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On Some Properties of the Astigmatic Beam for a Plane

On the basis of the Hopkins invariant $H_s = nu_s\eta_s$ it is shown that the sagittal foci of a parallel plane system lie on a straight line perpendicular to the planes. Making use of this property the changes in the displacement of sagittal focus and astigmatism of parallel plates with the changes in the plate thickness, refractive index and incidence angle have been obtained. The derived relations enable one to study the plate astigmatic foci without the necessity to recalculate the path of the rays. From these considerations it follows, among others, that the farther removed is the entrance pupil from the plate the smaller are the aberrations of the astigmatic beam. It is also shown that for small field angles the changes in the astigmatism are approximately equal to double values of the change in sagittal focus.

The optical imaging on a plane is relatively the simplest and it is easily noticeable that there are practically no reports on this subject. Flat surfaces are frequently used in optical systems; and for this reason any regularities found for them are of great value for the designers of optical systems. The starting point for this work has been the observation that the astigmatic sagittal foci for planes perpendicular to the optical axis lie on a straight line parallel to the optical axis, the distance between this line and the axis being equal to the distance between the object sagittal focus and the axis for the first plane. Obviously this conclusion can be generalized to apply to parallel planes at any angle to the optical axis (the sagittal foci will then lie on the straight line perpendicular to the considered planes).

The proof will commence with showing that the quantity $\bar{H}_s$ introduced by H. H. Hopkins [1, 2] remains invariant

$$\bar{H}_s = nu_s\eta_s,$$

where

n — refractive index,

u_s — paraxial sagittal aperture angle,

η_s — distance of the sagittal focus from the optical axis.

U_s (Fig. 1) is the angle made by a marginal ray of the astigmatic beam with the beam axis. The principal ray is regarded as the beam axis in what follows. The angle, in analogy to the normal paraxial angle, is not equivocally determined because the astigmatic beam is an elementary beam. Only the angle ratios are

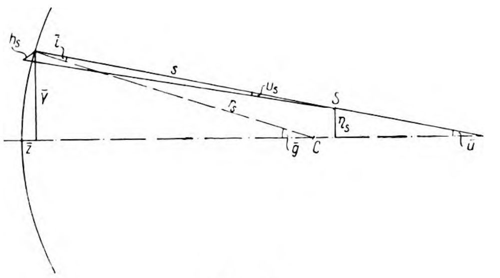

Fig. 1

determined. In analogy to the paraxial height of incidence h, the paraxial sagittal height h_s is also defined as equal to the product of the astigmatic sagittal focus distances (measured along the principal ray) and the paraxial sagittal aperture angle u_s

$$h_s = u_s s. \tag{1}$$

Also h_s, like u_s, is an undetermined quantity; only s has a direct physical meaning. Besides, these relations are identical with those for the paraxial image. If, however, we normalize one of these quantities, h_s or u_s, then owing to equation (1) and the equations for astigmatic foci for every surface, all the remaining paraxial sagittal quantities are already strictly determined. The assumption:

$$(h_s)_D = h_D, \tag{2}$$

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where

$(h_s)_D$ – paraxial sagittal height in the stop space,

$(h)_D$ – paraxial height of the aperture ray in the stop space,

leads to more simple relations [2, 3, 4] and will be consequently used in what follows. We will restrict our discussion to systems which have symmetry axis; the systems may be an optical system or a part of it. At the beginning we shall perform our considerations for single surfaces (Fig. 1). As seen in the figure

$$\eta_s = \bar{y} - s\sin\bar{u}. \tag{3}$$

Hence

$$\bar{H}_s = nu_s\eta_s = nu_s\bar{y} - nh_s\sin\bar{u}. \tag{4}$$

Let us calculate the change $\bar{H}_s$ caused by refraction on the k-surface

$$\begin{aligned}(\bar{H}'_s)_k - (\bar{H}_s)_k &= n'_k(u'_s)_k\bar{y}_k - n'_k(h_s)_k\sin\bar{u}'_k\\ &- n_k(u_s)_k\bar{y}_k + n_k(h_s)_k\sin\bar{u}_k\\ &= \bar{y}_k[n'_k(u'_s)_k - n_k(u_s)_k]\\ &- (h_s)_k[n'_k\sin\bar{u}'_k - n_k\sin\bar{u}_k].\end{aligned}$$

For the sagittal focus holds the equation [5]:

$$\frac{n'_k}{s'_k} - \frac{n_k}{s_k} = \frac{n'_k\cos\bar{i}'_k - n_k\cos\bar{i}_k}{(r_s)_k}, \tag{5}$$

where

$(r_s)_k$ – sagittal radius of the curvature for a given principal ray.

On multiplying both sides of Eq. 5 by $(h_s)_k$ and allowing for Eq. 1 we obtain:

$$\begin{aligned}n'_k(u'_s)_k - n_k(u_s)_k\\ = (h_s)_k(c_s)_k[n'_k\cos\bar{i}'_k - n_k\cos\bar{i}_k],\end{aligned} \tag{6}$$

where

$$(c_s)_k = \frac{1}{(r_s)_k}.$$

As seen in Fig. 1

$$\bar{y}_k = (r_s)_k\sin\bar{g}_k, \tag{7}$$

$$\bar{g}_k = \bar{u}'_k + \bar{i}'_k = \bar{u}_k + \bar{i}_k.$$

By using equations (6) and (7) it is easily seen that:

$$\begin{aligned}(\bar{H}'_s)_k - (\bar{H}_s)_k &= (h_s)_k\sin\bar{g}_k(n'_k\cos\bar{i}'_k\\ &- n_k\cos\bar{i}_k) -\\ &- (h_s)_k[n'_k(\sin\bar{g}_k\cos\bar{i}'_k - \cos\bar{g}_k\sin\bar{i}'_k)\\ &- n_k(\sin\bar{g}_k\cos\bar{i}_k - \cos\bar{g}_k\sin\bar{i}_k)] = 0.\end{aligned} \tag{8}$$

In the last equation the low of refraction has been used

$$n'_k\sin\bar{i}'_k = n_k\sin\bar{i}_k. \tag{9}$$

Since

$$n'_k = n_{k+1},$$
$$(u'_s)_k = (u_s)_{k+1},$$
$$(\eta'_s)_k = (\eta_s)_{k+1}.$$

Then

$$(\bar{H}'_s)_k = (\bar{H}_s)_{k+1}. \tag{10}$$

By using (8) and (10) one obtains:

$$\begin{aligned}(\bar{H}_s)_1 = (\bar{H}'_s)_1 = (\bar{H}_s)_2 = \ldots =\\ = (\bar{H}_s)_p = (\bar{H}'_s)_p\end{aligned} \tag{11}$$

where p denotes the number of the last surface in a system. Thus the invariant $\bar{H}_s$ is an invariant of a whole optical system with rotational

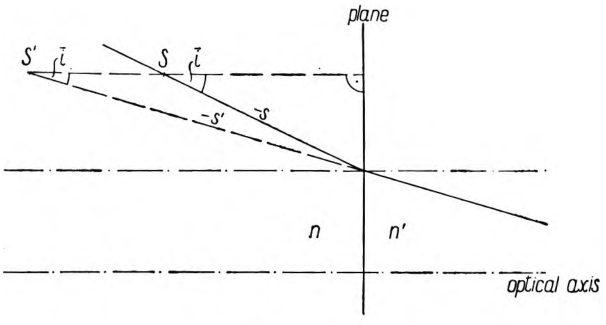

Fig. 2

symmetry. For flat surfaces it follows from Eq. 5 that:

$$n'_k(u'_s)_k - n_k(u_s)_k = 0. \tag{12}$$

This condition together with Eq. 11 leads to the relation:

$$\eta_1 = \eta'_1 = \eta_2 = \ldots = \eta'_p. \tag{13}$$

From Eq. 13 it is seen that all distances of the sagittal foci from the symmetry axis are identical for flat surfaces. So, if a system or its part consist exclusively of planes parallel to each other, then all sagittal foci lie on one straight line. This can also be verified starting from Eqs. 5 and 9. Let the straight line SS' be parallel to the optical axis (Fig. 2). As seen in the figure

$$\frac{s}{\sin\bar{i}'} = \frac{s'}{\sin\bar{i}} \quad \text{or} \quad \frac{s'}{s} = \frac{\sin\bar{i}}{\sin\bar{i}'} = \frac{n'}{n}.$$

The last equality follows from the refraction law. Thus we have

$$\frac{s'}{s} = \frac{n'}{n} \tag{14}$$

which corresponds to Eq. 5 for planes. The meridional astigmatic focus satisfies this relation in two trivial cases only ($\bar{i} = 0$ or $n' = n$). In other cases the meridional astigmatic focus does not lie on a straight line.

The derived relations will be applied for an investigation of the astigmatic beam behaviour after it has passed through a parallel plate.

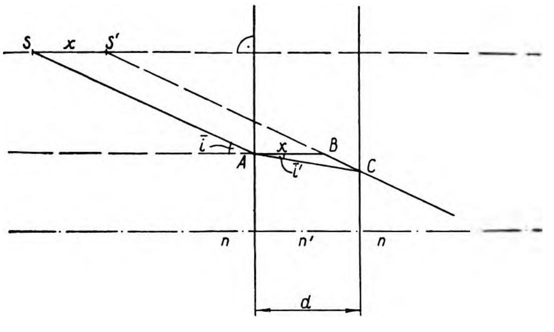

Fig. 3

This problem is of a fairly great practical importance due to the fact that the prisms used in optical systems are equivalent when expanded to parallel plates. Let a plate and its surrounding medium have their refractive indices n' and n respectively. Let us assume $n' \geqslant n$.

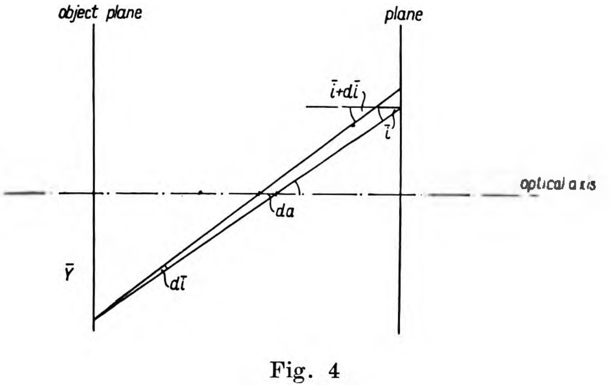

Fig. 4

At the beginning a formula will be derived for the distance of the astigmatic sagittal focus from the ideal point image (Fig. 3). According to the previous derivation, S' must lie on a straight line perpendicular to the plate and passing through the point S. From the properties of the astigmatic beam it is known that the point S' lies on the principal ray. The astigmatic sagittal image focus S' is thus determined by the intersection point of a normal to the plate, passing through the point S, and the image principal ray. In Fig. 3 the interfocus

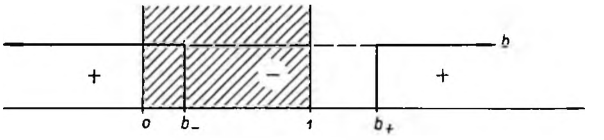

Fig. 5

distance is denoted by x. From the triangle ABC one obtains

$$\frac{x}{\sin(\bar{i}-\bar{i}')} = \frac{d}{\cos\bar{i}'\sin\bar{i}}.$$

Hence on rearranging

$$x = d\left(1-\frac{n\cos\bar{i}}{n'\cos\bar{i}'}\right). \tag{15}$$

The displacement for a nonaberration image would have the value

$$x_p = d\left(1-\frac{n}{n'}\right). \tag{16}$$

The equation 15 for very small incidence angles goes into the familiar form (16) which expresses the paraxial displacement for parallel plate. The displacement of the sagittal image focus, later on referred to as displacement, relative to an ideal image equals to

$$x - x_p = d\frac{n}{n'}\left(1-\frac{\cos\bar{i}}{\cos\bar{i}'}\right). \tag{17}$$

As seen from Eq. 17 it is proportional to the plate thickness. The change of the displacement with thickness will be given by:

$$\Delta_d(x-x_p) = \Delta d\frac{n}{n'}\left(1-\frac{\cos\bar{i}}{\cos\bar{i}'}\right). \tag{18}$$

Before examining the displacement dependence on the refractive index and incidence angle, the change of astigmatism will be calculated. This will allow us to study the changes in both the sagittal and meridional foci. As follows from (14)

$$s_1' = s_1\frac{n'}{n},$$

$$s_2 = s_1' - \frac{d}{\cos\bar{i}_1'} = s_1\frac{n'}{n} - \frac{d}{\cos\bar{i}_1'}, \tag{19}$$

$$s_2' = s_2\frac{n}{n'} = s_1 - \frac{n}{n'}\frac{d}{\cos\bar{i}_1'}.$$

In analogy for the meridional image focus, from the condition

$$\frac{n'\cos^2\bar{i}'}{t'} - \frac{n\cos^2\bar{i}}{t} = \frac{n'\cos\bar{i}' - n\cos\bar{i}}{r}, \tag{20}$$

one obtains

$$t_2' = t_1 - d\frac{n\cos^2\bar{i}_1}{n'\cos^3\bar{i}_1'}. \tag{21}$$

For simplicity index 1 will be ommited in what follows, this implying the following assumption:

$$\bar{i}_1' = \bar{i}' \quad \bar{i}_1 = \bar{i}.$$

From Eq. 19 and 21 one obtains:

$$t_2' - s_2' = t - s - \frac{dn}{n'\cos\bar{i}'}\left(\frac{\cos^2\bar{i}}{\cos^2\bar{i}'} - 1\right) \tag{22}$$

or

$$t_2' - s_2' = t - s + \frac{d\sin^2\bar{i}}{\cos^3\bar{i}'}\frac{n}{n'}\left[1 - \left(\frac{n}{n'}\right)^2\right]. \tag{22a}$$

The astigmatism changes also linearly with the plate thickness. The change can thus be expressed by

$$\Delta_d(t_2' - s_2') = \Delta d\frac{n}{n'\cos\bar{i}'}\left(1 - \frac{\cos^2\bar{i}}{\cos^2\bar{i}'}\right). \tag{23}$$

Now we shall examine how the displacement depends on the incidence angle $\bar{i}$

$$\frac{\partial(x - x_p)}{\partial\bar{i}} = -d\frac{n}{n'}\frac{\partial}{\partial\bar{i}}\left(\frac{\cos\bar{i}}{\cos\bar{i}'}\right)$$
$$= -d\frac{n\sin\bar{i}}{n'(\cos\bar{i}')^3}\left[\left(\frac{n}{n'}\right)^2 - 1\right]. \tag{24}$$

The expression in the bracket is not positive, and hence the displacement increases with the incidence angle regardless of the plate thickness and its refractive index. The variation of the astigmatism with the incidence angle is the following:

$$\frac{\partial(t_2' - s_2')}{\partial\bar{i}} = \frac{\partial(t - s)}{\partial\bar{i}} + \frac{nd}{n'}$$
$$\times\left[1 - \left(\frac{n}{n'}\right)^2\right]\frac{\partial}{\partial\bar{i}}\left(\frac{\sin^2\bar{i}}{\cos^3\bar{i}'}\right) = \frac{\partial(t - s)}{\partial\bar{i}} + \frac{nd}{n'}$$
$$\times\left[1 - \left(\frac{n}{n'}\right)^2\right]\frac{\sin\bar{i}\cos\bar{i}}{\cos^5\bar{i}'}$$
$$\times\left[2 + \frac{n}{n'}\sin^2\bar{i}\left(3 - 2\frac{n}{n'}\right)\right]. \tag{25}$$

The first term equals to zero if an object is placed just before the plate. This term equals to the change in the system's astigmatism with the change in the field angle if there is an optical system or its part, before the plate. The purpose of our considerations is to estimate the changes in the plate astigmatism which is expressed by the second term in Eq. 25. Since

$$\frac{nd}{n'} > 0, \quad 1 - \left(\frac{n}{n'}\right)^2 \geqslant 0, \quad \frac{\sin\bar{i}\cos\bar{i}}{\cos^5\bar{i}'} > 0,$$

the following expression decides the sign of the change in astigmatism:

$$2 + \frac{n}{n'}\sin^2\bar{i}\left(3 - 2\frac{n}{n'}\right).$$

As easily noticeable, it is also positive. Thus we have shown that with increasing incidence angle the displacement and astigmatism both increase, and consequently both foci depart from the ideal image given by the plate. If the rest of the optical system allows it, the diaphragram should be placed in a way which gives the maximal possible distance from the entrance pupil to the parallel plate. Let us displace the diaphragm by a small distance *da* and calculate how much the incidence angle will change. As seen in Fig. 4

$$\frac{\bar{y}}{\sin\bar{i}}d\bar{i} = da\sin(\bar{i} + d\bar{i}) = da\sin\bar{i}.$$

In the last equation we have neglected a quantity of the second order of smallness. Hence the calculated value $d\bar{i}$ is

$$d\bar{i} = \frac{\sin^2\bar{i}}{\bar{y}}da. \tag{26}$$

The variation of the displacement and astigmatism with the diaphragm position will be obtained from relations (24, 25, 26)

$$\frac{\partial(x - x_p)}{\partial a} = \frac{\partial(x - x_p)}{\partial\bar{i}}\frac{\partial\bar{i}}{\partial a}$$
$$= d\frac{n}{n'}\frac{\sin^3\bar{i}}{\bar{y}\cos^3\bar{i}'}\left[1 - \left(\frac{n}{n'}\right)^2\right] \tag{27}$$
$$\times\frac{\partial(t_2' - s_2')}{\partial a} = \frac{\partial(t - s)}{\partial a} + \frac{nd}{n'}\frac{\sin^3\bar{i}\cos\bar{i}}{\bar{y}\cos^3\bar{i}'}$$
$$\times\left[1 - \left(\frac{n^2}{n'}\right)\right]\times\left[2 + \frac{n}{n'}\sin^2\bar{i}\left(3 - 2\frac{n}{n'}\right)\right].$$

Now we shall calculate the change of the displacement and astigmatism with a change in the refractive index. In order to simplify the resulting formulae we introduce the following notation:

$$z = \frac{n}{n'}, \qquad b = \sin^2\bar{i}.$$

For displacement

$$\frac{\partial(x-x_p)}{\partial z} = d\frac{\partial}{\partial z}\left[z\left(1-\frac{\cos\bar{i}}{\sqrt{1-bz^{2'}}}\right)\right]$$
$$= \left[1-\frac{\cos\bar{i}}{\cos\bar{i}'}-z^2\frac{\sin^2\bar{i}\cos\bar{i}}{\cos^3\bar{i}'}\right]d \qquad (28)$$
$$= \left(1-\frac{\cos\bar{i}}{\cos^3\bar{i}'}\right)d.$$

For $z = 1$ this expression is negative regardless of the incidence angle $\bar{i}$ (excluding $\bar{i} = 0$). For other z values it is negative if $\cos^3\bar{i} < \cos\bar{i}$. Since in the considered interval cosines are always positive, this inequality is equivalent to $\cos^6\bar{i} < \cos^2\bar{i}$ which, using the new notation, can be rewritten as:

$$(1-bz^2)^3 < 1-b.$$

On rearranging this inequality takes the form:

$$(b-b_+)(b-b_-) < 0$$

where

$$b_\pm = \frac{3z\mp\sqrt{4-3z^2}}{2z^3}$$

According to our assumption the value of z is always smaller than unity. Thus two solutions exist, b_- being always positive whereas b_+ positive only for $z^2 > 1/3$ (for $z > 0{,}5778$). Let us examine whether b_+ can be greater than unity. To this end we will calculate the z derivative of b_+. On rearranging we obtain

$$\frac{\partial b_+}{\partial z} = \frac{-3z^2-3z\sqrt{4-3z^2}+6}{z^4\sqrt{4-3z^2}}.$$

For the considered values of z ($0{,}5 \leqslant z \leqslant 1$) this expression is always positive and hence the function b_+ is monotonic increasing, assuming its maximal value $b_+ = 1$ for $z = 1$. The areas with positive and negative displacement are marked in Fig. 5. Physical meaning can only be prescribed to those values of b which lie within the limits $0 \leqslant b < 1$ (the dashed area). Since $b_+ = 1$ for $z = 1$, there exists no area with positive derivative for this value. For other values of z we can read from the diagram and equations (29) those values of the incidence angles which correspond to positive derivative. For example for $z = 0{,}5$ the positive derivative area corresponds to all values of the incidence angle. Other values can be taken from the table 1.

Table 1

z	The area with positive derivative	The area with negative derivative
0,5	$0 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0$
0,55	$0 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0$
0,6	$0{,}21108 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}21108$
0,65	$0{,}54068 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}54068$
0,7	$0{,}74256 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}74256$
0,75	$0{,}86436 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}86437$
0,8	$0{,}93525 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}93525$
0,85	$0{,}94687 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}94687$
0,9	$0{,}99245 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}99245$
0,95	$0{,}99903 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}99903$
1,0	$1 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 1$

Now we shall calculate the varation of astigmatism with the refractive index

$$\frac{\partial(t_2'-s_2')}{\partial z} = d\sin^2\bar{i}\frac{\partial}{\partial z}\left(\frac{z(1-z^2)}{(1-bz^2)^{3/2}}\right)$$
$$= d\sin^2\bar{i}\frac{1+z^2(2\sin^2\bar{i}-3)}{\cos^5\bar{i}'}. \qquad (30)$$

Since $\dfrac{d\sin^2\bar{i}}{\cos^3\bar{i}'} > 0$ for all incidence angles, the sign of the derivative is decided by the following expression:

$$G = 1+z^2(2b-3) = 2z^2\left(b-\frac{3z^2-1}{2z^2}\right).$$

The expression G is not negative if

$$b \geqslant \frac{3z^2-1}{2z^2}.$$

For $z = 1$ it is not negative only for $b = 1$; for $0{,}5 \leqslant z \leqslant 0{,}5778$ it is positive for all values of b. The areas of permanent sign are listed in table 2. Comparing the two tables we see that for different incidence angles (different field angles) the change in the astigmatic focus position can be in both directions. For $z = 0{,}6$, that is for almost all glasses when $\sin^2\bar{i} \leqslant 0{,}1$, the change in foci position is the smaller the nearer to unity the relative refractive index. The condition $\sin^2\bar{i} \leqslant 0{,}1$ corresponds to the angle $\bar{i} \leqslant 19°20'$, which practically covers the whole application range of parallel plates.

Table 2

z	The area with positive derivative	The area with negative derivative
0,5	$0 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0$
0,55	$0{,}15289 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}15289$
0,6	$0{,}11111 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}11111$
0,65	$0{,}31656 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}31656$
0,7	$0{,}47959 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}47959$
0,75	$0{,}61111 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}61111$
0,8	$0{,}71875 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}71875$
0,85	$0{,}80795 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}80795$
0,9	$0{,}88271 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}88271$
0,95	$0{,}94598 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 0{,}94598$
1,0	$1 \leqslant b \leqslant 1$	$0 \leqslant b \leqslant 1$

It should be emphasized that the change in astigmatism is greater that the change in displacement, and thus the change in the plate thickness is accompanied by the following ratio of both changes (Eqs. 18 and 23):

$$\frac{\Delta_d(t_2 - s_2)}{\Delta(x - x_p)} = \frac{1}{\cos\bar{i}'}\left(1 + \frac{\cos\bar{i}}{\cos\bar{i}'}\right). \tag{31}$$

As easily seen, it is always greater than unity and approaches two for small angles. Similarly, making use of (28, 30), we obtain (for an object without astigmatism)

$$\frac{\partial(t_2' - s_2')}{\partial z} : \frac{\partial(x - x_p)}{\partial z} = \frac{\sin^2\bar{i}\,[1 + z^2(2\sin^2\bar{i} - 3)]}{\cos^5\bar{i}' - \cos\bar{i}\cos^2\bar{i}'}. \tag{32}$$

On rearranging and neglecting terms $\sin^k\bar{i}$ if $k \geqslant 6$, one obtains

$$\frac{\partial(t_2' - s_2')}{\partial z} : \frac{\partial(x - x_p)}{\partial z} = \frac{2[1 + z^2(2b - 3)]}{1 - 3z^2 + (1/4)b(1 - 4z^2 + 15z^4)}.$$

For small angles this expression also tends to two. By using formulae (24, 25) we obtain

$$\frac{\partial(t_2' - s_2')}{\partial\bar{i}} : \frac{\partial(x - x_p)}{\partial\bar{i}} = \frac{\cos\bar{i}}{\cos^2\bar{i}'}\,[2 + z\sin^2\bar{i}(3 - 2z)]. \tag{33}$$

So, again for small angles the value is near 2. As easily noticeable, for $\bar{i} \to 0$ formulae (31, 32, 33) approach 2. In practice we usually have to deal with small angles, and thus it can be assumed that the change in astigmatism (when estimated) approximately is equal to double displacement (of the sagittal focus).

The relations derived above enable us to study the astigmatic foci of parallel plates without the necessity to recalculate the path of rays and facilitate the correction analyses of prism containing systems.

Sur quelques propriétés du pinceau astigmatique pour les surfaces planes

En partant de l'invariant de Hopkins $\overline{H}_s = nu_s\eta_s$ on a démontré que les foyers sagittaux d'un système de surfaces planes et parallèles sont situés sur la normale aux surfaces. En utilisant cette propriété on a établi la variation du déplacement du foyer sagittal ainsi que de l'astigmatisme de la lame à faces parallèles en fonction de l'épaisseur de la lame, du coefficient de réfraction et de l'angle d'incidence. Les relations obtenues permettent d'étudier les foyers astigmatiques de la lame sans calculer le parcours des rayons. Il en résulte, entre autres, que plus la pupille d'entrée est éloignée de la lame, plus les aberrations du pinceau astigmatique diminuent. On a démontré aussi que pour les petits angles du champ visuel les variations de l'astigmatisme sont, à peu près, deux fois plus grandes que les variations du foyer sagittal.

О некоторых свойствах астигматического пучка для плоских поверхностей

Опираясь на инвариант Гопкинса $\overline{H}_s = nu_s\eta_s$ установлено, что сагитальные фокусы параллельных плоских поверхностей лежат на перпендикулярной прямой. Пользуясь этим свойством, выведены изменения перемещения сагитального фокуса и астигматизма плоско-параллельной пластинки вместе с изменением толщины пластинки, коэффициента преломления стекла, а также угла падения. Выведенные зависимости предоставляют возможность исследовать астигматические пучки пластинки без необходимости перерасчёта хода лучей. Из рассуждений этих следует, между прочим, что чем дальше расположен от пластинки входной зрачок, тем аберрации астигматического пучка меньшие. Установлено также, что для малых углов поля зрения изменения астигматизма приблизительно в два раза больше, чем изменения сагитального фокуса.

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