*Zbigniew Kraska, Marek Jerzy Matczak**

Analysis of Vibrations by the Method of Holographic Interferometry

The interferograms of a vibrating membrane obtained by the holographic method have been presented in the paper. An analysis of intensity distribution of the obtained images has been performed and the distribution determined as a function of amplitude of mechanical vibrations. The method was found to be applicable, among others, to the determination of amplitude distribution on a vibrating surface, that is — to the study of solutions of equations describing a given vibrational problem.

1. Introduction

One of the interesting applications of holography is the holographic interferometry. The wave front constructed from a hologram contains full and unequivocal information about the holographed object. When a hologram is put into the holographic system in which it was made, the constructed wave front coincides with the real wave front that is being sent off the object. Every change in the object that causes a deformation of the wave front gives rise to a set of bands that results from the interference of the deformed wave front with that of the hologram. The set of bands thus obtained allows to determine, usually unequivocally, the change in the studied object. Such a method enables us to observe, for example, deformations and distortions as they develop. This, however, requires a precise reconstruction of the configuration of the system in which the hologram was previously abtained.

The method of double exposition is more simples; two holograms of the same object in two different states are made on one photographic plate. During the reconstruction two coherent wave fronts are developed and the change in the object may be found from the result of their interference. Using this method we may study the deformation of light-diffusing objects of irregular shapes, where classical interferometric methods fail [1,5]. The method may be also used for studying phase objects. In this case the interferogram represents the change in the optical path of the object beam between the first and the second exposition. The application of a ruby laser with a modulated quality factor of the resonance chamber allows interferometric study of nonstationary phenomena, such as a shock wave, temperature phenomena in gases, transparent solids and liquids, etc. [3,6,7].

Holographic interferometry has also been applied to study small vibrations [8]. Interferograms of vibrating objects develop as a result of a change in the real path of the object beam. The band pattern determines the amplitude and the type of object vibrations. In this case the applied exposition lasts much longer than the frequency of object vibrations. The application of a synchronously modulated reference beam in the holographic system [2] or the stroboscopic way of exposure [9,10] results in an improvement of the interference image contrast; this in turn allows to broaden the range of studied amplitudes.

The present paper is concerned with the development and analytic description of interferograms of vibrating objects.

2. Experimental

The scheme of the holographic setup, in which the interferograms of vibrating objects were obtained, is shown in fig. 1. It is a typical setup used for making

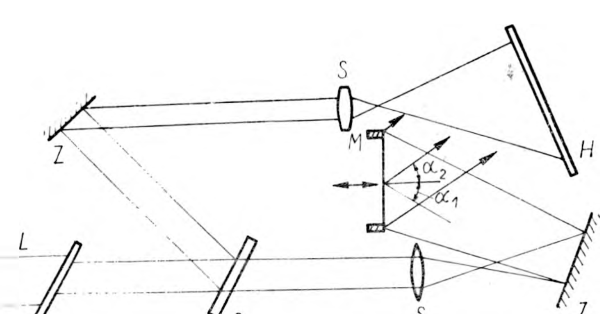

Fig. 1. Schematic diagram of the holographic setup; L — He-Ne laser, P — beam-spliting plate, Z — mirrors, S — lenses, M — vibrating membrane, H — holographic plate

*) Wojskowa Akademia Techniczna, Warszawa — 49, Poland.

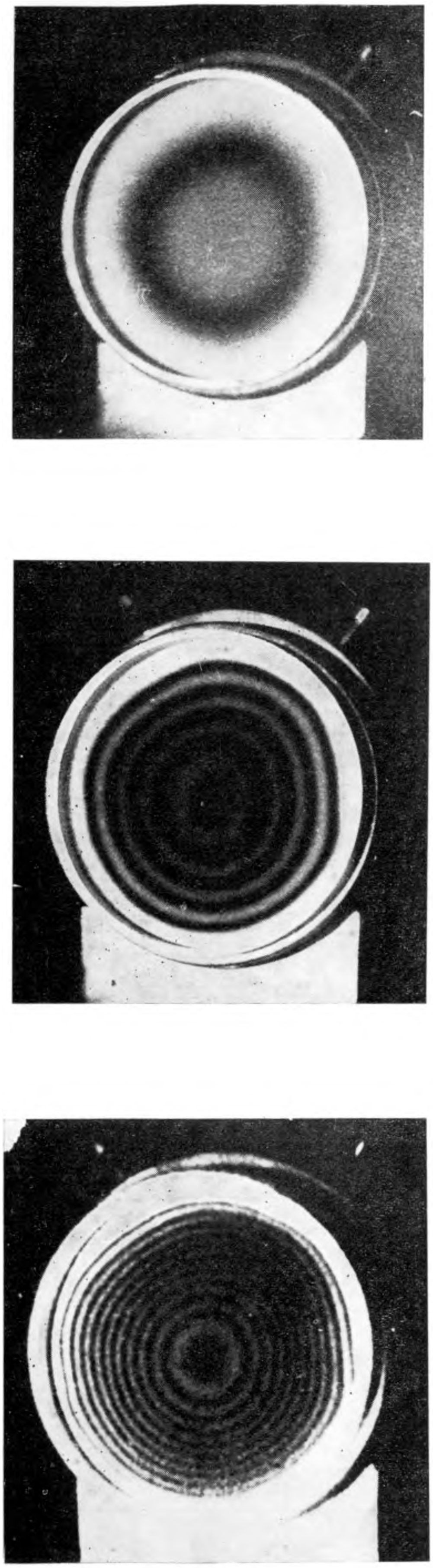

Fig. 2. Interferograms of earphone membrane vibrating with the frequency of 1 kHz and various increasing amplitudes

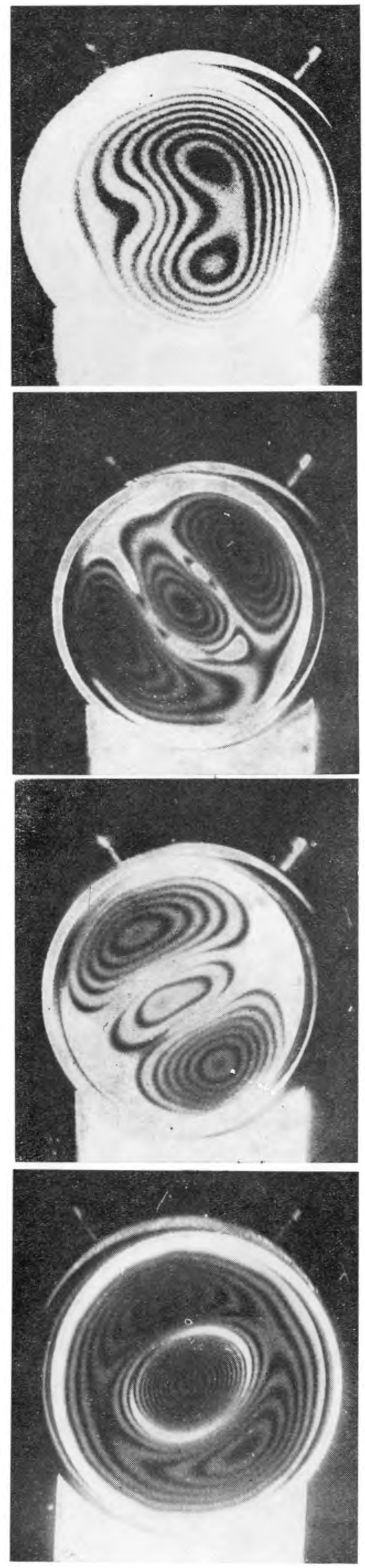

Fig. 3. Interferograms of earphone membrane vibrating with frequencies: a — 2 kHz, b — 3 kHz, c — 3.5 kHz, d — 4 kHz

holograms of reflective objects. A helium-neon laser operating at the 632.8 nm wavelength served as the coherent light source. The vibrations of an earphone membrane fed by a generator of acoustic frequencies were studied. Interferograms which are photographic reproductions of holograms of vibrating earphone membrane are presented in figures 2 and 3.

3. Theoretical description

The analysis of the interferograms obtained by the holographic method is based on the mathematical model of coherent light. The interferogram area represents the intensity distribution of the reconstructing beam modulated by the hologram. This modulation is approximate (when the ratio of the intensities of the object beam to the reference beam sufficiently small) a linear function of the object beam amplitude (e.g. [4]) and bears the name of the amplitude factor of hologram transmission:

$$\sigma \approx 1 - \frac{\gamma}{2} a_o^* a_p - \frac{\gamma}{2} a_o a_p^* , \qquad (1)$$

where γ is the slope coefficient of photographic emulsion characteristic in its working part, a_o — the normalized amplitude of the reference beam ($|a_o|^2 = 1$), whereas a_p — the amplitude of the object beam. Beam amplitude signifies in the present paper the complex quantity in front of expression e^{-iwt} used for describing the propagating disturbance (ω — angular frequency).

In the presented paper the subject of holographic recording are surfaces vibrating sinusoidally in time. Their period of vibration is much smaller than the time of exposure. The quantity a_p in expression (1) is, therefore the result of time averaging of amplitude of the beam scattered by the vibrating surface. The magnitude of the electric component E of the electromagnetic field that represents the object beam may be written in complex form as:

$$E = b(x, y) e^{i(\boldsymbol{kr} - wt)}, \qquad (2)$$

where $b(x, y)$ is the distribution of the complex amplitude modulus on the holographed surface, $\boldsymbol{k} = \frac{2\pi}{\lambda} \boldsymbol{\tau}$ — the wave vector (λ — the wavelength, $\boldsymbol{\tau}$ — versor coinciding with the direction of wave propagation), while $\boldsymbol{r}$ — the vector that describes the position of a point on the holographed vibrating surface relative to a recording point on a photographic plate. In the discussed case the quantity $\boldsymbol{r}$ may be expressed as a sum of the initial position $\boldsymbol{r}_o$ and a change of the position in time $\Delta\boldsymbol{r}(t)$:

$$\boldsymbol{r}(t) = \boldsymbol{r}_o + \Delta\boldsymbol{r}(t). \qquad (3)$$

Owing to a free choice of $\boldsymbol{r}_0$ we may require $e^{i\boldsymbol{kr}_0} = 1$. The direction of vector $\Delta\boldsymbol{r}$ coincides with $\boldsymbol{n}$ normal to the vibrating surface, whereas, two averaged beam directions $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ should be considered for the wave vector: that of the reflected and scattered in the direction of photographic plate and which form with the normal to the membrane surface angles α_1 and α_2, respectively, (fig. 1). For a surface vibrating with frequency Ω the vector $\Delta\boldsymbol{r}$ may be expressed as follows:

$$\Delta\boldsymbol{r} = \boldsymbol{n} u(x, y) \sin\Omega t, \qquad (4)$$

where $u(x, y)$ is the amplitude of mechanical vibrations. Thus expression (2) assumes the form:

$$E = b(x, y) \exp\left[i \frac{2\pi}{\lambda} (\cos\alpha_1 + \cos\alpha_2) \times \times u(x, y) \sin\Omega t\right] [e^{iwt}], \qquad (5)$$

where the factor

$$\cos\alpha_1 + \cos\alpha_2 = (\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2) \cdot \boldsymbol{n}$$

assumes a constant value β for a given holographic system.

Amplitude a_r of the reconstructed light beam which carries information about the holographed object, with the assumption that the reference beam occurring in the holographing process is now the reconstructing beam, is expressed as follows:

$$a_r = \sigma a_0 = a_0 - \frac{\gamma}{2} a_p - \frac{\gamma}{2} a_0^2 a_p^*, \quad (a_0^* a_0 = 1). \qquad (6)$$

The virtual image reconstructed from the hologram is described by the real part of the second term on the right side of expression (6). Denoting this term by $\tilde{a}_r$, assuming $b(x, y) = C$ (uniform illumination of the vibrating surface), and taking into account the averaging of amplitude of the object beam during the exposition, i.e. the factor standing in front of e^{-iwt} in expression (5), we obtain:

$$\mathrm{Re}\{\tilde{a}_r\} = -\frac{\gamma}{2} \frac{1}{T} \int_0^T \mathrm{Re}\{C \exp(ik\beta u(x, y) \sin\Omega t]\} dt) = -\frac{\gamma}{2} \frac{C}{T} \int_0^T \cos[k\beta u(x, y) \sin\Omega t] dt = -\frac{\gamma}{2} C J_0(k\beta u(x, y)), \qquad (7)$$

where $\mathrm{Re}\{z\}$ is the real part of the complex quantity z. T is the period of vibrations of the studied surface, and J_0 — Bessel function of the first kind and zeroth order. Thus the brightness distribution of the image of vibrating surface has the following form:

$$I(x, y) = J_0^2(k\beta u(x, y)). \tag{8}$$

The shape of this function is shown in fig. 4.

The obtained result in the form of expression (8) allows an experimental determination of the amplitude distribution for mechanical oscillations on a vibra-

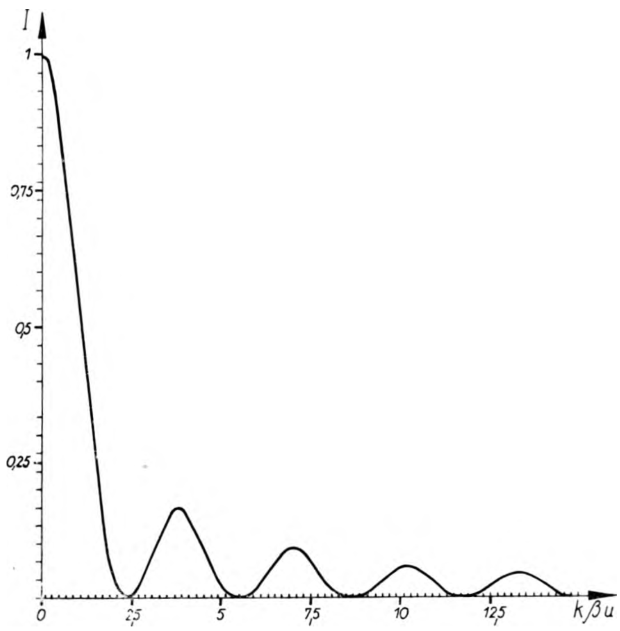

Fig. 4. Distribution of illumination intensity on interferograms as a function of amplitude of mechanical vibrations

ting surface. On interferograms presented in the previous part one may clearly see the places where the brightness is the greatest and, therefore, the amplitude equals zero. These places are the nodes of a standing waves. The dark places correspond to those values of amplitudes for which the Bessel function has zero values. In fig. 2 the node area is the fixed edge of the membrane, whereas in fig. 3, photographs b, c and d, node places lying inside the vibrating area may be seen.

4. Applications and conclusions

A study of vibrations by the holographic interferometry method may be extensively applied in various fields of technology. Some important advantages of the method are:

— great accuracy of the vibration measurement of the order of the wavelength used for recording,

— free from contact during measurement,

— the possibility of measuring in vacuum or in any other medium that is "transparent" for the applied radition.

The method may, for example, be applied to study loudspeakers, musical instruments, as well as the vibrating parts of machines and structuctions. The range of measured amplitudes dependes on the wavelength used for the measurement. This restricts to some extent the applicability range of the method, for in the case of vibrations with amplitude considerbly greater than the used wavelength, the interference image becomes obscure because of the big congestion of bands.

The presented method makes it possible to obtain amplitude distribution for vibrations of the studied object; thus providing us with a solution of equations describing a given vibrational problem. This approach may be useful when estimating how far theoretical calculations coincide with obtained technical effects and also for the determination of some material parameters which may be connected with the quantities appearing in equations which describe a given vibration problem.

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Анализ вибрации методом голографической интерферометрии

В работе приводятся полученные голографическим методом интерферограммы колеблющейся мемраны. Проводится анализ распределения интенсивности полученных изображений, а также определяется форма этого распределения в функции амплитуды механических колебаний. Установлена возможность применения этого метода, между прочим, для определения разложения амплитуды на колеблющейся поверхности, т. е. для исследований решений уравнения проблемы, описывающей данные вибрационные вопросы.

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