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Inverse Diffraction of the Electromagnetic Waves

Quite a large amount of work has recently been done on the inverse diffraction of wave fields [1, 2, 3, 4, 5, 6, 7]. However, the attention of research workers was almost entirely focused on the scalar wave fields. It is the aim of the present paper to give the basic formulae for the inverse diffraction of the vector electromagnetic waves. The electromagnetic theory of diffraction formulated by R. K. Luneburg [8] is taken as a starting point of our considerations. According to [8] the monochromatic, diffracted, electromagnetic field in the half-space $z > 0$ may by described by the following formulae:

$$E_x(x_1, y_1, z_1) = \frac{1}{2\pi}\int E_x(x_0, y_0, z_0)\frac{\partial}{\partial z_0}\left(\frac{e^{ikr}}{r}\right)dx_0\, dy_0,$$

$$E_y(x_1, y_1, z_1) = \frac{1}{2\pi}\int E_y(x_0, y_0, z_0)\frac{\partial}{\partial z_0}\left(\frac{e^{ikr}}{r}\right)dx_0\, dy_0, \quad (1)$$

$$E_z(x_1, y_1, z_1) = \frac{1}{2\pi}\int \left(\frac{\partial E_x}{\partial x_0} + \frac{\partial E_y}{\partial z_0}\right)\frac{e^{ikr}}{r}dx_0\, dy_0,$$

where $z_1 > z_0 > 0$; $E_x(x_0, y_0, z_0)$ and $E_y(x_0, y_0, z_0)$ are given functions in the plane $z = z_0$. Formulae (1) present a general solution to the direct diffraction problem. They express the electric field E at a point $x_1, y_1, z_1 > z_0$ in terms of its boundary values E_x (x_0, y_0, z_0), E_y (x_0, y_0, z_0) in a plane $z = z_0$.

Following the properly extended reasoning of Shewell and Wolf [2] we can obtain formulae for the inverse electromagnetic diffraction:

$$E_x(x_0, y_0, z_0) = -\frac{1}{2\pi}\int E_x(x_1, y_1, z_1)\frac{\partial}{\partial z_1}\left(\frac{e^{-ikr}}{r}\right)dx_1\, dy_1,$$

$$E_y(x_0, y_0, z_0) = -\frac{1}{2\pi}\int E_y(x_1, y_1, z_1)\frac{\partial}{\partial z_1}\left(\frac{e^{-ikr}}{r}\right)dx_1\, dy_1 \quad (2)$$

$$E_z\,(x_0, y_0, z_0) = -\frac{1}{2\pi}\int \left(\frac{\partial E_x}{\partial x_1} + \frac{\partial E_y}{\partial y_1}\right)\frac{e^{-ikr}}{r}dx_1\, dy_1.$$

Formulae (2) present evidently ($0 < z_0 < z_1$) a general solution to the inverse diffraction problem. They express the electric field E at a point x_0, y_0, z_0 in terms of its boundary values E_x (x_1, y_1, z_1), E_y (x_1, y_1, z_1) in a plane $z = z_1$. Formulae (2) have been obtained under assumption [2] that the contribution introduced by the inhomogenous plane waves in the angular-spectrum representation [3, 4] of E may be neglected.

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References

[1] Sherman G. C., J. Opt. Soc. Am. 57 (1965), 1490.
[2] Shewell R. J., Wolf E., J. Opt. Soc. Am. 58 (1968) 1596.
[3] Laror E., J. Opt. Soc. Am. 58 (1968), 1235.
[4] Laror E., J. Math. Phys. 9 (1968), 2001.
[5] Sherman G.C., Phys. Rev. Lett. 21 (1968), 761.
[6] Sherman G.C., J. Opt. Soc. Am. 59 (1969), 697.
[7] Jannson T., Karczewski B., Opt. Communications, 2, (1970), 201.
[8] Luneburg R. K., *Mathematical Theory of Optics*, Univ. of California Press, 1964.

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