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Variability of Birefringence Properties of the Optical Glass Blocks during Cutting them into Elementary Pieces for Optical Elements

The paper presents the results of experiments carried out to establish the rules of changes in double refractivity when altering both the magnitude and sizes of the rectangular blocks or rounded glass pieces. The measurements were carried out on samples made out of the glass melted inafire-clay pots and pored into quadrate forms. The results have been gathered and are presented in the graphs which allow to foresee the double refractivity at an arbitrary point lying on the symmetry axis of the surface of the.diminished glass blocks.

Introduction

Birefringence of the optical glass is caused by the volume stress determined by its thermal past. It is not the mechanical strength that makes people try to restrict the birefringence. The stress introduces waste heterogeneity of the refractive index. Some information about that effect may be gained from the graph shown in Fig. 1, the latter being reproduced here from the catalogue of the optical glass works, Spezial Glas G. M. B. H. Mainz. As can be seen the differences in the refractive index in an optical piece are greater than the birefringence determined with the polarimeter i.e. than the pure difference between the indices of refraction $\Delta n = n_1 - n_2$ for the rays, one of which (n_1) vibrates parallelly, while the other (n_2) perpendicularly to the stress vector direction σ in the sampling point. For example, the real differences in the index of refraction in a sample made of heavy flint are nearly 5 times as great as the birefringence $n_1 - n_2$.

It is well known from practice that the birefringence in optical elements is less than that in a block of glass used to make them. The authors tried to find an experimental relation which would enable to foresee the double refraction in an optical element on the base of the birefringence of the glass block. The calculations of the birefringence in the optical pieces would be very difficult because of the variety of their possible shapes. In this paper we would restrict our attention to the possibility of forecasting the birefringence in the elementary pieces of optical elements.

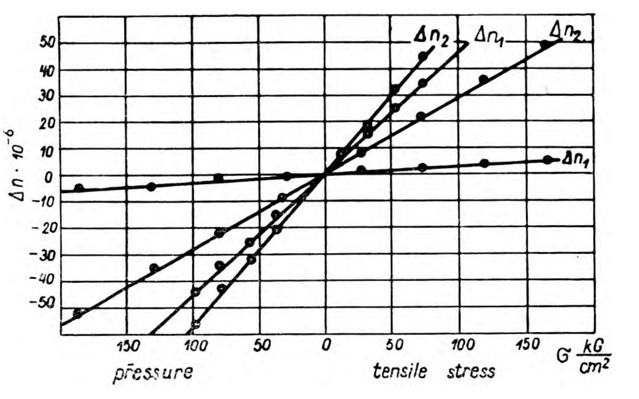

Fig. 1. Changes of the refractive indices in the glass subject to pressure or tensile stress perpendicular to the direction of the light beam

Δn_1 — changes of refractive index of light ray vibrating parallelly to the stress vector direction,
Δn_2 — changes of refractive index of light ray vibrating perpendicularly to the stress vector direction.
—.—.—.— glass of the BK type (Spezial Glas GMBH Mainz)
—o—o—o—o glass of the SF type (Spezial Glas GMBH Mainz)

Usually two types of elementary samples are considered: rounded pieces for lenses (Fig. 2) and rectangular pieces for prisms (Fig. 3). A rounded piece of glass, which is supposed to be a material for a lens, will be named unit rounded elementary sample. The corresponding

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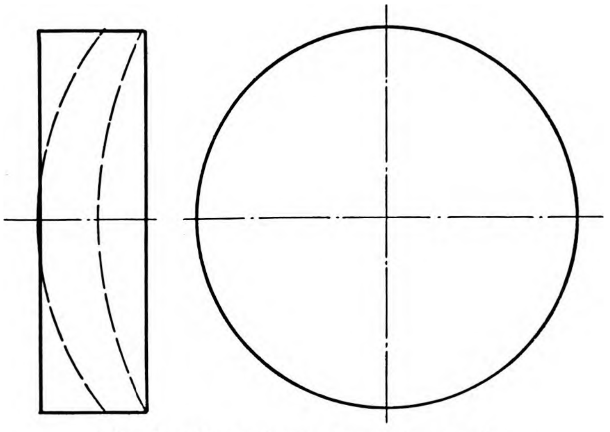

Fig. 2. Elementary rounded sample

sample of quadrate form circumscribing a future prism is called unit rectangular elementary sample.

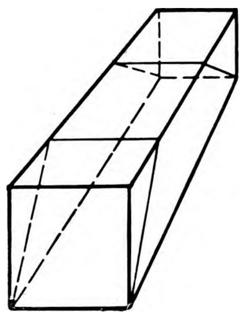

Fig. 3. Elementary quadrate sample

Material to be measured

Here, glass produced in J. Z. O (Poland) was chosen for measurements. In the said factory the glass is melted by a classical method, pored in rectangular forms and tempered in an electric bell-type furnace with automatically controlled temperature. Thus some rectangular glass plates of a thickness of about 18 cm and the other sizes of about 110 cm each are obtained (Fig. 4). After tempering, the plate is cut into rectangular blocks of the sizes of about $30 \times 20 \times 18$ cm, two opposite sides being polished afterwards. These polished sides render possible to examine the inside of the block so far as striaes and stress are considered. They will be called acting sides. This is that kind of blocks or their parts which were used to measure the birefringence changes during the glass block cutting process.

The double refraction measured at a given point of the acting surface depends not only on the internal stress, but also on the surface stresses occuring in both the untreated and grounded surfaces of the blocks. To avoid the surface stress also the unacting block, surfaces were polished or grounded with such tiny grinding powders that the surface stress became negligible.

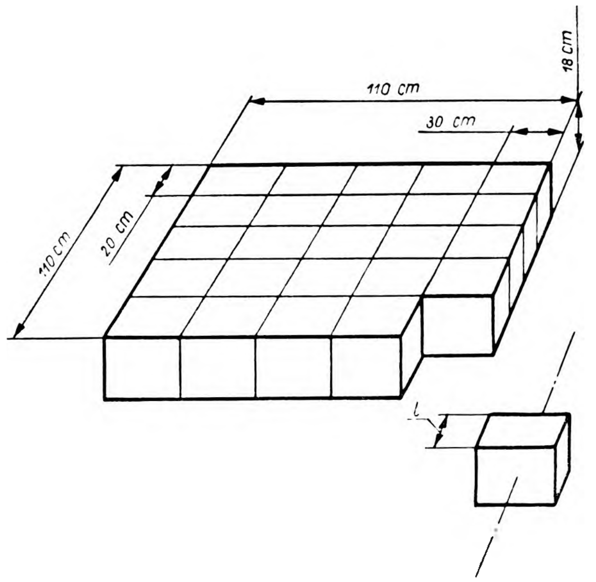

Fig. 4. The dimensions and the way of tracing of the glass plate

It will be convenient to state at the beginning of our considerations that the birefringence changes appeared to be independent of the viewing direction as well as the glass sort. Hence we will not specify either the glass sort or the viewing direction. However, the viewing direction in each measurement series was preserved.

Method of measurement

The measurements were carried out with the help of the linear polariscope using the Senarmonte compensator. The linear polariscope consists of the polariser P (Fig. 5a) and an analyser A crossed with the first. The examined glass block G is placed between the polariser and the analyser in such a way that one of its axes of transmission (identical with one of the main stress directions) makes an arbitrary angle α with the transmission axis of the polariser. To simplify the data presentation let us assume that the polariser transmission axis is vertical. The angle α grows

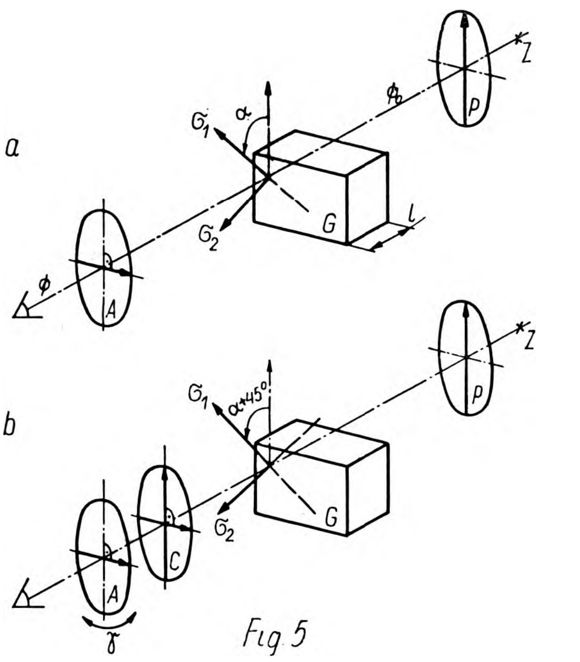

Fig. 5. a) Linear polariscope b) Linear polariscope with a $\lambda/4$ retarding compensator

in the direction opposite to the clock hand movement. Iluminator Z emits the monochromatic light beam of the wave length λ. In that mutual position of the set-up elements the light flux leaving the analyser is expressed by the formula $\phi = \phi_0 \sin^2 2\alpha \sin^2 \delta/2$ (1), where ϕ_0 denotes light flux transmitted through the polariser, ϕ — light flux transmitted through the analyser and δ is the phase difference between the rays vibrating in the directions consistent with the main stresses σ_1 and σ_2 at the measurement point.

When observing the glass block in the polariscope, two kinds of lines may be observed: isoclines and isochromatic lines. Isoclines are understood as the set of points on the sample acting surface at which one of the transmission directions makes the same angle α with the vertical. These angles are called isocline asimuths. In the up-to-now description it was assumed that there exists a plane state of stress acting in the directions perpendicular to the viewing direction. This assumption being justified by the fact that the forces projected on the viewing direction do not influence the polarisation status of the light. From the formula (1) it is clear that the isocline line is dark ($\phi = 0$) when $\alpha = 0$ or $\alpha = 90°$. To make the isocline, associated with the asimuth α to be a dark one, it is necessary to rotate the glass block to such a position that $\alpha = 0$, in other words so that the polariser and analyser transmission axes were identical with the main stress directions.

A typical isocline distribution corresponding to different asimuths is shown for a glass block in Fig. 6. It can be seen that each of the axes

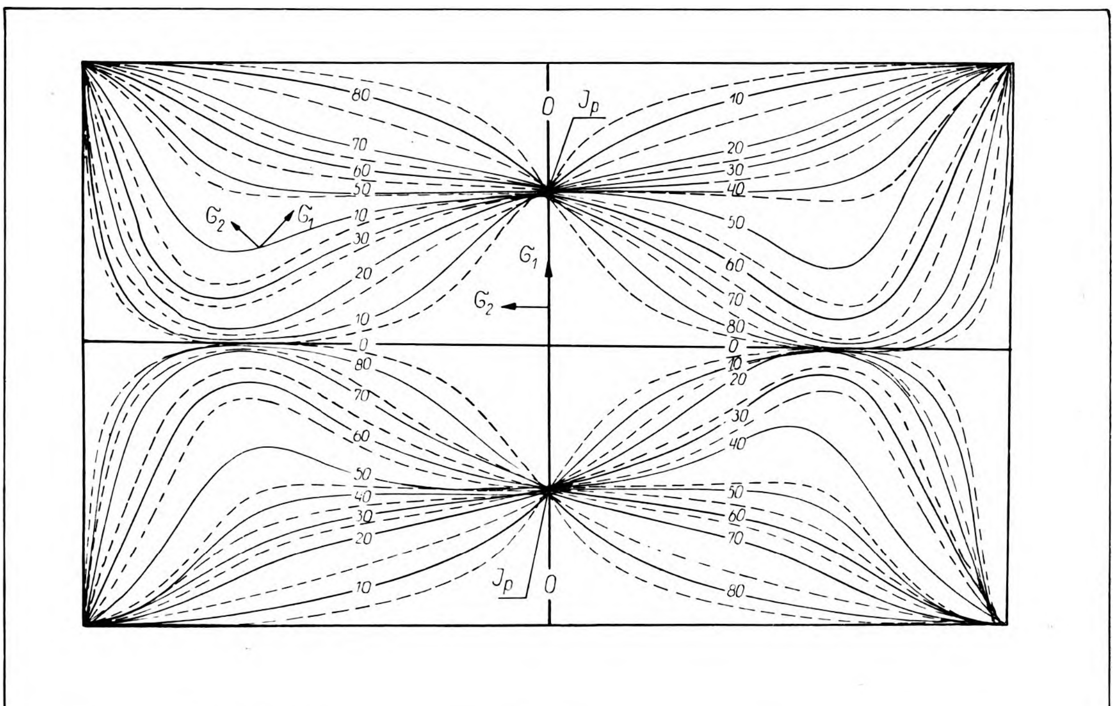

Fig. 6. Isocline lines distribution on the acting surface of the block of optical glass

of symmetry of an acting surface overlaps one of the main stress directions at the points lying on these axes.

The other kind of lines observed in a linear polariscope are an isochromatic stress pattern. An isochromatic line is a set of points of constant phase difference $\delta = 2k\pi = \text{const.}$ in the acting surface of the examined block. The whole value of k, denoted by k_c, is called an isochromatic line order. Isochromatic lines for integer value of k are dark if the monochromatic light is used (see Eq. 1). The points at which $k = 0$ are called isotropic (I_p on Fig. 6 and Fig. 7). The order of the isochromatic line at the measurement point is equal to the number of dark isochromatic lines between the isotropic and measuring points. The angle δ may be also represented in the form $\delta = 2\pi R/\lambda$ where R is the optical path difference in the glass between the rays vibrating in σ_1 and σ_2 directions. Thus the isochromatic line may be also defined as a set of points located on the acting glass block surface of constant optical path difference. Quotient rate of the optical path difference to the geometrical path $\Delta n = R/L$ is the measure of the double refraction in the medium.

The glass block birefringence was measured with the help of the Senarmonte's method. The corresponding arrangement (Fig. 5b) differs from the linear polariscope previously described in that there is a $\lambda/4$ retarder located between the glass block and the analyser, the transmission axes of the retarder being identical with those of the polariser and the analyser respectively. The main stress directions at the measurement point are bisecting lines for the angles between the transmission axes of the retarder. In that position the measurement point of a sample is not dark in general. To darken it, it is necessary to turn the analyser so that the darkening comes from the isotropic point. Denoting the analyser rotation angle by γ, we have for the birefringence at the sample point

$$\Delta n = \frac{\lambda}{l}\left(k_c + \frac{\gamma}{180}\right). \qquad (2)$$

Evidently, there is a nonuniform state of tension along the trace of the ray passing

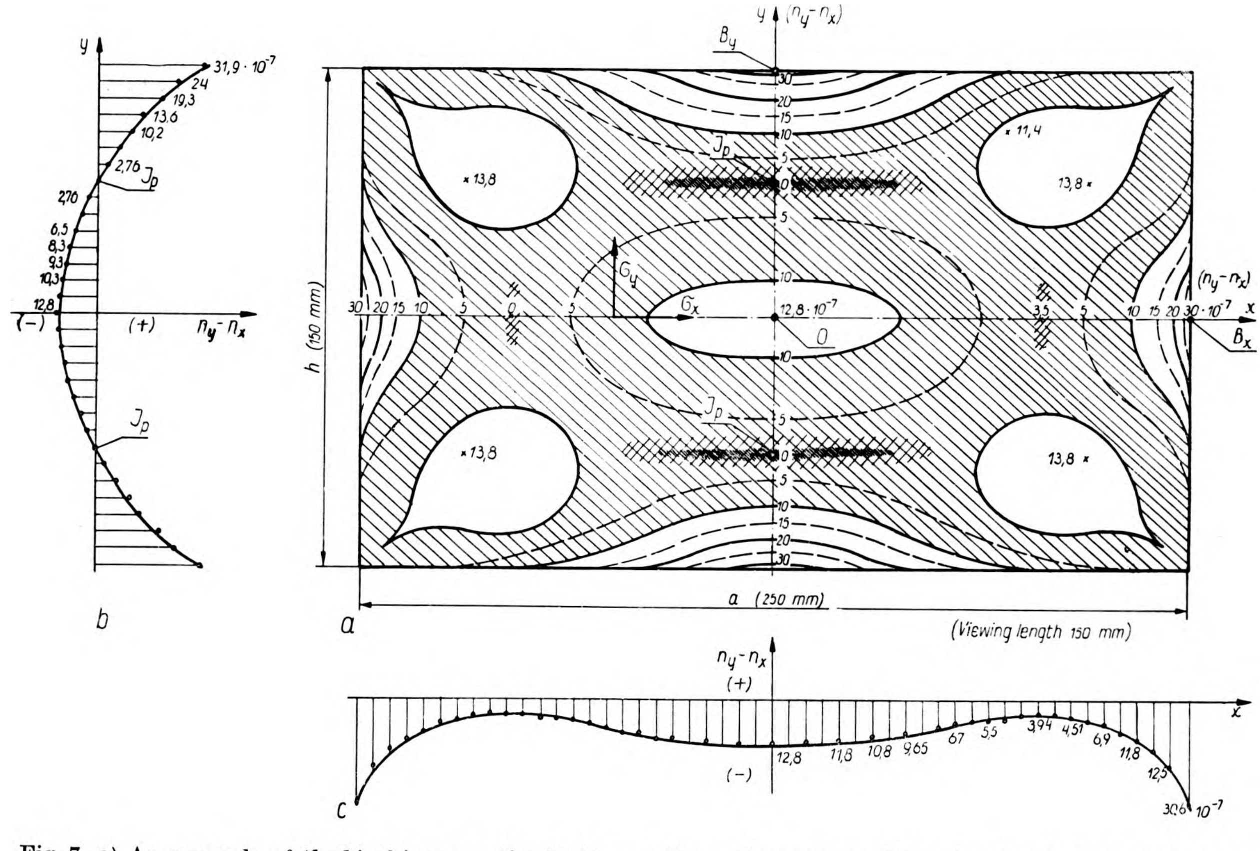

Fig. 7. a) An example of the birefringence distribution on the acting surface of the glass block, b) Birefringence distribution along the y-axis on the acting surface of the block, c) Birefringence distribution along the x-axis on the acting surface of the block

through the sampling point, and consequently the birefringence in various volume elements distributed along the ray trace is different. For this reason the birefringence estimated from the formula (2) is an average value. Also the notion of the sampling point used so far is a matter of convention, because it denotes in reality only a point of the acting side of the block lying on the ray trace.

Measurement results

The most important source of information about the changes of birefringence after cutting the block into smaller pieces appeared to be the analysis of the birefringence distribution on the acting surface of the examined glass. The birefringence distribution in the case of different glass sorts is similar. In Fig. 7a a typical pattern of the birefringence on the acting surface for some glass blocks made of BaK 569-56 (J. Z. O. production) is shown, x and y denoting the two axes of symmetry. As the symmetry axes cover one of the main stresses σ_1, σ_2, the birefringence may be put into the form $\Delta n = n_y - n_x$. With this definition of the birefringence in mind one can conclude whether the ray with the greater index of refraction vibrates along the x or y axes.

Birefringence distribution along the symmetry axis of the blocks

a) The distribution along the axis bisecting the longer edge of the acting surface.

The birefringence distribution along the y axis in the acting surface has been shown for some real case in Fig. 7b. The ray of the greater index of refraction in the space between the isotropic points (I_p) and the block edges vibrates along the y axis, while that between the singular points oscilates perpendicularly to this axis. The shape of the birefringence distribution curve depends upon the quotient a/h of the acting surface. The family of curves along the y axis for a randomly chosen glass block of CF 648-34 (J. Z. O.) is shown in Fig. 8. During the measurement the block height was kept constant, while its width a was being changed so that $a/h > 1$.

The least birefringence at the points on the y axis appears when the acting surface is quadratic ($a/h = 1$). Then the birefringence between the isotropic points is equal to zero. The birefringence at the point B, which is the

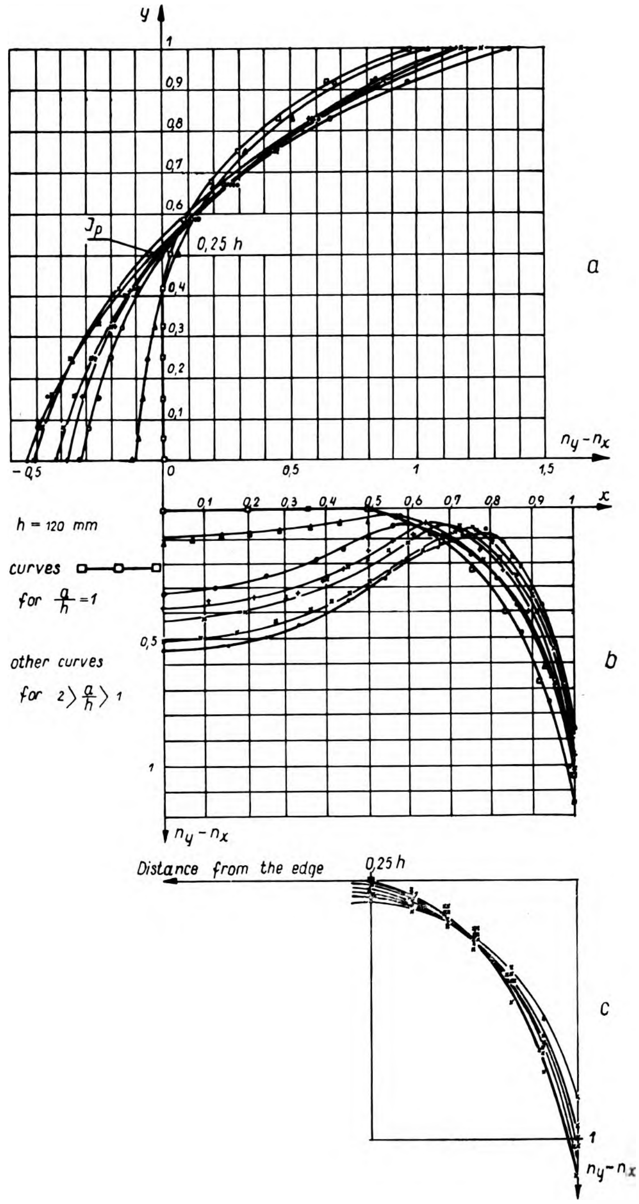

Fig. 8. The birefringence distribution along the symmetry axis y and the symmetry axis x of the acting surface of a block with constant high and variable width a

intersection point of the symmetry axis and the edge, is assumed to be equal to 1. When a/h is increased, the double refraction also increases and becomes constant for $a/h \geq 2{,}25$. The birefringence in the middle of the block is then twice smaller than that at the B_y point on the longer edge. The curves of the birefringence distribution along the y axis are presented for different values of a/h. All these curves intersect each other at the isotropic point located in the middle of the segment $O - B_y$. Similar relations have been established for all the examined sorts of optical glass. In the next two

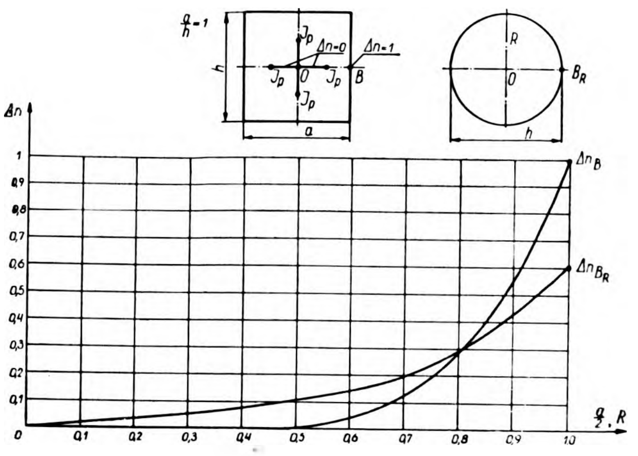

Fig. 9. a) Relative birefringence distribution along the axis of symmetry for the square acting surface b) Relative birefringence distribution along the axes of symmetry for the rounded acting surface

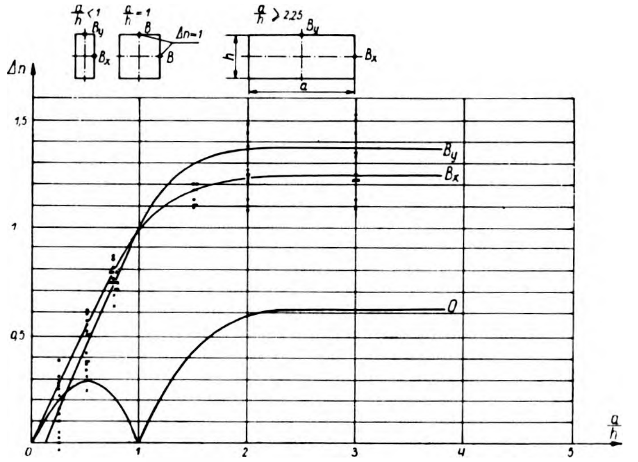

Fig. 10. Dependence of the relative birefringence at the middle points B_y and B_x of the edges and in the middle O of the acting glass block surface on the ratio a/h, h = const

graphs (Fig. 9 and 10) the average values of birefringence have been collected. In Fig. 9 an average distribution of the relative birefringence (i.e. assuming the birefringence at B on the edge to be equal to 1) along the symmetry axis of the quadrate acting surface is given. In the same graph the distribution of the relative birefringence (i.e. normalized by birefringence at the point B) along the radius of the rounded sample made of the said quadratic sample is given.

The next graph (see Fig. 10) ilustrates the change in the relative birefringence at the points B_x and B_y (Fig. 7a) and in the middle of the block of constant value of h depending on the quotient a/h.

b) The distribution of the birefringence along the symmetry axis halving the shorter side of the acting surface.

The birefringence along the axis denoted in Fig. 7a by x and passing through the shorter side of the acting surface is determined by another law. An example of such a distribution for some incidental value of a/h is presented in Fig. 7c. The whole family of the corresponding curves for other values of a/h is shown in Fig. 8b. The right hand part of the curves may be approximately transformed (preserving the constant value of h) into a common curve by puting on the x – axis the real distances of the sampling point from the point B_x lying on the block edge. The reduced distribution curve obtained in this way along the x axis tends to zero at the point located 0.25 h away from the edge. However in general, when $a/h > 1$ it does not reach the point and lowering its course, goes over to the middle of the block (see Fig. 7c).

Dependence of the birefringence on the block size

Several hundreds of measurements have been made to establish the dependence of the birefringence on the block size. The results may be formulated in two statements:

1. Birefringence in glass does not depend on the thickness 1 of the block (Fig. 4). When cutting the glass block perpendicularly to the direction of viewing no changes of birefringence were observed.

2. When diminishing the acting surface, with a/h kept constant, the birefringence is being reduced linearly at each point of the acting surface. As it has been shown in Fig. 11 the birefringence tends to zero faster then the length of the acting surface edge. In the block with the acting surface edge equal to several millimeters no birefringence was observed in the majority of cases with the help of a polarimeter. The behaviour of the birefringence in the case of rounded samples was identical.

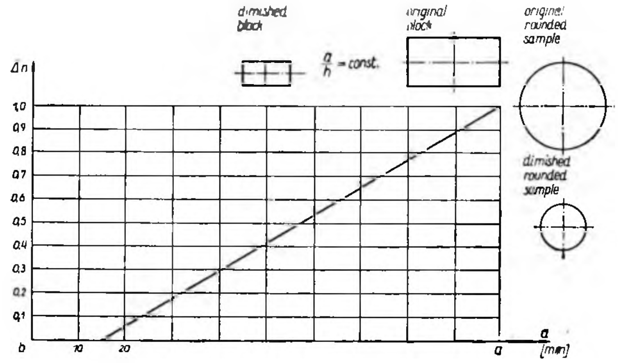

Fig. 11. Change of the birefringence at an arbitrary point of the acting surface during its diminishing, when keeping its original shape constant

Examples

Example 1. The graphs show in Figs. 9–11 are sufficient to find the birefringence in an arbitrary point on the symmetry axis of the reduced block, when knowing the birefringence at a determined point on the symmetry axis of the original block. As an example we will calculate the birefringence in the middle of the block with the quotient $a/h = 0.5$, knowing the birefringence Δn_{B_y} at the point B_y of the block with the same value of h and the ratio $a/h > 2.25$.

In the graph (Fig. 10) we find that in the point B_y of the edge for $a/h > 2.25$, the relative birefringence is equal to $\Delta n = 1.36$, while the birefringence in the middle of the block, normalized by birefringence at the point B, is equal to $\delta n = 0.28$ for $a/h = 0.5$. Thus, when passing from the point B_y of the greater block to the point O of the smaller block the birefringence becomes reduced 0.28/1.36 times and is equal to $\delta n = \Delta n_{B_y} \dfrac{0.28}{1.36}$.

Example 2. Birefringence in the middle of the block with the acting surface of the size 100×200 mm is equal to $25 \cdot 10^{-7}$. What will be the birefringence Δn_{B_r} at the edge of a rounded sample of the diameter 40 mm?

Solution:

Step one. We find the birefringence at the edge of the rounded sample of the diameter equal to the shorter edge of the original block, i.e. $\phi = 100$ mm. From the graph (Fig. 10) we find that the relative birefringence Δn_0 in the middle of the block with $a/h = 2$ is equal to 0.58, while that at the edge of the rounded sample of the diameter $\phi = h$ is $\Delta n_{B_r} = 0.6$. Thus the double refraction at the edge of the rounded sample of the diameter 100 mm amounts to $\Delta n_{B_r} = \dfrac{0.6}{0.58} 25 \cdot 10^{-7}$.

Step two. Knowing the birefringence at the edge of the rounded sample of a 100 mm diameter we find the birefringence at the edge of the rounden sample of a 40 mm diameter. Putting $a = 100$ we find from Fig. 11 that the value of birefringence corresponding to the diameter 40 mm is equal to 0.3 of the original value. Thus the birefringence at the edge of the 40 mm diameter rounded sample amounts to $\delta n_{B_r} = 0.3 \dfrac{0.6}{0.58} 25 \cdot 10^{-7} = 7.8 \cdot 10^{-7}$.

Concluding remarks

The accuracy of each single measurement differed from case to case depending on the measured block lengths 1. On average the accuracy was about $2 \cdot 10^{-7}$. The authors checked many times the agreement of the calculated expected values of the birefringence with those obtained by experiment. In 85 per cent of cases the agreement was satisfactory (i.e. not exceeding 10 per cent). For the remainder the deviation of the measurement results from the calculated values was greater and the authors feel that this is due to the assymetry of the stress distribution. However, the observer may state the lack of symmetry when viewing the original block, i.e. sufficiently early to take it into account.

Le changement de la biréfringence des blocs de verre optique taillés en pièces de forme des éléments optiques

L'article contient les résultats des travaux expérimentaux faits pour établir les formules de changement de la biréfringence du verre optique en fonction des dimensions des blocs de forme parallélépipédique ou cylindrique. Les mesures se rapportent au verre fondu dans des pots d'argile et recuit dans des formes à couler carrées. Les résultats sont présentés sur les graphiques permettant de prévoir la biréfringence dans un point quelconque de l'axe de symétrie de la surface du bloc ou du cylindre taillé dans un bloc de biréfringence connue et également dans un point quelconque de l'axe de symétrie. Les considérations sont illustrées par deux exemples.

Изменение двойного лучепреломления стеклянных оптических тел после пореза их на элементарные куски оптических элементов

В статье приводятся результаты экспериментальных работ, пытающихся установить правила изменения двойного лучепреломления во время изменения величины и пропорции оптических тел в виде прямоугольных параллелепипедов, а также кружков оптического стекла. Измерения эти относятся к стеклу, варенному в шамотных стеклоплавильных горшках и отпущенному в квадратиых формах. Результаты представлены в нескольких диаграммах, которые позволяют предвидеть двойное лучепреломление в произвольной точке, лежащей на оси симетрии поверхности оптического тела или кружка, вырезанного из большего оптического тела с известным двойным лучепреломлением — также в произвольной точке на оси симметрии. Рассуждения эти иллюстрируются двумя примерами.