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ROLE OF THE NORMAL LOGICAL FORM IN DECISION MAKING AND KNOWLEDGE MANAGEMENT

The normal form in logic has been considered. Any propositional function, i.e. any finite logical expression can be written in such a form. This indicates the possibility of an unequivocal logical representation of many different objects investigated in science and everyday life. The properties of the normal form give a new dimension to the management of processes examined in science. Understanding of the laws of logic and its calculus allows us to obtain this form in a finite number of logical transformations. In addition, this form indicates the cognitive essence and pragmatic dimension of logic. The paper considers axiomatization, and then optimization. Both of these formulations of logic reflect its essence. Shannon's theorem gives us only a modest signpost that reality has a complex nature, which is confirmed by the richness of logic in the form of its equivalent propositional functions. Knowledge about the behavior of these structures is ambiguous in terms of the complexity of the corresponding logical expressions, that is, two different or identical logical functions may be related to identical (similar) or quite different behaviors in relation to the processes or objects represented by these functions.

Keywords: *normal form, knowledge, management, logic, pragmatism, decision making*

1. Introduction

The paper aims to present the pragmatism of logic in a new dimension, and even the thinking of a man, in the process of management and decision making (in terms of knowledge). The concept encapsulated by the term knowledge management has grown

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up almost to the size of a scientific paradigm. In fact, there is nothing surprising in this. Without management, there is no order, and without knowledge regarding the human environment, only uncertainty or even mental chaos[4] exists. On the other hand, knowledge cannot be managed, just as it is not possible to manage the weather. However, as highlighted, this term was created to denote a scientific concept. Generally speaking, you can manage the ways of obtaining knowledge using research methods. Such methods provide knowledge to man in an organized way. All this, in turn, is closely linked to the logical dimension of management processes, and so to knowledge about them.

However, in practice, we operate with information and knowledge. Do these two concepts differ in terms of content and, if so, how? This is a question that modern science is unable to definitively answer. The natural language corresponding to both these concepts does not assign any specific (independent) syntactic phrases. In addition, there is no strict scientific definition of either of these concepts. However, the distinction is often relatively easy to carry out in intuitive terms. Information about something (a certain object or process) is poorer in terms of content than knowledge about that thing. Furthermore, owing to the less specific content, it is easier to manipulate and control information than knowledge – truth transmitted as false and vice versa with respect to any object and the process of actual or abstracted reality. When, however, we previously had some knowledge, and also in some sense information, about a subject, it is much easier, in turn, logically and practically to decide what is the truth, and which information is untrue, what is important and what is less important, etc.

Fundamental knowledge is obtained from scientific research, or at least, science through its research methods allows us to decide what information that we possess is important – what is likely to be true and what is not. Science, or rather its research methods, constitutes a complicated system. Complex systems and their products, which in the case of science is, of course, knowledge, also require more sophisticated management methods (compared to simple systems). The major factor of such management is undoubtedly logic. However, logic also is a product of science. It was already being used in the times of ancient Greece[5]. The significant pragmatic aspect of this field of knowledge has been perceived from the very beginning.

[4]In the field of psychology, research has been carried out for a long time in the area of human behavior (from the age of childhood) associated with disorders of equilibrium states. For example, a person who finds himself in a new situation, or even in an unfamiliar environment, behaves in a nervous manner. This demonstrates a lack of balance between the environment and knowledge about that environment. See e.g. [7, p. 44 and the following] also [8, p. 16 and the following]. Both of these books should be considered as important source material for research into disorders of the equilibrium state between a Subject and his Surroundings (known in psychology as the classic P-O layout), which were conducted by J. Piaget over many years and published, among others, in the abovementioned positions.

[5]Aristotle developed its formalized form.

Therefore, the main objective of this paper is to describe the advantages of the pragmatic dimension of logic, although it is generally regarded as an abstract science. Only some basic tools of logic will be presented, but these are important from a pragmatic point of view, supporting both the management of knowledge acquired by science and those who have not cultivated any science, using knowledge and decision making.

2. Pragmatic dimension of logic

The outstanding Polish logician, K. Ajdukiewicz, was the first person in the world to publish a book [1] in which he discussed the pragmatism of logic in a very diversified, in terms of content, formulation. This means that the book analyzes the pragmatic dimension and logical structure of many individual problems, as well as branches of science such as the theory of statistical hypothesis testing in terms of the logical structure of decision making using statistical tests. You can ask whether such pragmatism is necessary for science and generating knowledge? For fields of knowledge that have their own conceptual system, there exists the question of whether in addition to specific, and often very particular, methods of inference, further logical analysis is required? The answer to this question essentially comes down to the following reasoning.

For any prospective researcher of any topic or field of knowledge, especially of mathematics, it is very difficult to master the skills of formulating strict and logical thinking, correct conclusions and clearly defining on what correct reasoning should be based, and what logical structure it has. Before this can be done, however, there is a need to adopt, and especially to understand, the basic concepts of logic. These difficulties have different sources. The most important is a lack of proper preparation in the field of logic and in its pragmatic dimension. That is to say mastering its conceptual apparatus, and not only from the formal, but also practical point of view, i.e. from a proper understanding of what the basic concepts of logic are. This proper understanding of the concepts of this important field of knowledge provides the basis for its use in practice, which is the essence of its pragmatism. This very pragmatism of logic can generally be expressed as follows:

Logic captures and then defines methods of inference used in practice, especially in science, which are considered to be correct, and organizes them into logical systems. Such systems are given by a set of laws and rules, which when respected, make inferences that we spontaneously recognize as flawless [1]. This field of logic, understood as science, and related to inference, covers three disciplines which should be strictly separated from each other, because each of them has different areas of scientific inquiries and methods. These are:

• Formal logic – this generally considers the so-called the laws of logic (logical sentences in particular) according to which one obtains logically true expressions from other expressions known to be logically true.

• Methodology – this is the theory of the rules of applying logic to various fields of science and life.

• Philosophy of logic – this poses general questions concerning logic itself and the nature of its laws.

The above introduction to the place of logic in knowledge management and making associated decisions at its most general level has an essentially philosophical and methodological dimension with an emphasis on the pragmatism of logic in science. In the following considerations, we will focus on the fundamentals concerning logical inference, useful in the practical management of knowledge, decision making in scientific research[6] and the operational dimension of logic.

3. Knowledge or tautologies and logical inference schemes

The basis of knowledge as (scientific) truth comes from a logical concept that is called a tautology. In practice, a tautology is understood as the expression of one thing by something equivalent, that is, colloquially speaking, an object that represents itself. But in logic, there is an operation which possesses the concept of tautology in another sense, namely as a logical expression (propositional function) that is always true from the point of view of the logical value of the propositional variables included in it. Therefore, the following definition of the logical understanding of tautology can be provided:

Definition. A logical expression is called a tautology when it is true for every possible combination of logical variables connected by functors (logical conjunctions). Such truth must hold in every non-empty field of knowledge, that is, in the domain to which the given expression is referred to.

Thus, in relation to Aristotle's logic, expressions are named tautologies[7] if they are always true, that is, for any combination of logical variables occurring in it, i.e. of variables taking the logical values of 0 (false) or 1 (true).

[6]It should be emphasized here that not all people are *scientists***.** The role of scientists is to familiarize (translate) scientific findings into everyday language, and the carrier of this translation is our natural language. One can operate very precisely with concepts defined within a certain field of knowledge, but in turn, these concepts are difficult to translate into everyday language. The aim of science is to recognize and explain reality in the best possible way to so-called ordinary people. This is a difficult task, as shown in practice.

[7]The concept of tautology is also present in the theory of quantifiers but this has not been analyzed in the paper.

Various operational methods have been developed in the field of logic for checking whether something – a certain process, object – occurs according to the law of logic, namely, whether it has the logical structure of a tautology, and therefore is a law of logic, or it is not.

Let, therefore, our data be logical expressions (propositional functions), for example:

$$\alpha = \{(p \to q) \to [(q \to r) \to (p \to r)]\} \tag{1}$$

which logically (structurally) represents a thought (or real) process that is taking place in the reality surrounding a particular person. Now let the following expression be another piece of data

$$\beta = [p \to (q \vee r)] \tag{2}$$

The expression β also represents the logical structure of a certain mental or real event. Knowing the rules for creating propositional functions, an infinite number of them may be generated. However, logic knows only three categories of propositional functions (tautologies, anti-tautologies and satisfiable expressions). The most important of these are theorems of logic (tautologies)[8].

In relation to the two expressions mentioned above, it should be indicated here that the first of them is a law of logic, whereas the other is not. However, these expressions are so constructed that they contain premises and a conclusion. In research practice, most frequently we have situations in which we only have a certain set of premises, called facts. It has already been remarked that logic operates by exhaustive evaluation of whether a given expression is a law of logic or not.

Speaking about knowledge management, two methods which are well suited to such management can be related to it, especially with regard to the pragmatic approach to logic[9] that is, the operationalization of logic.

One of them is called natural deduction or the propositional method[10]. The second is defined by the so-called resolution rule [6] [11]. The essence of natural deduction lies in the fact that:

It enables to check in a very efficient manner, in a logical and operational sense, whether the result obtained by a given thought process can be considered as a new

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[8]It should now be operationally decided whether these two expressions α and β are laws of logic or not.

[9]Recently, more has been spoken about the management of human capital in education in the wide sense rather than knowledge management, see, e.g. [11].

[10]This is a method developed by the Polish logician L. Jaśkowski in 1934, see, e.g. [2, 5, 6].

[11]This monographic lecture about resolution theory includes the following position [12].

element of knowledge or as the result of a research process. The conclusion obtained from premises (given facts) is just a new element of the system of logic, and the premises (facts) and conclusion together form a law of logic[12].

Example. Let some known facts have the following logical representation:

$$\gamma = (p \to q) \wedge (r \to s)$$

As a conclusion – knowledge from these facts can be gained in the form of the following expression:

$$\delta = (p \wedge r) \to (q \wedge s)$$

We now ask whether this knowledge is properly obtained in terms of logic. That is, whether the facts and the conclusion together logically constitute any law of logic. Natural deduction and the principle of resolution can operationally answer this question[13].

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[12]Known facts constitute prior knowledge, but the conclusion is new knowledge, and is verifiable within the framework of logic. However, this approach is sometimes labor intensive and quite difficult to use in the operational dimension. In addition to primary rules (the standard laws of logical inference), it requires the use of yet other laws of logic, which are specific to the subject matter. This method also uses two approaches to creating a primary proof (rule of direct propositional proof or ordinary rule of direct proof). In addition, any proof of a logical statement can also be carried out as an indirect proof, but there is no rule declaring which method of proof is easier to perform (direct or indirect).

[13]Because natural deduction is described in logic textbooks, the proof of this proposition is given in this footnote.

$$S : \begin{cases} (p \to q) \\ (r \to s) \\ \hline (p \wedge r) \to (q \wedge s) \end{cases}.$$

We use direct proof and it is necessary to begin by writing the propositions in rows, and these are:

$$\left.\begin{array}{ll} 1. & (p \to q) \\ 2. & (r \to s) \\ 3. & (p \wedge r) \end{array}\right\} \text{assumptions of the proof}$$

As a result of the proof, we have to obtain the expression $(q \wedge s)$. To obtain this expression as a row in the proof, the strict procedures of logical analysis should be used, i.e., a suitable law of logic should be used in each step of the proof. The step-by-step proof is as follows:

$$\left.\begin{array}{ll} 4. & p \\ 5. & r \end{array}\right\} \text{(the elimination rule of conjunction in 3)}$$

6. $[(p \to q) \wedge p\ ]$ (the introduction rule of conjunction)

7. q (the detachment rule applied to 4 and 6)

However, as has been emphasized, such proofs, especially the propositional method, are sometimes quite difficult to apply in operational terms because it should be known which rules and laws are to be used at each step.

A brief outline of the essence of the resolution principle is given below. This principle gives very great pragmatic benefits and, furthermore, is easier to use in practice than the propositional method.

The principle of resolution[14], in contrast to the propositional method, uses one and the same law from the beginning to the end of the proof, i.e., this inference scheme has the nature of indirect proof. It is represented, symbolically speaking, by the following inference scheme:

$$\frac{(x \vee y) \wedge (\sim x \vee z)}{(y \vee z)} \tag{3}$$

The expressions $(x \vee y)$ and $(\sim x \vee z)$ are called clauses in this scheme, and in turn, the expression $(y \vee z)$ – the resolvent. We see that the resolvent obtained relies on the removal of the same variable from both clauses but in one of them it occurs without negation and in the other, with negation. We connect what remains as alternatives and recognize this as a conclusion resulting from the conjunction of two clauses. The line in the diagram replaces the implication sign in standard logical inference. The simplicity of this rule of inference is its strong point. It serves several important purposes in the practice of logical analysis. These are:

- checking the consistency of a set of premises,
- checking the reliability of a logical scheme (law of logic),
- in relation to knowledge management, the most important goal of this law is to obtain conclusions resulting from a given set of premises.

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8. $(r \rightarrow s) \wedge r$ (as in 6)

9. s (the detachment rule applied to 5 and 8)

$(q \wedge s)$ **(**the introduction rule of conjunction applied to 7 and 9)

The last row obtained in the proof ends this formal proof of our proposition. The conclusion $(p \wedge r) \rightarrow (q \wedge s)$ constitutes new knowledge, but the scheme *S* represents the law known in logic as the law of implication multiplication. The rules applied in the proof to go from one line to the next are called primary rules, and the way of carrying out the proof is called direct propositional proof.

[14]This principle was used by J.A. Robinson for automatic theorem proving [12]. In addition, this can be used to verify conclusions in *knowledge bases*, which provide formally written knowledge. This principle is widely described in the artificial intelligence and computer science literature. Textbooks of classical logic normally present the sequent calculus (of sequences) created by G. Gentzen in 1934, or Jaśkiewicz's natural deduction, also published in the same year.

Example. Applying the principle of resolution, investigate what direct conclusions result from the following set of premises:

$$S = \{p \to q, \quad p \to r, \quad \sim q \vee \sim r\}$$

Solution. Using the appropriate laws of logic, we obtain the clauses:

$$\sim p \vee q \quad (1)$$
$$\sim p \vee r \quad (2)$$
$$\sim q \vee \sim r \quad (3)$$

Clauses (1) and (2) were obtained based on the definition of implication in terms of logical arithmetic:

$$(p \to r) \equiv (\sim p \vee r)$$

and correspondingly

$$(p \to q) \equiv (\sim p \vee q)$$

Using the resolution scheme, we now obtain the following conclusions (the clauses written under the line delimiting the set of clauses):

$\sim p \vee q$		(1)
$\sim p \vee r$		(2)
$\sim q \vee \sim r$		(3)
$\sim p \vee \sim r$	(1; 3)	(4)
$\sim p$	(2; 4)	(5)
$\sim p \vee \sim q$	(2; 3)	(6)

The purpose of this paper is not to describe the practical use of logical analysis, i.e. logical calculus, so we only present the above example. We note that if we add to our set of clauses the negation of any of the conclusions, then we must obtain a contradiction as the final result (an empty clause of the type ($p \wedge \sim p$)). This shows the relation of the resolution rule to indirect proof. This is a proof, that the resulting conclusion is logically correct.

It seems, therefore, that the most important role of the resolution rule in operational terms is the ability to check the consistency of a set of premises. This is also its strong pragmatic side. The knowledge generated from a logically contradictory set does not constitute any value in either theoretical or practical terms, and the emphasis placed on the third point of applying the resolution rule refers to knowledge management.

4. Role of normal form in logic and knowledge analysis

The resolution rule implicitly contains in its operational dimension a very important conceptual category for classical logic[15] that allows us to formulate any logical expression in one and the same logical form, known as normal conjunctive-disjunctive (n.c.d.) or disjunctive-conjunctive (n.d.c.) form. This plays an important, and even extremely important organizational, operational, cognitive and optimizing function in a logical representation of any of the analyzed (classical) processes, and therefore also in the process of acquiring knowledge and its management. Logic has two main dimensions. One of them is especially favored by us and is called its pragmatic dimension because it enables us to obtain specific information about an examined object in the form of conclusions. The second dimension is the abstraction of logic, essentially its essence, i.e., its cognitive dimension which somehow constitutes the pure knowledge of logic – science.

We now will deal in turn with other dimensions of logic. We will start with pragmatics. However, for this purpose, it is first necessary to give a definition of the normal form of any logical function in propositional calculus. The concept of normal form, whose definition is given below, is used for many practical purposes and for creating the theory of logic. One of the practical objectives is associated with inference. The above example of obtaining conclusions using the resolution principle does not answer the question of whether any tautology can be proved,, and if not what tautologies is it possible to prove, or how to prove them. These questions lead to the following definition:

Definition. The expression α has a normal conjunctive-disjunctive form if and only if α is the conjunction of a number of alternatives $\alpha_1,\ldots,\alpha_n$. The elements of these alternatives are propositional variables or the negation of propositional variables:

$$\alpha = \underbrace{(p \vee q \vee {\sim}q \vee \ldots \vee r)}_{\alpha_1} \wedge \underbrace{(p \vee q \vee s \vee \ldots \vee r)}_{\alpha_2} \wedge \ldots \wedge \underbrace{(p \vee t \vee {\sim}s \vee \ldots \vee z)}_{\alpha_3} \qquad (4)$$

[15]Classical logic can be split into two sections: propositional calculus and quantifier calculus (first order predicate calculus).

Note. By switching the role of the conjunction and disjunctive functors, in an analogous manner we can define the normal disjunctive-conjunctive form.

If the principle of the excluded middle holds for every element of the conjunction corresponding to the above definition, then the expression represented by the n.c.d. form is a tautology. Thus, the normal form can serve as a method of checking whether a logical expression is a tautology. This is one of the most important properties of this logical form. This form also plays many other very important functions. Previously, we noted two of its dimensions, and both of them fulfill several functions. We will start with the cognitive dimension.

5. Normal form in the axiomatics of propositional calculus

We already know that the propositional calculus can be formulated in many ways. One such representation is an axiomatization of propositional calculus. But how does this relate to the concept of the normal form? Axiomatization of a given field of knowledge is a representation of its ideal. In practice, only some fields of knowledge can be presented as a set of axioms. With respect to the axiomatization of propositional calculus, the concept of the normal form plays a very important role. This concept is the basis of deriving whether a given logical expression is a tautology, though in a slightly different way than that defined above. Before propositional calculus was formulated in its axiomatic form, the principle formulated below on the derivability of tautologies (laws of logic) as follows:

Theorem (about decidability): There is an effective method (algorithm) allowing us to check in a finite number of steps, whether a logical expression (scheme) composed of logical conjunctions, propositional variables (letters), and parentheses is a law (a tautology of classical propositional calculus), or it is not. This approach is also called the zero-one approach, because each variable takes a value of zero (0) or one (1). Using this theorem, in practice, we are able to decide whether any logical scheme is a tautology. In logic, this approach is called synthetic.

Usually, however, theories – especially mathematical ones – are created a little differently. The trend is towards axiomatization. Therefore, at the beginning of the development of a given theory (field of knowledge), fundamental properties of the objects considered are highlighted. These are generally very obvious properties, that is, the most certain. Therefore, sentences expressing these properties are called certainties. They are also called assumptions or axioms. These axioms form the initial theorems. All other theorems are derived from them by logical inference. This procedure relates to various fields of knowledge. So an axiomatic structure can also be given to logical systems. Therefore, therein can be distinguished certain basic laws of logic, the most obvious and primarily appealing to intuition. These are accepted as axioms and all other laws are derived from this set of axioms in a way that guarantees that the derived laws are as true as the accepted axioms.

Classical propositional calculus is one of those branches of science that has been expressed in axiomatic form. There are even a lot of different systems of axioms for propositional logic which are equivalent[16]. It should be emphasized once again that the concept of the normal form plays a very important role in the axiomatization of propositional calculus. Thanks to this, the following important extremely important theorem, not just for logic but also science as a whole, has been proven.

Theorem. The set of tautologies of classical propositional calculus is finitely axiomatizable[17]. This means that by adopting a number of primary terms, axioms and introducing the ability to define other concepts, and further attaching the rules of substitution, detachment and definitional replacement, a set of consequences (conclusions) can be derived, which is identical to those of classical propositional calculus[18].

Definition. D is the proof of a proposition B based on a set of formulas X accepted as assumptions if and only if D is a finite sequence of formulas

$$D = \{D_1, D_2, ..., D_n\}$$

such that the last formula of the sequence is identical to the proposition $B : D_n = B$ and any formula D_k of the series D ($1 \leq k \leq n$) either (1) belongs to the set X, (2) arises from the formula D_j of this set such that $j < k$ by appropriate substitution, or (3) arises from two formulas A: D_j, D_i such that $j < k$, $i < k$ by detachment: $D_j = (D_i \rightarrow D_k)$.

Example (of a proof). We have the theorems:

$$p \rightarrow (q \rightarrow p) \quad (1)$$

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)) \quad (2)$$

$$(p \rightarrow (q \rightarrow p)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow p)) \quad (3)$$

$$(p \rightarrow q) \rightarrow (p \rightarrow p) \quad (4)$$

$$(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow p) \quad (5)$$

$$p \rightarrow p \quad (6)$$

[16]You can even specify axiomatics of propositional calculus consisting of only one axiom, but such an axiom, in terms of the essence of axiomatics, is non-transparent. With more axioms, it is easier to grasp the role of each axiom.

[17]The proof of this theorem can be seen, e.g. in [3].

[18]Important statements about the axiomatization of c.p.c. are: theorems about independence, consistency and completeness.

The set of theorems (1)–(6) constitutes a proof of Eq. (6). Axioms (1) and (2) are the starting point. Theorem (3) is obtained from (2) by substituting *r*/*p*. As is easy to notice, Eq. (4) is the consequence of the rule of detachment from (1) and (3). Theorem (5) arises by substituting $q/(q \to p)$. Finally, theorem (6) is obtained from (1) and (5) by applying the rule of detachment. Using this example, the following definition can be given:

Definition (Tarski). Formula *A* is a consequence of the set *X* (symbolically: $A \in Cnq(X)$ if and only if there is a finite sequence of inscriptions *D*, such that *D* is a proof of the proposition *A*, based on the set of formulas *X*.

Formulas (3)–(6) are consequences of the set $X = \{1, 2\}$ and they constitute the proof. The cognitive theorem is another important result for propositional calculus:

Theorem. The set of tautologies of classical propositional calculus is a theory. This means that every c.p.c. theorem is a direct consequence of some of its other theorems. All the theorems of propositional calculus are in some way related to each other but the theorem about independence says that there are such finite sets of theorems that may constitute systems of axioms of this theory, and in this set none of the theorems results from the others. The following theorem combines the concepts of consequence, theory and axiomatics into a whole:

Theorem. The set *Y* is finitely axiomatizable if and only if there exists such a finite set *X*, that $Y = Cnq(X)$, and if furthermore the set *X* constitutes the axiomatics of the set *Y*, then set *Y* is a theory.

These statements are important cognitive information related to the properties of logic and especially relevant to knowledge obtained by science.

An example of the axiomatics of c.p.c. is given below (Łukasiewicz's axiomatics):

1. Primary terms: ~, →.
2. Axioms:

$$A_1 : (p \to q) \to \big((q \to r) \to (p \to r)\big)$$
$$A_2 : p \to (\sim p \to q)$$
$$A_3 : (\sim p \to p) \to p$$

3. Definitions:

$$d_1 : (p \lor q) \equiv (\sim p \to q)$$
$$d_2 : (p \land q) \equiv \sim (p \to \sim q)$$
$$d_3 : (p \equiv q) \equiv \big((p \to q) \land (q \to p)\big)$$

Furthermore, the following rule of inference may be defined:

In any c.p.c. theorem, any part equiform to one side of the definition can be replaced by a part equiform to the other side of that definition.

When applied to axioms, definitions and other theorems, new theorems can be derived. Furthermore, this law, called the rule of definitional replacement, together with the rule of substitution, elimination and the three axioms above, generates the same set of consequences as classical propositional calculus.

6. Generating conclusions from premises and premises from a given conclusion

Axiomatization and the normal form enabled the formulation of propositional calculus in the form of a finite system of objects from which any theorem of this calculus can be derived, although it should be emphasized that this method has a non-algorithmic character, we only ask: from which laws does the law of direct interest to us derive? In practice, we either know or do not know how to answer such a question. With respect to the pragmatics of logic, this means that classical propositional calculus is generative but it does not have an algorithmic character from the operational point of view. However, one can ask about the character, and in particular the extent of this generativity. Does it concern only theorems (tautologies) or manifest itself at the level of expressions that are not tautologies, but the conjunction of a set of such elements generates such an expression, which together with its cause (predecessor), form a law of logic? The answer to this question is positive. This procedure in logic is called the creation of conclusions from a given set of premises. What is the mechanism, the algorithm defining this process?

In propositional calculus, apart from the normal form there is another form called the full normal form which serves to generate any true conclusion from a given set of premises in an algorithmic way and derive (reproduce) premises, when any of their conclusions is given. In order to present these two logical processes operationally, we should first give a definition and understanding of the conclusion and the (conjunctive -disjunctive) full normal form[19].

Definition. Expression F of propositional calculus is called a logical conclusion of expressions F_1, F_2, ..., F_k, if the implication $(F_1 \wedge F_2 \wedge ... \wedge F_k) \rightarrow F$ creates a law of

[19]In the practical application of logic, both the full normal conjunctive-disjunctive form (f.n.c.d.) and its full normal disjunctive-conjunctive form (f.n.d.c.), are used. However, tautologies cannot be written in full normal f.c.d. form, and in turn, anti-tautologies cannot be written in its dual form, that is, f.d.c.

logic, and furthermore, the conjunction $F_1 \wedge F_2 \wedge ... \wedge F_k$ of premises is not a contradiction.

Another important definition for logic and especially its pragmatics is:

Definition. The disjunctive-conjunctive form which satisfies the conditions listed below is called the full normal disjunctive-conjunctive form of the expression $F(p_1, p_2,..., p_n)$, containing n various propositional variables:

- it contains only various components,
- each component is a logical conjunction of different variables,
- none of the components contain the same variable together with its negation,
- each component contains all the variables in the expression $F(p_1, p_2, ..., p_n)$ but in only one form: either with or without negation.

A procedure for writing an expression in f.n.d.c. form is given below:

1. Firstly, the n.d.c. form is obtained from a given expression $F(p_1, p_2, ..., p_n)$.
2. If the variable p_i is not present in any component, then we supplement it using the always-true expression $p_i \vee \sim p_i$ and then we use the separation law of conjunction with respect to disjunction. In this way, we finally obtain the desired f.n.d.c. form for the expression $F(p_1, p_2, ..., p_n)$.

Example. To find the f.n.d.c. form for the implication $p \rightarrow q$.

According to point 1, we have $(p \rightarrow q) \equiv (\sim p \vee q) \equiv (\sim p) \vee (q)$. The variable q is missing from the first component, and the variable p from the second one. So we supplement these components by the expressions $(q_i \vee \sim q_i)$ and $(p_i \vee \sim p_i)$, respectively. So we have:

$$(\sim p \wedge (q \vee \sim q)) \vee (q \wedge (p \vee \sim p)) \equiv (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$

Thus, the implication $p \rightarrow q$ has the following equivalent form:

$$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$

If in the definition of the f.n.d.c. form given above we swap the roles of disjunction and conjunction, and the roles of components and segments, we obtain the f.n.c.d. form of the expression $F(p_1, p_2,..., p_n)$.

Both definitions of the full normal form were given primarily in order to use them in the practice of decision making. Hence, what is the algorithm for generating conclusions from a given set of premises and what other practical and operational capabilities do these two forms give?

This algorithm is described below:

1. Let the expression $F_1(p_1, ..., p_n), ..., F_k(p_1, ..., p_n)$ be represented by premises composed from the variables $p_1, ..., p_n$ (not all of these variables have to be present in any given premise; and one propositional variable can occur several times in a premise both with or without negation).

2. To obtain all the logical conclusions from a given set of premises, you must create the conjunction of these premises and the expression obtained leads to the f.n.c.d. form.

3. The conclusions of these premises are given firstly by single parts of the conjunction, then combinations of pairs, then combinations of triplets and so on, until at the end we obtain the whole f.n.c.d. form (compare with the reversibility of the implication $\alpha \rightarrow \alpha$.

If we have n conjunction segments, the number of conclusions is of exponential order and is exactly $L = 2^n - 1$.

Example. We have a set of premises: $P = \{p \rightarrow q, \sim q\}$. Find all the conclusions that logically result from this set.

The conjunction of these premises is of the form:

$$\left[(p \rightarrow q) \sim q\right] \equiv \left[(\sim p \vee q) \wedge \sim q\right]$$

After applying the logical procedures of the algorithm for obtaining the f.n.c.d. form, we obtain:

$$(p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$$

The conclusions are given by the expressions:

$$p \vee \sim q \quad (1)$$

$$\sim p \vee q \quad (2)$$

$$\sim p \vee \sim q \quad (3)$$

$$\left[(p \vee \sim q) \wedge (\sim p \vee q)\right] \equiv (p \equiv q) \quad (4)$$

$$\left[(p \vee \sim q) \wedge (\sim p \vee \sim q)\right] \equiv \sim q \quad (5)$$

$$\left[(\sim p \vee q) \wedge (\sim p \vee \sim q)\right] \equiv \sim p \quad (6)$$

$$\left[(p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)\right] \equiv (\sim p \wedge \sim q) \quad (7)$$

This algorithm may seem to have little use in practice when the number of premises and variables is large. However, practice imposes limitations on this number. Thus, in fact, commonly such inference processes contain up to four variables (factors).

In management practice, very frequently knowledge about a given process is general, but we do not know its specific logical structure. However, we know how single components of this process behave, that is, their logical values are known. The following question arises: how from such incomplete knowledge, can we recreate the logical structure of such a process in an unambiguous way?

Instead of general considerations, let us take a simple example: Find an explicit form of the function $F(p, q, r)$ of the variables p, q, and r, where only its logical values are known for each possible combination of the logical variables p, q, r as presented in Table 1

Table 1. Logical values of $F(p, q, r)$ for each possible combination of p, q, r

p	q	r	$F(p, q, r)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

Considering the rows for which the expression $F(p, q, r)$ is true, we can create the f.n.d.c. form given below:

$$\begin{aligned} F(p,q,r) = {} & (p \wedge q \wedge r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \\ & \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r) \end{aligned} \tag{5}$$

By using the relevant laws of logic to the individual segments, it is easy to prove that

$$F(p,q,r) = \left[\left(\sim p \vee (q \wedge r) \right) \equiv \left(p \rightarrow (q \wedge r) \right) \right] \tag{6}$$

Thus, these two equivalent logical expressions represent the propositional function $F(p, q, r)$ in an explicit form.

One can also look for another operational dimension of the normal form. Now that we know how to algorithmically create conclusions from premises, one can also ask how to algorithmically find a set of premises, when one of its conclusions is known?[20]

[20]One can also formulate very detailed tasks, for example: defining the relation of the logical variables in premises to a conclusion of a specified form, etc.

This algorithm is described below:

1. To determine what the premises are when their logical conclusion is given in the form of the expression $W(p_1, p_2, ..., p_n)$, the f.n.c.d. form should be created from this conclusion.

2. In the next step, we generate the premises by conjoining the expression $W(p_1, p_2, ..., p_n)$ with subsets of the segments of the f.n.c.d. form obtained.

3. The premises are conjunctions of the conclusion with, in turn, a single segment, then particular pairs of segments, etc. and finally with all the segments.

Example. Find the premises from which one of the conclusions is the expression $W(p, q) = (p \equiv q)$.

According to the above algorithm, we have:

$$(p \equiv q) \equiv \left[(p \rightarrow q) \wedge (q \rightarrow p)\right] \equiv \left[(\sim p \vee q) \wedge (\sim q \vee p)\right]$$

The segments containing both variables p and q in the expression $W(p, q)$ presented in f.n.c.d. form are:

$$(\sim p \vee \sim q) \text{ and } (p \vee q)$$

Now in accordance with the above algorithm, we create the combinations:

$$\left[(p \equiv q) \wedge (p \vee q)\right] \equiv (p \wedge q) \qquad (1)$$
$$\left[(p \equiv q) \wedge (\sim p \vee \sim q)\right] \equiv (\sim p \wedge \sim q) \qquad (2)$$
$$\left[(p \equiv q) \wedge (p \vee q) \wedge (\sim p \vee \sim q)\right] \equiv 0 \qquad (3)$$

The last premise is false, and hence any logical statement follows from it, including the expression $p \equiv q$.

7. Logically undefined forms, optimization, logical networks and black boxes

It can be said in general that the normal form plays an important role in the representation of the logical structure of important and, above all, simple processes related to management, as well as decision making. A decision is clear, when the logical values of all the factors that constitute a given process are known. If the logical shape of the process of interest is unknown, but we know its logical values, we can express this process through its logical form. But in reality, the physical conditions of a process often do not allow certain combinations of logical variables. This is a common situation in technical or production practice. In such cases, the truth matrix of propositional variables will only be partially defined and excludes *undetermined* combinations, that is, from a physical point of view, forbidden ones. Logically, they can take any value, so such a physical process can represent many sets of function values $F(p_1, p_2,..., p_n)$.

Example. The behavior of a given object depends on three variables *p*, *q* and *r*, and a partially specified function of these variables is of the form (Table 2)[21]:

Table 2. Partially specified function of the variables *p*, *q* and *r*

p	*q*	*r*	*F*(*p*, *q*, *r*)
0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	0

Table 3. Full array of combinations of the variables *p*, *q* and *r* of the expression *F*(*p*, *q*, *r*)

p	*q*	*r*	*F*(*p*, *q*, *r*)
0	0	0	–
1	0	0	–
0	1	0	–
1	1	0	1
0	0	1	–
1	0	1	1
0	1	1	1
1	1	1	0

The full normal d.c. form is

$$F(p, q, r) = (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge q \wedge \sim r) \tag{7}$$

The complete form of the table is shown in Table 3.

Note. Dashes (–) in the array mean that any logical value (0 or 1) can be taken by the expression *F*(*p*, *q*, *r*).

This kind of knowledge is not indifferent to the issue of optimizing the logical structure of a technical object or production process. Full knowledge of the logical

[21]More specifically, see [6, p. 46–48], [4, p. 164 and the following].

structure of each element in the considered object allows us to develop the optimal form of logical links between the layout elements, and thus obtain the complete logical structure of the functioning of such an object. This requirement is also important for two other reasons that connect the technical and economic optimization criteria for design to the description of the logical structure of the analyzed object or process, in general a *system*.

- The technical criteria require that a given layout (system) should perform the desired action and act in a reliable manner for a given time.
- The economic criteria in turn require that the resulting system (device, manufacturing process), should be the least expensive of its technically (logically) equivalent variants.

We now consider the problem of optimization. Any kind of project or analysis of a given set of elements should be expressed logically in the simplest possible way. Therefore, the optimization of logic circuits is the most widely used method, especially in electronics. However, our goal is not to introduce this process in too detailed a way, but only indicate that an understanding of logical calculus can be used to optimize a particular form of logical circuit – logical network. In other words, this calculus relies on replacing the data (output) of a logical network with a simpler form, which is thus more economical in the sense of physical realization. It should be stressed, however, that we need to know the value of the logical expression for all the possible combinations of variables. This is a basic condition for meaningful logical optimization.

Example. By applying the relevant laws of logic (logical calculus), it can easily be shown that the logical expression

$$\alpha = \left\{ \left[\sim (p \vee r) \wedge q \right] \vee \left[\left((\sim r \wedge q) \vee s \right) \vee p \right] \right\}$$

can be converted into an equivalent form that contains, in this case, even four fewer elements (logical functors, e.g. in electronics the functors ~, $\wedge$, and $\vee$ correspond to negators, multiplicators and sumators, respectively), namely:

$$\beta = \left\{ (\sim r \wedge q) \vee (s \wedge p) \right\}$$

The optimal configuration is in normal disjunctive-conjunctive form.

The analytical proof of this equivalence $(\alpha \equiv \beta)$ is given below:

$$\alpha = \{[\sim(p \vee r) \wedge q] \vee \{[(\sim r \vee q) \vee s] \wedge p\}\} \equiv \{(\sim p \wedge q \wedge \sim r) \vee [(p \wedge s) \vee (p \wedge q \wedge \sim r)]\}$$

$$\equiv [(\sim p \wedge q \wedge \sim r) \vee (p \wedge q \wedge \sim r)\ (p \wedge s)] \equiv \{\{[\sim p \wedge (q \wedge \sim r)] \vee [p \wedge (q \wedge \sim r)]\}$$

$$\vee (p \wedge s)\} \equiv \{[(q \wedge \sim r) \wedge (p \vee \sim p)] \vee (p \wedge s)\} \equiv [(q \wedge \sim r) \vee (p \wedge s)] = \beta$$

In turn, we move on to discuss issues regarding black boxes and their connection with logical networks. Black boxes appear in many fields of science. This concept was created within the framework of cybernetics and plays a very important role in the identification of the logical structure of investigated systems[22]. We are often able by deduction to conclude certain logical relationships-couplings inside a black box, that is, inside the system examined. Getting to know the complete set of couplings depends on the set of inputs and outputs, that is, the behavior of a system. In turn, the complexity of a system is often evaluated from the complexity of the set of inputs and outputs to the system.

The creator of information theory, C.E. Shannon, proved, however, that any behavior of a system can be obtained in the logical sense in an infinite number of ways, namely by infinitely many possible logical networks [9].

Example. The following three logical expressions represent three different logical networks in terms of their possible implementation, but from the point of view of propositional calculus they are all equivalent.

$$\text{A}: \alpha = x \wedge y$$

$$\text{B}: \beta = x \wedge (y \vee \sim x)$$

$$\text{C}: \gamma = y \wedge \{x \vee [\sim y \wedge (y \vee \sim x)]\}$$

By applying the relevant laws of logic, it is easy to demonstrate their equivalence. We will show that

$$\gamma \equiv \alpha$$

$$\gamma = \{y \wedge [x \vee (\sim y \wedge (y \vee \sim x))]\} \equiv \{y \wedge [(x \vee \sim y) \wedge (x \vee \sim x \vee y)]\}$$

$$\equiv \{y \wedge [(x \vee \sim y) \wedge y]\} \equiv [y \wedge (x \vee \sim y)] \equiv [(y \wedge x) \vee (y \wedge \sim y)] \equiv (x \wedge y) \equiv \alpha$$

Comment. It is often claimed in science that the structure of a process is understood but then it turns out that it acts differently, although it is not clear why. Perhaps

[22]The structure of a system (layout) is often tested, not only from the point of view of logic but also its analytical form, see, e.g. [10].

two equivalent logical structures, in terms of behavior, are different. We will give two examples to confirm our suppositions. One is connected to practical knowledge and the other with general knowledge (based on an example from game theory, because people operate with both practical and theoretical knowledge).

In particular, practical knowledge, e.g. all instruction manuals for devices, should be presented to their users in a simple, clear, unambiguous and easy to understand way.

Example. The ignition of an engine α can be manual or automatic. Automatic ignition occurs when a remote-controlled switch operates and, at the same time, there is no personnel in the cabin[23].

Let us therefore denote individual phrases with logical symbols in the following way:

- manual start – p,
- personnel present in the cabin – q,
- no personnel in the cabin – $\sim q$,
- remote control operation – r.

The logical normal form of engine ignition according to the above stated instructions are of the following form:

$$\alpha = \left[p \vee \left(\sim q \wedge r \right) \right]$$

The disjunction sign clearly separates the two possible ways of igniting the engine, and the expression presented is clearly in n.d.c. form, but the content of the instruction does not specify a clear way of understanding the disjunction as inclusive (standard) or exclusive. Therefore, the expression α corresponds to a number of different propositional functions $F(p_1, p_2, ..., p_n)$ but only one of them should correspond to our operational instructions, that is, represent the physical behavior of the device, and so possibly an equivalent form of the expression α should represent the behavior of this object.

One of the concepts of how humans behave and obtain (general) knowledge about the reality that surrounds them is given by the concept of a game[24]. Game theory considers games with different analytical forms[25]. There are always at least two participants. We consider zero sum two person games with players G_1 and G_2. Both have at

[23]Here we have knowledge which is specific and important to a user. It should be expressed logically and optimally with respect to its content, which is important for the trouble free utilization of the device.

[24]In particular, two person zero sum games.

[25]Game theory brings to science many theoretically and practically important benefits. One of the important dimensions of this theory is *statistical games*, which belong to the group of games against Nature.

their disposal a number of strategies. Even if one of the players is Nature, the normal form of such a game gives a very simple and clear logical image. Let a game be represented by its payoff matrix:

$$\mathbf{M} = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{i1} & \cdots & \vdots & \cdots & \vdots \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{m1} & \cdots & \cdots & \cdots & a_{mn} \end{bmatrix}.$$

The first player can choose from m strategies, and the second from n strategies. Let, therefore, player 1's choice of strategy be represented by the logical variables $p_1, p_2, \ldots, p_m$, and player 2's choice by the variables $q_1, q_2, \ldots, q_n$. The logical form of this game is shown in Table 4.

Table 4. Logical form of the under consideration

G_1/G_2	q_1	...	q_j	...	q_n
p_1	a_{11}	...	a_{1j}	...	a_{1n}
$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
p_i	a_{i1}	...	$\vdots$	...	a_{in}
$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
p_n	a_{m1}	...	a_{mj}	...	a_{mn}

The logical expression representing the above game is the following:

$$\alpha = \{[p_1 \wedge (q_1 \vee q_2 \vee \ldots \vee q_n)] \vee \ldots \vee [p_m \wedge (q_1 \vee q_2 \vee \ldots \vee q_n)]\} \tag{8}$$

By applying the relevant laws of logic, it can be easily shown that this is equivalent to the following expression:

$$\beta = [(p_1 \vee p_2 \vee \ldots \vee p_m) \wedge (q_1 \vee q_2 \vee \ldots \vee q_n)] \tag{9}$$

The expression β represents the logical structure of our game, and it is in normal conjunctive-disjunctive form. However, this is not a tautology, which is quite normal, because players only play the game from time to time, but when it is played, it takes the value 1 (one), as one of the variables p_i (i = 1, 2, ..., m) must take the value one (true), together with one of the variables q_j (j=1, 2, ..., n). In addition, it should be pointed out that the logical expression β implements, for each player separately, eliminating disjunction. Moreover, the equivalent expressions α and β can represent many games of different natures, though their logical representation is the same. In a way, this confirms our remark that the same logical form in terms of equivalence in physical implementation can represent different behaviors of the objects analyzed (with reference to logical networks). We mention game theory here because science obtains fundamental knowledge about the reality surrounding us, as well as knowledge about ourselves, from studying Nature, and logical operationalization may help greatly in the process of acquiring knowledge.

At the end, we refer to the application of the normal form to one of the most important aspects of logic, namely checking the consistency of a set of premises. While discussing the essence of the principle of resolution, it was stressed that the consistency of premises can be checked by applying the principle of resolution, but in order to do this, the normal conjunctive-disjunctive form of the premises should be obtained from their conjunction.

Example. We want to check the consistency of the following set of premises:

$$S = \{p \to q,\ r \vee \sim s,\ p,\ \sim r,\ \sim q \vee s\}$$

We create their n.c.d. form:

$$\alpha = (\sim p \vee q) \wedge (r \vee \sim s) \wedge (p) \wedge (\sim r) \wedge (\sim q \vee s)$$

Now each part of the conjunction forms a separate clause and applying the principle of resolution to them we obtain:

$\sim p \vee q$		(1)
$r \vee \sim s$		(2)
p		(3)
$\sim r$		(4)
$\sim q \vee s$		(5)
q	$(1, 3)$	(6)
s	$(5, 6)$	(7)
r	$(2, 7)$	(8)
contr.	$(4,8)$	(9)

A contradiction has been obtained, so our set of premises cannot be used for inference. The consistency of the set of premises S can be also checked using the laws of logic. Applying the conjunction separation law relative to disjunction several times to the expression α, we obtain the following expression:

$$\beta = \left(p \wedge q \wedge {\sim}q \wedge {\sim}r \wedge {\sim}s \right) \tag{10}$$

which is a contradiction because the variable q appears once without negation and secondly with negation. However, the method of resolution allows us to prove this contradiction in a simpler way. It does not require the application of any other laws of logic.

8. Conclusions

The idea connected with pragmatic dimension of propositional logic has been presented. This approach generally emphasizes: inference rules, inference schemes, role of normal forms in logical investigations, generating conclusions from premises and premises from conclusion, logical optimization and a some other important for logic concepts.

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