## Two-Simplet Superachromatic Objective with Single Lens

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The possibility of obtaining the superachromatic correction in the case of two lenses with air space between them has been analysed. It is shown that the superachromatic correction depends on the choice of glasses only. Then, one of the computed systems has been used for calculation of a more complicated superachromatic system composed of a three-lens simplet and a single lens. The aberrations of the calculated systems have been compared with those of an earlier computed system.

Chromatic aberration is one of the more difficult to correct, and it is to be corrected at the beginning of optical system calculation. The problem of colour correction in optical systems has been the subject of interest since long ago. The first achromatic systems were built by J. Dollond. These systems were made of two lenses of crown and flint glass and corrected for two colours (red and violet). The systems had a big secondary spectrum. This was a great disadvantage unabling their application to optical systems which were required to give faithful colour reproduction. Optical systems with better chromatic correction were first built by ABBE. They had the colour correction for three colours and were called "apachromats". For a long time apochromatic systems have been the best chromatically corrected systems. It was so until 1960, when the paper by STEPHENS [1] appeared. There it was shown that the chromatic correction is possible for four colours. Such systems were called "superachromats". They were investigated in greater detail by Herzberger who designed superachromatic systems composed of three lenses [2]. Herzberger formulated conditions which lenses must fulfil in order to form a superachromatic system. He also derived formulas for lens powers and additional conditions insuring the powers to be small.

The problem of superachromatic correction in optical systems has also been treated by SCHULTZ. He showed [3] that the superachromatic correction is possible for two lenses. This problem was fully considered in paper [4] where a simple condition, which glasses must fulfill, was derived and used for objective design calculation (f' = 100 mm, aperture 1:4.5, field of vision 6°). The first lens of this objective was made of fluorite and the second of Lak 11. The computed system had chromatic aberration comparable with that of the three--lens system published in paper [5], but the field aberrations were much greater. Additional restriction imposed on the system was that, practically, the first lens had to be made of fluorite, otherwise the values of lens power became comparatively large and practically prevented its application. Therefore, a system composed of two-lens simplets was developed, with the second lens being quasisimmetrical with respect to the first. It turned out that such a system had smaller aberrations, especially field aberrations, than the system composed of two simplets with the glasses applied in the same sequence.

The purpose of the present paper is to investigate the possibility of obtaining better correction using the same number of lenses. A system composed of one three-lens simplet and a single lens located at a distance has been investigated.

At the beginning we will consider the possibility of obtaining the superachromatic cor-

OPTICA APPLICATA I, 1 51

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rection using only two lenses separated by a distance d. The chromatic aberration of the system will disappear when the following conditions are fulfilled [6]

$$\sum_{k=1}^{n} h_k U_k = 0, (1)$$

where

$$U_k = \frac{a_{k+1} - a_k}{\mu_{k+1} - \mu_k} \left( \frac{1 - \mu_{k+1}}{\nu_{k+1}} - \frac{1 - \mu_k}{\nu_k} \right) \tag{2}$$

and

 $\mu = \frac{1}{n}$  — the reciprocal of the refractive index,  $\nu$  — the Abbe's number,  $\alpha$  — paraxial aperture angle.

We assume infinitely thin lenses, so that  $h_1 = h_2$ ,  $h_3 = h_4$ . The paraxial incidence height of the ray on the first lens is  $h_1 = 1$ , and the paraxial angle of the aperture ray after leaving the system is  $a_{n+1} = 1$  (it corresponds to the system focal length equal 1). The height  $h_3$  can be calculated from:

$$h_3 = h_1 - da_3. \tag{3}$$

Applying condition (1) for the system we get

$$\frac{\varphi_1}{\nu_1} + (1 - d\varphi_1)^2 \frac{\varphi_2}{\nu_2} = 0 \tag{4}$$

where

 $\varphi_i$  — power of the lenses.

The Abbe's number  $\nu$  defined as in the Herz-Berger's paper

$$v_{\lambda} = \frac{n_F - 1}{n_F - n_{\lambda}}. (5)$$

Superachromat is usually corrected for four lines  $\lambda_* = 1.014 \,\mu$ ,  $\lambda_c = 0.653 \mu$ ,  $\lambda_F = 0.486 \mu$ ,  $\lambda_{**} = 0.365 \mu$ . Writing the condition explicitely for four lines we have:

$$\frac{\varphi_1}{\nu_{1c}} = -(1 - d\varphi_1)^2 \frac{\varphi_2}{\nu_{2c}},$$

$$\frac{\varphi_1}{\nu_{1*}} = -(1 - d\varphi_1)^2 \frac{\varphi_2}{\nu_{2*}},$$

$$\frac{\varphi_1}{\nu_{1**}} = -(1 - d\varphi_1)^2 \frac{\varphi_2}{\nu_{2**}}.$$
(6)

The condition on glasses for superachromat can be derived from these equations:

$$\frac{n_{F_1} - n_{**_1}}{n_{F_1} - n_{*_1}} = \frac{n_{F_2} - n_{**_1}}{n_{F_2} - n_{*_2}}. (7)$$

The above condition is analogous to that which two glasses must fulfill in order to make superachromat without air separating two lenses. Therefore, the same two glasses may be used to calcutate superachromat regardless of the distance between lenses. Superachromat with air-filled space has bigger powers for all lenses, which is undesirable as far as higher order aberrations are concerned. Systems of this type usually serve as parts of bigger systems, and it is difficult to say about their usefullness without a detailed analysis. For comparison, a superachromat made of the same glasses is analysed in paper [4] (made of FK50, SK20 and Fluorite, Lak 11 respectively).

Equation (4) permits to express d as function of  $\varphi_1$ 

$$d = \frac{\nu_1(1-\varphi_1) + \nu_2}{\nu_1(\varphi_1 - \varphi_1^2)}.$$
 (8)

The power of the second lens can be found from:

$$\varphi_2 = \frac{1 - \varphi_1}{1 - d\varphi_1} \,. \tag{9}$$

The power of the first lens  $\varphi_1$  we assume to be a known parameter  $(\varphi_1 - \text{positive lens power})$  of a superachromat without air space).

These values are given for various systems and for glasses FK50 SK20 and Fluorite Lak 11. Each of those systems has been calculated in two different ways. Systems 1 and 3 were calculated strictly according to the formulas which insure superachromatic correction, assuming the thickness of the lens to be 1 and 5 mm, respectively. The air space in system No. 1 was 1.2 mm, while in system No. 3 was 3.1. mm. It turns out that in this type of system the allowance for thicknesses of lenses, particularly the thickness of the negative lens, deteriorates the chromatic aberration correction. In the table the difference between distances from the last lens surface, for different wavelengths, to the foci of the system are given for focal length f' = 100 mm. In superachromat without the air space the thicknesses of lenses did not practically influence the chromatic aberration correction. Systems 2 and 4 are calculated for the lens thickness equal zero. It is seen that the correction, especially for systems No. 2, is really superachromatic.

The differences between the distances from the last lens surface to the foci of the system, for different wavelengths

Glass	$S_D - S_\lambda$ [mm]							
	*	.1'	C	D	F	g	h	**
FK50 SK20	-0.1	-0.1	-0.03	0	0.11	0.18	0.23	0.3
77	0.06	-0.02	0	0	0	0	-0.01	-0.06
Fluorite Lak 11	0.03	-0.11	-0.01	0	0.03	0.06	0.13	0.01
11	0.08	-0.08	0.01	0	0	0.01	0.06	- 0.09
FK50 SK20 Fluorite Lak 11	-0.06	-0.01	-0.02	0	0.05	0.15	0.19	0.21
10	0.05	0.06	0	0	0.02	0.03	0.02	-0.03

When designing such a system better correction can not be gained by assuming zero thicknesses but by a small change in power of the second lens  $\varphi_2$ . Two-lens system has no important application in practice, and we will not consider it in what follows. Instead of it we will calculate the above mentioned system composed of four lenses. We shall compute the first simplet for glasses FK50, SK20, Fluorite, and the second composed of one lens made of Lak 11. The distance between the simplet and the last lens in the computed system is 6.1 mm. The difference between distances from the last lens surface to the foci of the calculated system are given for different wavelengths in the 5-th row of table. The chromatic aberration correction deviates from the required one, but a small change in the power of the last lens makes the correction sufficiently good (system No. 6). This system has been assumed as the starting point. We corrected the spherical aberration the sine condition, the coma and the field curvature by changing the radii, but preserving the focal length.

At the end, slight changes of focal lengths were used as a correction factor.

The aberrations of so calculated systems are shown in Fig. 1:

- a) spherochromatic aberrations for 5 selected wavelengths,
- b) deviation from the sine condition for line  $\lambda_D$ ,
  - c) curvature of image,
  - d) coma.

The deviation from the sine condition for the other wavelengths and full aperture angle is not greater than  $0.04~^{\rm o}/_{\rm o}$  at the worst case. For comparison, the aberrations of a quasisymmetric system from paper [4] have been quoted in Fig. 2. It is seen that the spherochromatic aberration of the calculated system is better.

It is true that the correction for the A' line deviates from the rest, but the correction for the line next to A', i. e. line \* (shown in the diagram), is satisfactory, and that of line C (absent in the diagram) is practically the same as that of line D. If we change the aperture to 1:4, then the spherochromatic aberration of the computed system will be about the same

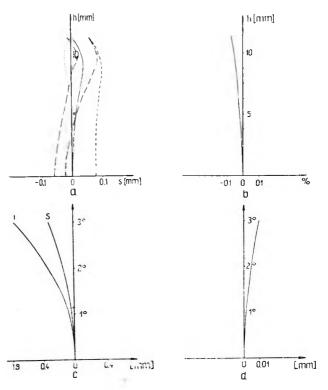


Fig. 1. Aberration curves of two-simplet superachromatic objective with single lens



as for the earlier computed system but for 1:4.5 aperture. The coma and deviation from the sine condition are better by an order of magnitude, only the field curvature is twice as worse. It is seen that for such a type of system

the field should be decreased to 3°. Without detailed analysis of suitable glasses it is difficult to assess to what extent the obtained corrections are optimal. It was not the aim of this work to calculate a system with the smallest aberrations, but to show that the

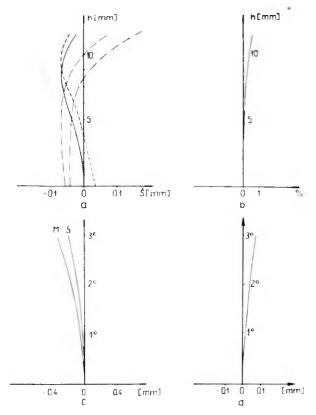


Fig. 2. Aberration curves of the system computed in paper [5]

superachromatic correction can be achieved also in the type of system described here. Besides, it turned out that this type of system has better correction of the aperture aberrations and more symmetric light beam. The derived condition on glasses of a two-lens superachromatic system does not depend on the lens spacing. The spacing influences only the powers of lenses, and may increase the influence of lens thickness on the superachromatic correction. Hence, it follows that the glasses which fulfill the condition of superachromatic correction for two- and three-lens systems [2], [4] or for their arbitrary combination, should be the basis for designing a superachromat. A superachromat i. not to be necessarily composed of superachromatic elements. In the calculated system none of the simplets is superachromat, not even achromat.

## Двухсимплетный суперахроматический объектив с одной линзой

В работе проанализирована возможность получения суперахроматической коррекции для двух линз с воздушным интервалом между ними. Показано, что суперахроматическая коррекци зависит только от подбора стёкол. Затем одна из так рассчитанных систем используется для расчёта более сложной суперахроматической системы, состоящей из трёхлинзового симплета и одной отдельной линзы. Сравниваются абберации так рассчитанной системы с абберациями системы расчитанной раньше.

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