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An Experimental Comparison of the Diffraction Theories as Formulated by Rayleigh - Sommerfeld and Miyamoto - Wolf Respectively

In the paper an attempt has been made for an experimental verification of the numerical results obtained by applying to the same problem two different diffraction theories, the one based on the Rayleigh-Sommerfeld integral and the other represented by the Miyamoto-Wolf formula. For this purpose a diffraction pattern by produced a half plane illuminated by a laser beam working in the basic mode TEM_{00} has been examined. An observation of the diffraction-interference pattern within the light zone allows to solve the problem.

1. In the papers [1, 2] it has been pointed out that, when starting with the Rayleigh-Sommerfeld integral, the laser beam propagating along the positive direction of the axis with the complex amplitude represented in the form

$$\Psi = \exp\left\{ i\left[P(z)+\frac{k}{2q}\varrho^2\right]\right\} \tag{1}$$

where $\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi\omega^2}$, $p(z)$ denotes a slowly changing phase parameter, $\varrho^2 = x^2+y^2$ denotes a distance from the optical axis z, $R(z)$ is a laser beam (the amplitude I_0 has been normalized to 1 to simplify the notation). When using the Fresnel approximation justified by the small cross section of the laser beam, the following formula for the intensity distribution in the light zone can be obtained

$$I(p) = K\exp\left(-\frac{\varrho^2}{\alpha^2}\right)\left[1+\frac{\sin\left(\frac{\pi}{2}u^2-\frac{\pi}{4}\right)}{\frac{\pi}{\sqrt{2}}u}\right], \tag{2}$$

where

$$\alpha^{-1} = \frac{R\sqrt{2}}{\omega(R+r_0)}, \quad u = y\frac{k}{r_0}\left[\pi k\left(\frac{1}{r_0}-\frac{1}{R}\right)\right]^{-1} \tag{3}$$

r_0 is a distance between the diffracting plane and a detecting screen.

Next the authors considered the diffraction on a half plane of the laser beam with a higher order term TEM_{mn} using the Miyamoto-Wolf diffraction formula (4) and applying the stationary phase method. It happens that the light intensity distribution in the light zone may be represented by the formula

$$I_{mn}(p) = KH_m^2\left(\frac{x}{\alpha}\right)H_n^2\left(\frac{y}{\alpha}\right)\exp\left(-\frac{\varrho^2}{\alpha^2}\right)\left[H_n\left(\frac{y}{\alpha}\right)+ +H_n(0)\exp\left(\frac{y^2}{2\alpha^2}\right)\frac{\sin\left(\frac{\pi}{2}u^2-\frac{\pi}{4}\right)}{\frac{\pi}{\sqrt{2}}u}\right], \tag{4}$$

which for the mode TEM_{00} is reduced to the form

$$I_{00}(p) = K\exp\left(-\frac{\varrho^2}{\alpha^2}\right)\left[1+\exp\left(\frac{y^2}{2\alpha^2}\right)\frac{\sin\left(\frac{\pi}{2}u^2-\frac{\pi}{4}\right)}{\frac{\pi}{\sqrt{2}}u}\right]. \tag{5}$$

When comparing the formula (2) with that of (5) we seen an exponential term $\exp\left(\frac{y^2}{2\alpha^2}\right)$ appearing in front of the oscillating term in the formula (5).

2. Experiments. The experimental comparison was reduced to verification whether the interference — diffraction pattern in the light zone is described by formula (3) rather than (5) or vice versa. The experi-

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mental method consisted in observing the isophots (curves of equal optical density D) registered on a photographic plate of arbitrary characteristics $D = f(I)$; the plate being used as an intensity detector. The method of the experimental treatment has been described in the papers [1], [2], [5] and [8]. The investigations of the oscillating term behaviour appearing in the light zone has been based also on the isophots registration by the experimental method of double diffraction suggested in the paper [6] (Fig. 1).

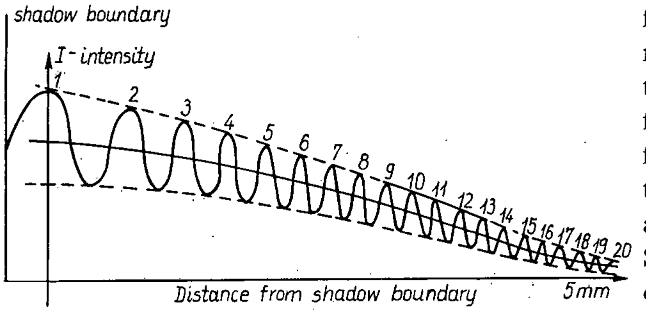

Fig. 1

To compare the theoretical results with those obtained by the experiment the ratio of the intensity in the successive interference fringes to the intensity

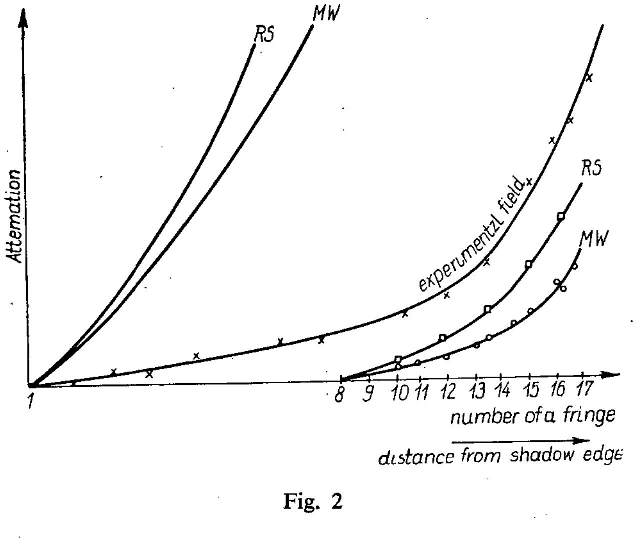

Fig. 2

of the first and next fringes has been experimentally established. A characteristic damping curve, i.e. a curve evidencing the oscillation disappearence have been determined. The same ratios have been numerically evaluated on the base of the two mentioned theories. The experiments have proved that if the intensity of the first fringe lying close to the shadow boundary is accepted as a reference value, which corresponds to very small diffraction angles, the both theories insufficiently describe the phenomenon of diffraction. A better agreement with the experiment is achieved for greater angles of diffraction of order of few minutes of arc. The results obtained indicate (see Fig. 2) that the behaviour of the diffraction interference field in the light zone are described better by the formula (5) resulting from the Miyamoto-Wolf treatment. The difference between the formulae (2) and (5) occurs due to the fact that the Rayleigh-Sommerfeld diffraction formula was derived by using one half of the Kirchhoff diffraction integral while for derivation of the Miyamoto-Wolf formula, which is equivalent to the Young-Rubinowicz transformation [7], the whole Kirchhoff diffraction integral was applied. Nevertheless in the shadow region both the theoretical descriptions are equally well correlated with the experiment; the result being demonstrated in the paper [8].

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