho_1 = \mathrm{SNR}$ of a single channel system are shown in Fig. 2 for both MIMO and single channel systems. It is evident that the MIMO transmission potentially gives a better spectral efficiency than single channel transmission only above a certain ρ_1 value, which varies from ~9 dB for 2×2 MIMO, ~10.9 dB for 3×3 MIMO and ~12.3 dB for 4×4 MIMO. Figure 2 refers to the ideal case when no crosstalk is present between received channels. Unfortunately, this is rather impossible in real systems. In the presence of crosstalk, the transfer matrix H is no longer diagonal, and the total spectral efficiency of the MIMO system may be calculated according to [7, 10] as

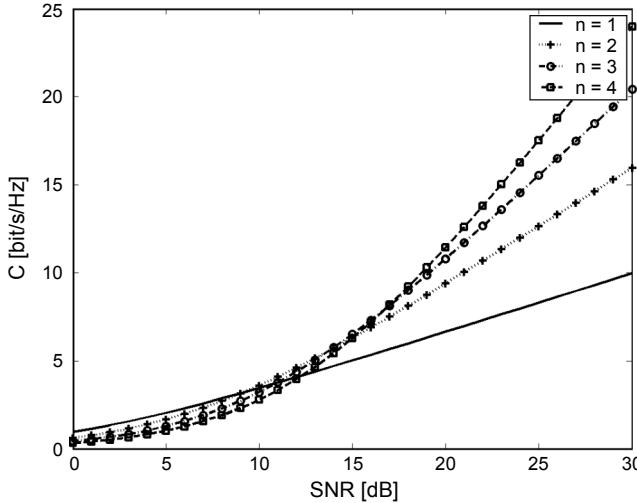$$C = \log_2 \det \left(I_n + \frac{\rho_1}{n^2} H \cdot H^+ \right) \tag{6}$$

Fig. 2. Maximum spectral efficiency C of incoherent optical MIMO system against SNR of a single channel system ρ_1 for various numbers of channels n.

Here det stands for determinant, I_n is a unit $n{\times}n$ matrix, H is the transfer matrix containing the (complex) power transfer coefficients at the receiver side (the fiber loss is included in ρ_1/n^2), and H^+ is the conjugate matrix transposition.
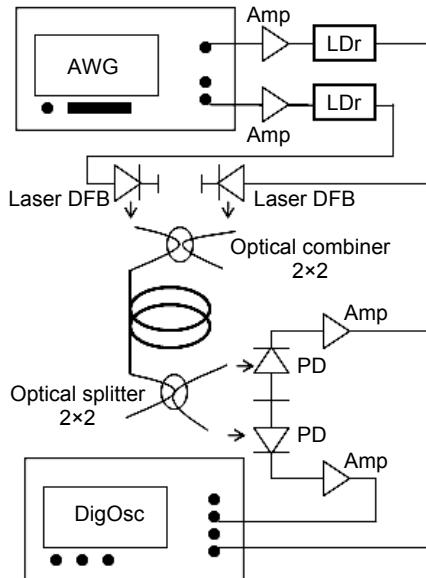
In the same way as was done in [7, 10], Eq. (6) may be rewritten to yield

$$C = \sum_{i=1}^{n} \log_2\left(1 + \frac{\rho_1}{n^2}\lambda_i\right) \qquad (7)$$

where λ_i are positive eigenvalues of $H \cdot H^+$.
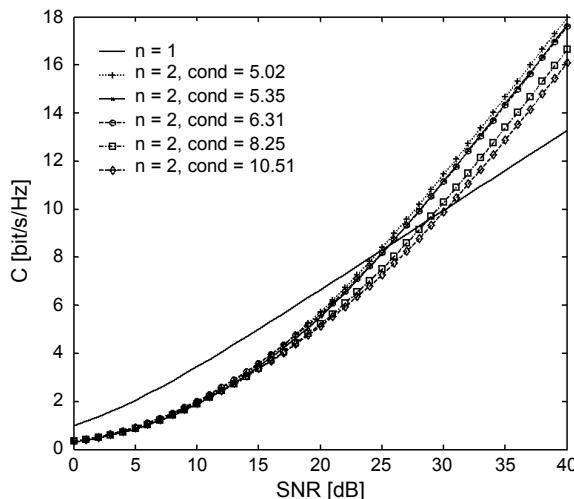
3. Experiment

The H matrices of real incoherent optical $2{\times}2$ MIMO system were measured in an experimental setup shown in Fig. 3. Arbitrary wave generator (AWG) was employed as a signal source for two channels of the MIMO system. The carrier frequency might be selected between 0.8 GHz and 1.5 GHz, which was well beyond the baseband of the MM fiber used (160 MHz), when generating signals oversampling ratio > 8 with regard to a carrier was employed. After amplification, the two signals from AWG drove two lasers (DFB, 1310 nm), which were coupled to two input ports of a $2{\times}2$ coupler/splitter. One of its output port was connected to the transmission MM fiber (OM2 grade, 50 µm core, 4.4 km long). At the receiving node, the optical signal from the fiber was fed to an input port of the second coupler/splitter, whose output ports were connected to two $p–i–n$ photodiodes. After photodetection and preamplification, two signals from the photodiodes were fed to a digital sampling

◀ Fig. 3. Block scheme of the experimental setup.

oscilloscope, where they were sampled and recorded for further (off-line) processing. In order to calculate elements of the transfer matrix *H* during the initialization stage, the unmodulated carriers were transmitted alternatively in both channels (*i.e.*, at a given time only one channel was on sending an unmodulated carrier). Simultaneously, the signals at both receivers were sampled and saved. This made it possible to determine the transfer matrix *H* elements during the off-line post processing.

Fig. 4. Real maximum spectral efficiencies calculated according to Eq. (7) against SNR of a single channel system for different real 2×2 MIMO system matrices that have various condition numbers (cond).

An additional 3rd channel was used as a reference that transmitted the carrier signal directly from AWG to the oscilloscope. The procedure was repeated a few times as the matrix H was not stable and slightly changed over time.

The values of maximum spectral efficiency calculated according to Eq. (7) for examples of measured real H matrices of a 2×2 system are shown in Fig. 4. They are compared with a single channel spectral efficiency. It was assumed that the transfer coefficient of a single channel system was an average taken from H matrix coefficients for a 2×2 system (sum of absolute values of matrix H elements divided by two). It is clearly visible that the 2×2 MIMO system has potentially better spectral efficiency only above a certain SNR value, which is fairly high (25–30 dB) for real systems. The corresponding values of spectral efficiency are above 10 bit/s/Hz. It means that incoherent optical MIMO systems perform better than single channel systems (in terms of spectral efficiency) only for very high SNR values at the receiver.

4. Conclusions

It was proved that under the best circumstances, the theoretical spectral efficiency of incoherent optical MIMO systems exceeds that of a single channel system above $\rho_1 = 9\ldots13$ dB (single channel SNR). For $\rho_1 = 20\ldots25$ dB the spectral efficiency increase (as compared to single channel) is 40%–110%, depending on the number of channels n, and ρ_1 itself. Unfortunately, the spectral efficiency of MIMO transmission depends on the matrix H realization and is not known *a priori*. For experimentally measured matrices, the MIMO advantage appears only for rather high values of $\rho_1 > 20\ldots25$ dB, whereas the single channel system offers better spectral efficiency for lower signal-to-noise ratios. Furthermore, due to the H matrix instability its elements should be frequently measured and updated, which involves system reinitialization. This adds to the higher cost and complicated signal processing of the MIMO systems.

The fundamental problem with incoherent MIMO transmission is high correlation of the matrix H elements in real systems. Unless some method of reliable decorreletion of matrix H elements is employed, the incoherent optical MIMO transmission is not a viable alternative to a single channel system operating beyond the MM fiber baseband. Unfortunately, such a method is not obvious. The solution may be to apply some form of modal multiplexing at the transmitter along with modal filtering at the receiver side, which corresponds to the transmission of different data over different modes (modal groups) and their separation at the receiver. Here, mode selective couplers/splitters may be employed at the output together with mode selective excitation at the input.

References

[1] RYF R., SIERRA A., ESSIAMBRE R., GNAUCK A.H., RANDEL S., ESMAEELPOUR M., MUMTAZ S., WINZER P.J., DELBUE R., PUPALAIKIS P., SUREKA A., HAYASHI T., TARU T., SASAKI T., *Coherent 1200-km 6×6 MIMO mode-multiplexed transmission over 3-core microstructured fiber*, 37th European Conference and Exhibition on Optical Communication (ECOC), 2011, article Th.13.C.1.

[2] STUART H.R., *Dispersive multiplexing in multimode optical fiber*, Science **289**(5477), 2000, pp. 281–283.

[3] TARIGHAT A., HSU R.C.J., SHAH A., SAYED A.H., JALALI, B., *Fundamentals and challenges of optical multiple-input multiple-output multimode fiber links*, IEEE Communications Magazine **45**(5), 2007, pp. 57–63.

[4] XU R., LAU F.C.M., *Performance analysis for MIMO systems using zero forcing detector over fading channels*, IEE Proceedings – Communications **153**(1), 2006, pp. 74–80.

[5] TSEKREKOS C.P., MARTINEZ A., HUIJSKENS F.M., KOONEN A.M.J., *Design considerations for a transparent mode group diversity multiplexing link*, IEEE Photonics Technology Letters **18**(22), 2006, pp. 2359–2361.

[6] VAN ZEMST A., *MIMO OFDM for Wireless LAN*, Ph.D. Thesis, Eindhoven University of Technology, Netherlands, 2004.

[7] HOLTER B., *On the capacity of the MIMO channel: a tutorial introduction*, [In] *Proceedings of IEEE Norwegian Symposium on Signal Processing*, Trondheim, Norway, October 18–20, 2001, pp. 167–172.

[8] SCHLEGEL C., BAGLEY Z., *MIMO Channels and Space-Time Coding*, WOC 2002, Banff, AB, Canada, July 18, 2002.

[9] LINDROOS P., *Postgraduate course in radio communications*, Communications Laboratory, Helsinki University of Technology, Finland, 2005.

[10] FOSCHINI G.J., GANS M.J., *On limits of wireless communications in a fading environment when using multiple antennas*, Wireless Personal Communications **6**(3), 1998, pp. 311–335.