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Hanna BORZĘCKA*

MULTI-CRITERIA DECISION MAKING USING FUZZY PREFERENCE RELATIONS

When dealing with multi-criteria decision making problems, the concept of Pareto-optimality and Pareto-dominance may be inefficient (e.g. generally multiple solutions exist), especially when there is a large number of criteria. Our paper considers the fuzzy multi-criteria decision making problem based on Zadeh's linguistic approach to *P*-optimality and *P*-dominance. The construction, analysis and application of a model of multi-criteria decision making using a fuzzy preference relation are considered. The paper is dedicated to the problem of modeling preferences in terms of fuzzy binary relations and provides an introduction to the important problem of forming fuzzy preference relations to analyze models of multi-attribute decision making. The key features of the multi-criteria evaluation, comparison, choice and ordering of alternatives in a fuzzy environment using fuzzy preference relations have been introduced.

Key words: *fuzzy sets, optimization, Pareto-optimal set*

1. Introduction. Concept of *P*-optimality in multi-criteria decision making problems

The concept of models of and solutions to multi-criteria decision making problems in a fuzzy environment was first presented by Bellman and Zadeh [5]. The most important advantage of this approach is its symmetry with respect to goals (criteria) and constraints which eliminated the differences between them and makes it possible to relate the concept of a decision process in a simple way as the intersection of goals and constraints. The clarity of the concept of "optimal solution" as the maximum de-

*Doctoral Student at the Cybernetics Department Military University of Technology, ul. Gen. Sylwestra Kaliskiego 2, 00-908 Warszawa 49, e-mail: hanna.borzecka@autograf.pl

gree of implemented goals (criteria) is very important in the Bellman–Zadeh approach. This enables finding the best solution in a clear and accessible way and avoids the paradigm of "ideal point" [34].

The aggregation of criteria is one concept of solution in multi-criteria decision making (MCDM) problems. The concept of Pareto-optimality in MCDM problems provides a classification of solutions to a multi-criteria optimization problem as dominated and non-dominated (Pareto-optimal).

In general, the concept of Pareto-optimality does not give a satisfactory solution (lack of a unique solution) to a MCDM problem and it may be impossible to derive the set of non-Pareto dominated alternatives. The concept of Pareto-optimality only gives a partial ordering in a space of alternatives. The decision maker must indicate the best solution and he has to take a decision "manually" from the *P*-optimal set. The concept of Pareto-optimality is inefficient in modeling a multi-criteria decision making process for economic and management science. It is often suitable and useful in engineering and projects where on average the number of criteria is small. The concept is less appropriate in many decision making problems, for instance in economics, management and social science where the number of criteria might be large. The portion p of the M-dimensional search space occupied by P-optimal solutions increases as the number of criteria increases as follows:

$$p = \frac{2^M - 2}{2^M} \tag{1}$$

where M is the number of criteria.

Hence, if $M \to \propto$, then $p \to 1$, i.e. essentially all the search space is P-optimal. It is clear that the concept of P-optimality is ineffective for a high number of criteria.

Inefficiency of this concept results also from the lack of the following information:

- The number of criteria according to which one of two solutions is better or equivalent,
- weights for the importance of each criterion.

The above consideration may be illustrated by an example of multi-criteria decision making with 30 criteria. This number of criteria does not occur frequently in technical problems but is often met in economics and social science. Suppose that in the set of possible decisions we have two decisions a_1 and a_2 such that according to 29 criteria a_1 is better than a_2 (i.e. $f_i(a_1) > f_i(a_2)$) and according to just one, the j-th criterion $f_j(a_1) < f_j(a_2)$) (for instance by a small amount δ). There is no doubt that all decision makers would choose a_1 as a better solution than a_2. However, according to Pareto's definition they are equivalent.

2. Concept of a solution to a multi-criteria decision making problem in a fuzzy environment based on Zadeh's fuzzy preference relation

The aggregation of fuzzy criteria is not the only solution to a multi-criteria decision making problem in a fuzzy environment. Zadeh proposed [33] a linguistic approach to the concepts of *P*-optimality and *P*-dominance. We are looking for a linear order in a *P*-optimal set or at least a restriction of this set, which becomes tighter as the number of criteria increases. Current methods are arbitrary and difficult to evaluate. Using the linguistic approach, the *P*-optimal set is fuzzified and its size is reduced. Zadeh's linguistic approach to *P*-optimality is based on a fuzzy preference relation determining the degree to which a solution is preferred to another according to a given criterion. Each element of a *P*-optimal set has a degree of membership to this set and the set becomes fuzzy *P*-optimal. The degree of membership to the *P*-optimal set is the complement of the degree of dominance by other solutions. The fuzzification of the *P*-optimal set has the result of eliminating those solutions which have a low degree of membership in the set.

The linguistic approach describes the dependences between decisions better than the AHP method. Zadeh's linguistic approach to decision analysis under multiple criteria is adapted to very complex systems or to systems which are not appropriate for quantitative analysis. The linguistic approach characterizes the fuzzy set of *P*-optimal solutions using a solution's degree of membership expressing the complement of the degree to which it is dominated by other elements.

A fuzzy preference relation denotes the degree ρ to which decision x_j – from the set of Pareto-optimal decisions – is preferred to x_k. Zadeh described ρ for $M = 2$ (two criteria):

If $\mu_{G1}(x_j)$ is much greater than $\mu_{G1}(x_k)$ and $\mu_{G2}(x_j)$ is approximately equal to $\mu_{G2}(x_k)$ and $\mu_{G1}(x_j)$ is approximately equal to $\mu_{G1}(x_k)$ and $\mu_{G2}(x_j)$ is much greater than $\mu_{G1}(x_k)$ then ρ is large.

Zadeh underlined that the linguistic approach to decision analysis is in the initial stages of development [33].

In the literature, the solution of multi-criteria optimization problems using fuzzy preference relations has been considered in a number of papers [3, 12, 14, 16, 17, 19, 21, 22, 23, 30, 32, 33, 35]. More detailed presentations of various aspects of the construction and modeling of fuzzy preference relations are given in [1, 2, 8, 15, 18, 20, 24–28, 31]. Some of the applications of fuzzy preference relations in solving FMDM problems may be found in [4, 6, 9, 11, 13].

3. Fundamental properties – the binary fuzzy relation

Fuzzy relations generalize the concept of relations in the same way as fuzzy sets generalize the idea of sets-by allowing a partial association between elements of a universe. The binary fuzzy relation between two non-empty and non-fuzzy sets is a fuzzy subset of the Cartesian product $X \times Y$, i.e.:

$$R = \left\{ \left\langle \left\langle x, y \right\rangle, \mu_R(x, y) \right\rangle : x \in X, y \in Y \right\} \tag{2}$$

where: $\mu_R : X \times Y \to [0, 1]$ is a membership function which assigns to each ordered pair $\langle x, y \rangle$, $x \in X$, $y \in Y$ the degree of membership $\mu_R(x, y)$, interpreted as the strength of the relationship between the elements $x \in X$ and $y \in Y$. $\mu_R(x, y) = 1$ means that the two elements x and y are fully related. $\mu_R(x, y) = 0$ means that the two elements x and y are completely unrelated.

If the sets X and Y are finite $(X = \{x_1, x_2, ..., x_m\}, Y = \{y_1, y_2, ..., y_3\})$, then the fuzzy relation can be expressed by a fuzzy matrix of dimension $m \times n$:

$$R = \begin{array}{ccccc} & (y_1) & (y_2) & \cdots & (y_n) \\ (x_1) & \mu_R(x_1, y_1) & \mu_R(x_1, y_2) & \cdots & \mu_R(x_1, y_n) \\ (x_2) & \mu_R(x_2, y_1) & \mu_R(x_2, y_2) & \cdots & \mu_R(x_2, y_n) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (x_m) & \mu_R(x_m, y_1) & \mu_R(x_m, y_2) & \cdots & \mu_R(x_m, y_n) \end{array} \tag{3}$$

where: $\mu_R(x_i, y_j) \in [0,1]$ is the degree of association between elements x_j and y_j in R.

If $X = Y$, then the fuzzy relation R is defined over the Cartesian product $X \times X$ with membership function:

$$\mu_R : X \times X \to [0, 1] \tag{4}$$

The fuzzy relation R on $X \times X$ may be presented as a directed graph with edges symbolizing the degree of association μ_{ij} between node x_i and x_j. A lack of edges between two nodes means that the degree of association between this pair of elements is zero (they are unrelated).

An n-ary fuzzy relation R between n non-empty and non-fuzzy sets $X_1, X_2, ..., X_n$ is defined as a fuzzy set over the Cartesian product $X_1 \times X_2 \times ... \times X_n$, i.e.:

$$R = \left\{ \left\langle \left\langle x_1, x_2, ..., x_n \right\rangle, \mu_R(x_1, x_2, ..., x_n) \right\rangle : x_1 \in X_1, x_2 \in X_2, ..., x_n \in X_n \right\} \tag{5}$$

where:

$$\mu_R : X_1 \times X_2 \times ... \times X_n \to [0, 1] \tag{6}$$

is the membership function of the relation R.

4. A discrete model of a fuzzy multi-criteria decision making problem

A non-fuzzy set A of decisions and a fuzzy binary preference relation R defined over the set A are given. The FMDM problem can be formulated as the problem of choosing a decision which has the maximum degree of non-dominance by others and can be presented as an ordered pair:

$$\langle A, R \rangle \tag{7}$$

where: $A = \{a_i\}_{i=\overline{1,N}}$ is non-fuzzy set of decisions, $\mathbf{R} = [R_1, R_2, ..., R_k, ..., R_K]$ – vector of fuzzy preference relations, K – the number of criteria, reflected by appropriate fuzzy preference relations, R_k $\left(k = \overline{1, K}\right)$ – binary fuzzy relation, hereafter called a fuzzy preference relation, defined over the Cartesian product $A \times A$ as follows:

$$R_k = \left\{ \left\langle \left\langle a_i,\ a_j \right\rangle, \mu_{R_k}(a_i,\ a_j) \right\rangle \ :\ \left(a_i,\ a_j\right) \in A \times A, \qquad i, j = 1, 2, ..., N \right\} \tag{8}$$

where $\mu_{R_k} : A \times A \to [0, 1]$ is the membership function of the k-th fuzzy preference relation, $\mu_{R_k}(a_i, a_j)$ – the degree of membership to the k-th fuzzy preference relation, (indicates the degree to which decision a_i is at least as good as decision a_j, according to relation R_k).

5. Solving the FMDM problem expressed by the model $\langle A, \boldsymbol{R} \rangle$

Solving the FMDM problem defined by the model $\langle A, R \rangle$ requires finding the fuzzy set of non-dominated decisions which indicates the degree of non-dominance of each decision by other decisions under all preference relations corresponding to the fuzzy criteria and then selecting a decision with the highest level of non-dominance.

The optimality criterion is to maximize the degree of the non-dominance of a decision which satisfies all the criteria reflected by the fuzzy preference relations.

In order to build a fuzzy set of non-dominated decisions, one should first create the following relations:

I. A fuzzy indifference relation

$$R^I = R \cap R^{-1} \tag{9}$$

where: R – a binary fuzzy preference relation, R^{-1} – the inverse (transpose) relation to R, $R^{-1}(a_i, a_j) = R(a_j, a_i)$ – according to Fodor, Rubens [16].

Following Orlovsky [22], the intersection of fuzzy sets is defined as a t-norm operator minimum, as follows:

$$\mu_{R^I}(a_i, a_j) = \min\left\{\mu_R(a_i, a_j), \mu_R(a_j, a_i)\right\} \tag{10}$$

Following other authors (for instance de Baets [1]), the intersection of fuzzy sets can be defined as any t-norm operator.

A fuzzy indifference relation is reflexive, symmetric and transitive.

II. A fuzzy strict preference relation is given by:

$$R^S = \frac{R}{R \cap R^{-1}} = \frac{R}{R^{-1}} \tag{11}$$

with a membership function given below:

$$\mu_{R^S}(a_i, a_j) = \max\left\{\left(\mu_R(a_i, a_j) - \mu_R(a_j, a_i)\right), 0\right\} \tag{12}$$

After the pioneering research of professor Orlovsky, some of his work was continued and developed by other authors. The following are among the most relevant contributions to approaches to defining the fuzzy preference relation:

- by Ovchinikov [24]:

$$\mu_{R^S}(a_i, a_j) = \begin{cases} \mu_R(a_i, a_j) \;\; if \;\; \mu_R(a_i, a_j) > \mu_R(a_j, a_i) \\ 0 \end{cases} \tag{13}$$

• by Roubens [28]:

$$\mu_{R^S}(a_i, a_j) = \mu_R(a_i, a_j) \wedge_T \left(1 - \mu_R(a_j, a_i)\right) \tag{14}$$

where: $\wedge_T$ – is a *t*-norm operator of conjunction.

A fuzzy strict preference relation is anti-reflexive and transitive.

III. A fuzzy non-dominance relation. Consider a single preference relation $R = [R]$, which can be built into a strict preference relation R^S. If $\left\langle a_i, a_j \right\rangle \in R^S$, then a_i is strictly better than a_j or element a_i dominates element a_j (according to the relation R). The membership function $\mu_{R^S}(a_i, a_j)$ indicates the degree to which a_i dominates a_j (according to the relation R). $\mu_{R^S}\left(a_j, a_i\right)$, $\forall a_i \in A$ denotes the membership function of the fuzzy set of all elements a_i, strictly dominated by a_j. The complement of this fuzzy set (i.e. $\overline{R^S}(a_j, a_i) = 1 - R^S(a_j, a_i)$, $\forall a_i \in A$) is the fuzzy set of decisions not dominated by a_j. In consequence, the intersection of all $\overline{R^S}(a_j, a_i)$, $\forall a_j \in A$ represents the fuzzy set of decisions which are not strictly dominated by others. This fuzzy set is called a fuzzy non-dominance relation R^{ND} with membership function (following Orlovsky [22]):

$$\mu_{R^{ND}}(a_i) = \min_{a_j \in A} (1 - \mu_{R^S}(a_j, a_i)) = 1 - \max_{a_j \in A} (\mu_{R^S}(a_j, a_i)) \tag{15}$$

The value $\mu_{R^{ND}}(a_i)$ represents the degree to which the element a_i is not dominated by any of the elements of the set A. In this way, Eq. (15) allows us to evaluate the degree of non-dominance of each decision by other decisions.

Optimal decision. An optimal decision a^{ND} in the set A is a decision which achieves the highest value of the membership function (highest level of non-dominance) in the fuzzy set of non-dominated decisions $R^{ND} = \left\{\left\langle a, \mu_{R^{ND}} \right\rangle : a \in A\right\}$ and following Orlovsky [22] can be expressed as:

$$a^{ND} = \left\{a_i^{ND} : a_i^{ND} \in A; \mu_{R^{ND}}(a_i^{ND}) = \max \mu_{R^{ND}}(a_i)\right\} \tag{16}$$

In particular, if $\max \mu_{R^{ND}}(a_i) = 1$, then the set of decisions:

$$a^{NFND} = \left\{ a_i^{NFND} : a_i^{NFND} \in A, \mu_{R^{ND}}(a_i^{NFND}) = 1 \right\} \tag{17}$$

is non-fuzzy and non-dominated.

Example 1 (by Orlovsky [22])

Let us consider the fuzzy preference relation R, defined over the set of decisions $A = \{a_1, a_2, a_3, a_4\}$. The optimal decision is defined to be the decision providing the highest level of non-dominance by other decisions

$$\mu_R(a_i, a_j) = \begin{array}{c|cccc} & (a_1) & (a_2) & (a_3) & (a_4) \\ \hline (a_1) & 1 & 0.2 & 0.3 & 0.1 \\ (a_2) & 0.5 & 1 & 0.2 & 0.6 \\ (a_3) & 0.1 & 0.6 & 1 & 0.3 \\ (a_4) & 0.6 & 0.1 & 0.5 & 1 \end{array}$$

Using Eq. (12), we obtain the membership function of the fuzzy strict preference relation R^S:

$$\mu_{R^S}(a_i, a_j) = \begin{array}{c|cccc} & (a_1) & (a_2) & (a_3) & (a_4) \\ \hline (a_1) & 0 & 0 & 0.2 & 0 \\ (a_2) & 0.3 & 0 & 0 & 0.5 \\ (a_3) & 0 & 0.4 & 0 & 0 \\ (a_4) & 0.5 & 0 & 0.2 & 0 \end{array}$$

Applying Eq. (15), we obtain the membership function of the fuzzy set of the non-dominance relation R^{ND}:

$$\mu_{R^{ND}} = \begin{bmatrix} 0.5 & 0.6 & 0.8 & 0.5 \end{bmatrix}.$$

On the basis of Eq. (16), we have:

$$a^{ND} = \{a_3\}$$

The optimal decision is decision a_3, because it attains the highest degree of non-dominance $\mu_{R^{ND}}(a_3) = \max\limits_i(\mu_{R^{ND}}(a_i)) = 0.8$ in the fuzzy set of non-dominated decisions R^{ND}.

On the basis of Eq. (8), we have:

$$a^{NFND} = \{\varnothing\}$$

None of decisions attain the degree of non-dominance $\mu_{R^{ND}} = 1$ in the fuzzy set of non-dominated alternatives. This means that there is no non-fuzzy solution to the problem expressed in terms of fuzzy sets.

Expressions (11), (15) and (16) may be also considered for a vector **R** of a fuzzy preference relation in the following way:

Method I (following Orlovsky [23]). The global (aggregated) preference relation R^G is a fuzzy set defined to be the intersection of all preference relations:

$$\mathbf{R}^G = \bigcap_{k=1}^{K} R_k \tag{18}$$

with a membership function:

$$\mu_{\mathbf{R}^G}(a_i, a_j) = \min_{1 \le k \le K} R_k(a_i, a_j), \qquad a_i, a_j \in A \tag{19}$$

When using the minimum as the operator defining the intersection of fuzzy sets $R_k, \left(k = \overline{1, K}\right)$, the set a^{ND} plays the role of a Pareto-optimal set [23].

Method II. Following other authors, e.g. Ekel, Neto [11], Ekel, Martini, Palhares, [12], applying Eq. (15) as follows:

$$\mu_{R^{ND}}(a_i) = 1 - \max_{a_j \in \mathrm{A}}(\mu_{R_k^S}(a_j, a_i)),\ k = \overline{1, K} \tag{20}$$

allows us to construct the membership function of the fuzzy set of non-dominated decisions for each fuzzy preference relation **R**.

The fuzzy sets $R_k^{ND}(a_i),\ k = \overline{1, K}$ may next be aggregated using the intersection operation for all K criteria:

$$\mu_{R^{ND}}(a_i) = \min_{k = \overline{1, K}} \mu_{R_k^{ND}}(a_i) \tag{21}$$

Example 2

$$\mathbf{A} = \{a_1, a_2, a_3\}$$

The decisions from set **A** are evaluated according to three criteria ($K = 3$), which correspond to the following preference relations:

$$\mu_{R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0.88 & 0.88 & 1 \end{bmatrix}, \quad \mu_{R_2} = \begin{bmatrix} 1 & 0.88 & 0.24 \\ 1 & 1 & 0.88 \\ 1 & 1 & 1 \end{bmatrix}, \quad \mu_{R_3} = \begin{bmatrix} 1 & 1 & 1 \\ 0.88 & 1 & 1 \\ 0.88 & 1 & 1 \end{bmatrix}$$

One should select the optimal decision, which in the maximal way fulfils all the criteria expressed by the fuzzy preference relations.

Solution by method I (following Orlovsky [23])

$$\mathbf{R}^G = \bigcap_{k=1}^{3} R_k \tag{22}$$

Applying (19) we obtain:

$$\mu_{R^G} = \begin{bmatrix} 1 & 0.88 & 0.24 \\ 0.88 & 1 & 0.88 \\ 0.88 & 0.88 & 1 \end{bmatrix}$$

Using Eq. (12) we have:

$$\mu_{R^S} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.64 & 0 & 0 \end{bmatrix}$$

Then using Eq. (15), we have:

$$\mu_{R^{ND}} = \begin{bmatrix} 0.32 & 1 & 1 \end{bmatrix}$$

On the basis of Eq. (16), the set of non-dominated solutions a^{ND} is given below:

$$a^{ND} = \{a_2, a_3\}$$

According to (17) :

$$a^{NFND} = \{a_2, a_3\}$$

Decisions a_2 and a_3 are equivalent in the sense of relation $\mathbf{R}^G$.

Solution by method II

$$\mathbf{R} = [R_1, R_2, R_3]$$

$$\mu_{R_1^S} = \begin{bmatrix} 0 & 0 & 0.12 \\ 0 & 0 & 0.12 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mu_{R_2^S} = \begin{bmatrix} 0 & 0 & 0 \\ 0.12 & 0 & 0 \\ 0.76 & 0.12 & 0 \end{bmatrix}, \qquad \mu_{R_3^S} = \begin{bmatrix} 0 & 0.12 & 0.12 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mu_{R_1^{ND}} = [1 \quad 1 \quad 0.88], \qquad \mu_{R_2^{ND}} = [0.24 \quad 0 \quad 1], \qquad \mu_{R_3^{ND}} = [1 \quad 0.88 \quad 0.88].$$

Using Eq. (21), we have:

$$\mu_{R^{ND}} = [0.24 \quad 0 \quad 0.88]$$

Then according to (16), we obtain:

$$a^{ND} = \{a_3\}$$

Thus using Eq. (8), we conclude that:

$$a^{NFND} = \{\varnothing\}$$

6. Weighting the importance of criteria

In the case of criteria of differing importance in a decision making process, it is possible to express them using the following fuzzy preference relation:

$$R^{\Lambda} = \left\{ \left\langle \left\langle k_i, k_j \right\rangle, \mu_{R^{\Lambda}}(k_i, k_j) \right\rangle : \ \left(k_i, k_j\right) \in K \times K, \qquad i, j = 1, 2, \ldots, K \right\} \tag{23}$$

where: $K = \left\{k_i\right\}_{i=\overline{1,K}}$ – the set of criteria, K – the number of criteria, reflected by appropriate fuzzy preference relations, $\mu_{R^{\Lambda}} : K \times K \to \left[0,1\right]$ – membership function of a fuzzy preference relation, $\mu_{R^{\Lambda}}(k_i, k_j)$ – degree of membership according to a fuzzy preference relation, R^{Λ} indicates the degree to which the decision k_i is at least as good as decision k_j according to a criterion.

Taking into consideration the weights of the criteria expressed in (23) and on the basis of (18), we obtain:

$$R^{\Lambda} = R \cap R^{\Lambda}$$

where: R^{Λ} – fuzzy preference relation taking into account the weights of the criteria.

The intersection of fuzzy sets is implemented by Orlovsky [22, 23] using the minimum function as a t-norm operator.

7. Application of the solution to a FMDM problem with a fuzzy preference relation in management

Example 3

Ranking a set of companies according to which have the best position on the market and the highest quality of products (for simplicity we consider only 2 criteria):

- the set of companies

$$\mathrm{A} = \left\{a_1, a_2, a_3, a_4\right\}$$

- the set of criteria for assessing companies

$$\mathbf{K} = \left\{k_1, k_2\right\}$$

k_1 – position on the market, k_2 – quality of product.

Table 1 presents qualitative (expressed in linguistic terms) expert assessments of the companies for both criteria.

Table 1. Expert evaluation of firms according to both criteria

Company	k_1-Position	k_2-Quality
a_1	strong	very high
a_2	weak	low
a_3	medium-strength	very high
a_4	weak	high

Author's own work, previously unpublished.

The assessments of the companies' activities are described by linguistic variables. The membership functions for each linguistic term are defined by trapezoidal fuzzy numbers.

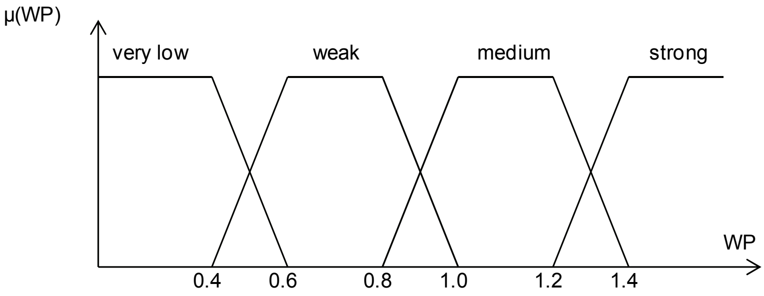

Fig. 1. Membership functions of the linguistic variable *The_position_on_the_market*; WP – indicator of market position, linguistic variable: Market position = {very weak, weak, medium-strength, strong} (author's work, first time published)

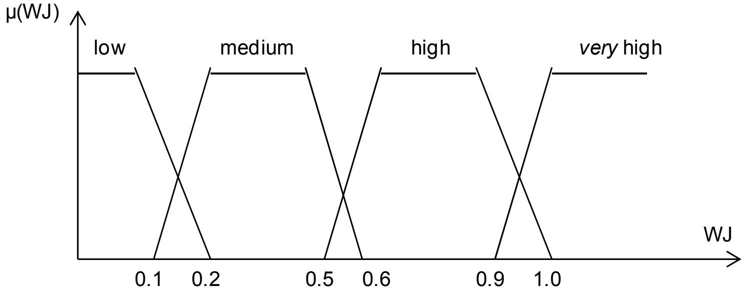

Fig. 2. Membership functions of linguistic variable *Quality* WJ – indicator of product quality, linguistic variable: Quality = {low, medium, high, very high} (author's work, first time published)

Based on this expert opinion, we created the following fuzzy preference relations corresponding to the appropriate criteria:

$$\mu_{R_1}(a_i,a_j)=\begin{array}{c|cccc} & (a_1) & (a_2) & (a_3) & (a_4) \\ \hline (a_1) & 1 & 1 & 1 & 1 \\ (a_2) & 0 & 1 & 0.3 & 1 \\ (a_3) & 0.3 & 1 & 1 & 1 \\ (a_4) & 0 & 1 & 0.3 & 1 \end{array}$$

$$\mu_{R_2}(a_i,a_j)=\begin{array}{c|cccc} & (a_1) & (a_2) & (a_3) & (a_4) \\ \hline (a_1) & 1 & 1 & 1 & 1 \\ (a_2) & 0.25 & 1 & 0.25 & 1 \\ (a_3) & 1 & 1 & 1 & 1 \\ (a_4) & 0.25 & 1 & 0.25 & 1 \end{array}$$

According to Eq. (22), the global preference relation $\mathbf{R}^G$ has a membership function of the form:

$$\mu_{R^G}(a_i,a_j)=\begin{array}{c|cccc} & (a_1) & (a_2) & (a_3) & (a_4) \\ \hline (a_1) & 1 & 1 & 1 & 1 \\ (a_2) & 0 & 1 & 0.25 & 1 \\ (a_3) & 0.3 & 1 & 1 & 1 \\ (a_4) & 0 & 1 & 0.25 & 1 \end{array}$$

In accordance with Eq. (12), the membership function of the strict preference relation $\mathbf{R}^S$ has the following form:

$$\mu_{R^S}(a_i,a_j)=\begin{array}{c|cccc} & (a_1) & (a_2) & (a_3) & (a_4) \\ \hline (a_1) & 0 & 1 & 0.7 & 1 \\ (a_2) & 0 & 0 & 0 & 0 \\ (a_3) & 0 & 0.75 & 0 & 0.75 \\ (a_4) & 0 & 0 & 0 & 0 \end{array}$$

From Eq. (6), the membership function of the non-dominance relation $\mathbf{R}^{ND}$ is:

$$\mu_{R^{ND}}(a_i,a_j)=\begin{bmatrix}1 & 0 & 0.3 & 0\end{bmatrix}$$

Based on the membership function of the non-dominance relation, the rank of companies (from the best to the worst) is as follows:

$$a_1$$
$$a_3$$
$$a_2, a_4$$

The companies (decisions) a_2, a_4 are equivalent according to relations $\mathbf{R}^G$, $\mathbf{R}^S$ and R^{ND}.

8. Conclusions

The concept of Pareto-optimality is not appropriate when considering a multi-criteria decision making problem with many criteria. In this paper, we have attempted to outline the main idea of the linguistic approach to multi-criteria decision making problems. The use of fuzzy functions could be very profitable for decision makers when making decisions in complex and multi-attribute problems. The linguistic approach may well be useful in real world decision making problems. We considered a special class of non-conventional binary relations for decision making in a fuzzy environment. This is a vector of fuzzy preference relations. A discrete model of decision making problems in a fuzzy environment was introduced. Fuzzy optimality was applied within the framework of this model. Simple methods of calculating solutions were described. Solutions obtained using fuzzy preference relations reflect real world problems better than solutions given by crisp, non-fuzzy tools.

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