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Phase of the Optical Transfer Function for Defocusing and Asymmetrical Apodization in Incoherent Light

The properties of the phase of the optical transfer function for systems with a defect of focus and asymmetrical apodization are theoretically determined. The conclusions are compared with the geometrical point of view. Numerical calculations for the exponential function of apodization are presented and on experimental verification are given. Theoretical and experimental consideration indicates that the Stiles-Crawford's phenomenon influences the parallactic error of visual instruments.

1. Introduction

For a transversal displacement of the head with regard to an optical instrument, the objects situated in different distances from the observer show an apparent shift in the visual field. This fact is known in technical literature [1], [2] as a parallactic error.

From the geometrical point of view it is assumed that the iris of the eye shades partly the exit pupil of the instrument, hence the angle between the principal rays of objects changes and consequently the images of these objects undergo a relative dislocation in a fixed plane of a detector.

The displacement of the eye with regard to the exit pupil of the instrument influences the resolution [3] and the author investigating this fact noted, that the parallactic phenomenon appeared also when the exit pupil of an instrument displaced inside the pupil of the eye without their mutual covering. One can perceive this phenomenon in white and monochromatic light, but it is more distinct in the first instance.

It seems that the reason foı apparent image shifts could also be Stiles-Crawford's phenomenon [4]. The eye is an optical system with axial apodization. The transmittance of the eye, the highest on the axis, decreases when we pass to the edge of the iris. If a transversal displacement of the eye with regard to the pupil of the instrument occurs, the combined optical system of the observer and the instrument becomes a system with asymmetrical apodization. In this case the center of the perspective does not coincide with the center of the pupil of the instrument. From the geometrical point of view the center of the perspective shifts in the direction of the higher transmittance of the pupil. The change of this center causes, therefore, a relative shift of the observed images.

The present work purports to explain that phenomenon on the basis of wave optics and the comparison of results with the geometrical calculation. The properties of the optical transfer function were examined with regard to a system with a defect of focus and asymmetrical apodization. To simplify our consideration and obtain explicit functions we studied the rectangular pupil with variation of the coefficient of apodization in one of the axial directions. If we limited our consideration to the examination of one-dimensional objects with the variable distribution of the intensity in the direction of variation of apodization, the study of the optical transfer function was reduced to the consideration of one variable function. Only the numerical way might yield results for the circular pupil and the determination of the defocusing influence [5].

2. General consideration

Let u'_ς , u'_η be the angular coordinates of exit pupil in a rectangular form (fig. 1a). Denotation of the coordinates is represented by fig. 1b. π' — Gaussian image plane; ζ', η' — plane of exit pupil.

According to (3.143) [2] the normalized optical transfer function for one-dimensional objects (variable intensity distribution in x direction) has been done by equation

$$d_n(\tilde{x}) = \frac{\iint\limits_{-\infty}^{\infty} V(u'_\varsigma, u'_\eta)\, V^*(u'_\varsigma+\lambda\tilde{x}, u'_\eta)\, du'_\varsigma\, du'_\eta}{\iint\limits_{-\infty}^{\infty} V(u'_\varsigma, u'_\eta)\, V^*(u'_\varsigma, u'_\eta)\, du'_\varsigma\, du'_\eta}, \quad (1)$$

where

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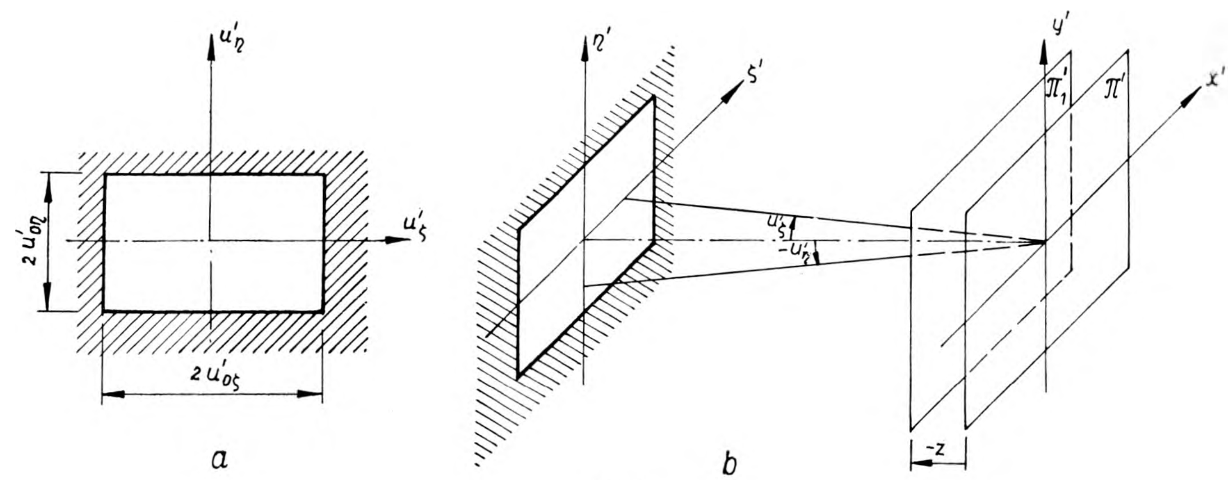


Fig. 1

$V(u'_\zeta, u'_\eta)$ — pupil function describing amplitude and phase distribution in the pupil plane,
$\tilde{x}$ — spatial frequency of intensity,
λ — wavelength.

The pupil function for the variable apodization in ζ' direction may be expressed in the form

$$V = V_0 f(u'_\zeta)\exp(ik\Delta'), \tag{2}$$

where

V_0 — pupil function at $u'_\zeta = 0$ and $|u'_\eta| < u'_{o\eta}$,
$f(u'_\zeta)$ — function of apodization representing the changes of amplitude in pupil plane (real function),
Δ' — wave aberration of the optical system,
$k = 2\pi/\lambda$.

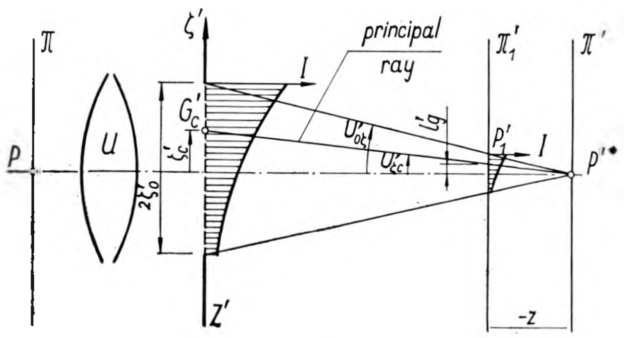


Fig. 2

To examine the transfer function for the plane π'_1 displaced by a distance z from the Gaussian plane π' for sufficiently small aperture angle u' we can write $\Delta' = -u'^2 z/2$. If we denote

$$\phi = \frac{k u'^2_{o\zeta} z}{2}, \tag{3}$$

because $u'^2 = u'^2_\zeta + u'^2_\eta$, equation (2) may be rewritten in the form

$$V = V_0 f(u'_\zeta)\exp\left[-i\phi\left(\frac{u'_\zeta}{u'_{o\zeta}}\right)^2\right]\exp\left[-i\phi\left(\frac{u'_\eta}{u'_{o\zeta}}\right)^2\right]. \tag{4}$$

Substituting (4) to (1), because out of pupil area $V(u'_\zeta, u'_\eta) = 0$, we have

$$d_n(\tilde{x}) = \frac{\exp\left[i\phi\left(\frac{\lambda\tilde{x}}{u'_{o\zeta}}\right)^2\right]\int\limits_{-u'_{o\zeta}}^{u'_{o\zeta}-\lambda\tilde{x}} f(u'_\zeta+\lambda\tilde{x}) f(u'_\zeta)\exp}{\int\limits_{-u'_{o\zeta}}^{u'_{o\zeta}} f^2(u'_\zeta)\,du'_\zeta} \times$$

$$\times \frac{\left(2i\phi u'_\zeta\frac{\lambda\tilde{x}}{u'^2_{o\zeta}}\right)du'_\zeta}{\int\limits_{-u'_{o\zeta}}^{u'_{p\zeta}} f^2(u'_\zeta)\,du'_\zeta}. \tag{5}$$

To transform this expression it is convenient to introduce new variables. In the numerator by

$$u'_\zeta = (s - \tilde{x}_n)u'_{o\zeta} \tag{5a}$$

and in the denominator

$$u'_\zeta = s u'_{o\zeta}, \tag{5b}$$

where

$$\tilde{x}_n = \frac{\tilde{x}}{\tilde{x}_g}, \tag{6}$$

$$\tilde{x}_g = \frac{2u'_{o\zeta}}{\lambda} \tag{7}$$

$\tilde{x}_n$ — normalized line space frequency with regard to limiting frequency $\tilde{x}_g$ (evidently for $|\tilde{x}| \leqslant \tilde{x}_g$ we have $0 \leqslant |\tilde{x}_n| \leqslant 1$).

Hence

$$d_n(\tilde{x}_n) = \frac{\int\limits_{-m}^{m} g \exp(4i\phi \tilde{x}_n s)\, ds}{\int\limits_{-1}^{1} g_0\, ds}, \tag{8}$$

where

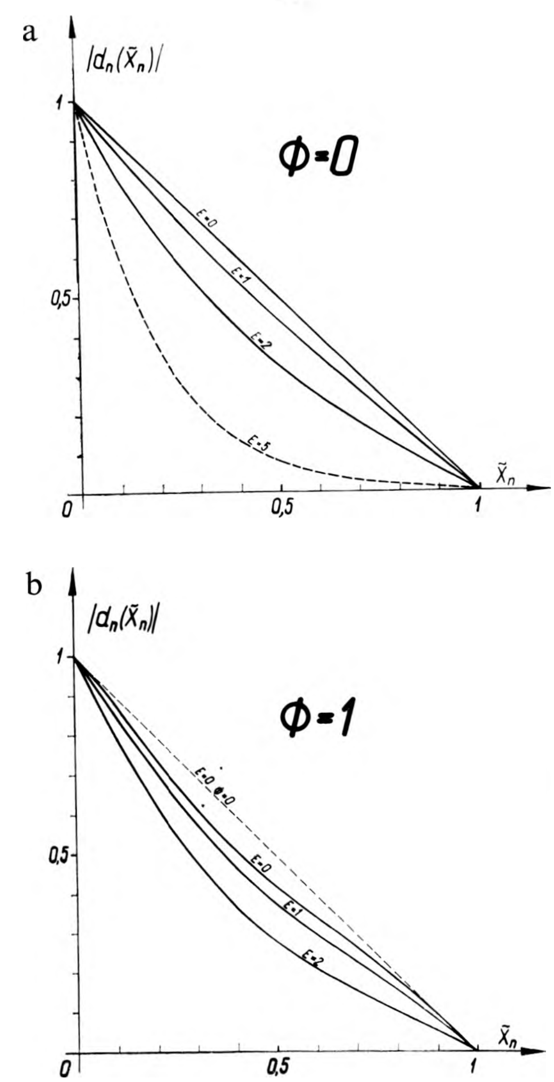
$$g = f[u'_{os}(s - \tilde{x}_n)]\, f[u'_{os}\,(s + \tilde{x}_n)], \tag{9}$$

$$m = 1 - \tilde{x}_n \tag{10}$$

$g_0 = f^2(u'_{os}\, s)$ is g function for $\tilde{x}_n = 0$.

As g is real, we can separate the module and phase of the transfer function, and the equation (8) may be rewritten

$$d_n(\tilde{x}_n) = \frac{\int\limits_{-m}^{m} g \cos(4\phi \tilde{x}_n s)\, ds + i \int\limits_{-m}^{m} g \sin(4\phi \tilde{x}_n s)\, ds}{\int\limits_{-1}^{1} g_0\, ds}. \tag{11}$$



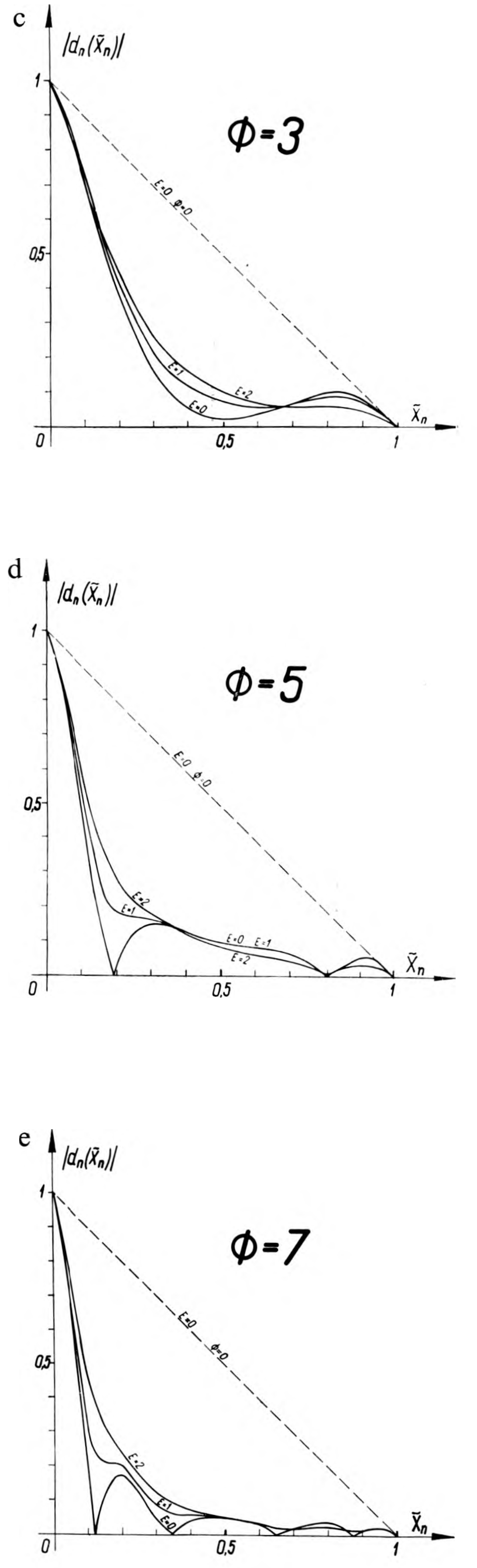


Fig. 3

If

$$d_n(\tilde{x}_n) = |d_n(\tilde{x}_n)| \exp[-i\Theta(\tilde{x}_n)] \tag{12}$$

the modul of the transfer function (contrast transfer function) is

$$|d_n(\tilde{x}_n)| = \frac{\sqrt{[\int_{-m}^{m} g\cos(4\phi\tilde{x}_n s)\,ds]^2 + [\int_{-m}^{m} g\sin(4\phi\tilde{x}_n s)\,ds]^2}}{\int_{-m}^{m} g_0\,ds} \tag{13}$$

and the phase Θ

$$\tan\Theta = -\frac{\int_{-m}^{m} g\sin(4\phi\tilde{x}_n s)\,ds}{\int_{-m}^{m} g\cos(4\phi\tilde{x}_n s)\,ds}. \tag{14}$$

Using the exponential functions equation, (14) may be represented in the more useful form sometimes

$$\tan\Theta = i\,\frac{\int_{-m}^{m} g\exp(4i\phi\tilde{x}_n s)\,ds - \int_{-m}^{m} g\exp(-4i\phi\tilde{x}_n s)\,ds}{\int_{-m}^{m} g\exp(4i\phi\tilde{x}_n s)\,ds + \int_{-m}^{m} g\exp(-4i\phi\tilde{x}_n s)\,ds}. \tag{15}$$

3. Properties of the phase of optical transfer function

It follows immediately from (14) that the phase shift is a non-zero quantity only outside the Gaussian plane ($\phi \neq 0$). Moreover it is sufficient to study the phenomenon of parallax for $\phi > 0$, because the change of the defocusing sign influences only the phase sign.

If the function of apodization is symmetrical as regards $u'_\varsigma = 0$, ei. if $f(-u'_\varsigma) = f(u'_\varsigma)$, then — independently of the quantity of defocusing — we have $\tan\Theta = 0$.

To prove this we shall study the numerator of expression (15). Substituting $-s$ in the place of s in the second integral and changing the limits of integration we obtain

$$\int_{-m}^{m} g\exp(4i\phi\,\tilde{x}_n s)\,ds - \int_{-m}^{m} g\exp(-4i\phi\tilde{x}_n s)\,ds =$$

$$= \int_{-m}^{m} [g - g(-s)]\exp(4i\phi\tilde{x}_n s)\,ds.$$

By (9)

$$g - g(-s) = f[u'_{o\varsigma}(s-\tilde{x}_n)]f[u'_{o\varsigma}(s+\tilde{x}_n)] -$$

$$-f[-u'_{o\varsigma}(s+\tilde{x}_n)]f[-u'_{o\varsigma}(s-\tilde{x}_n)].$$

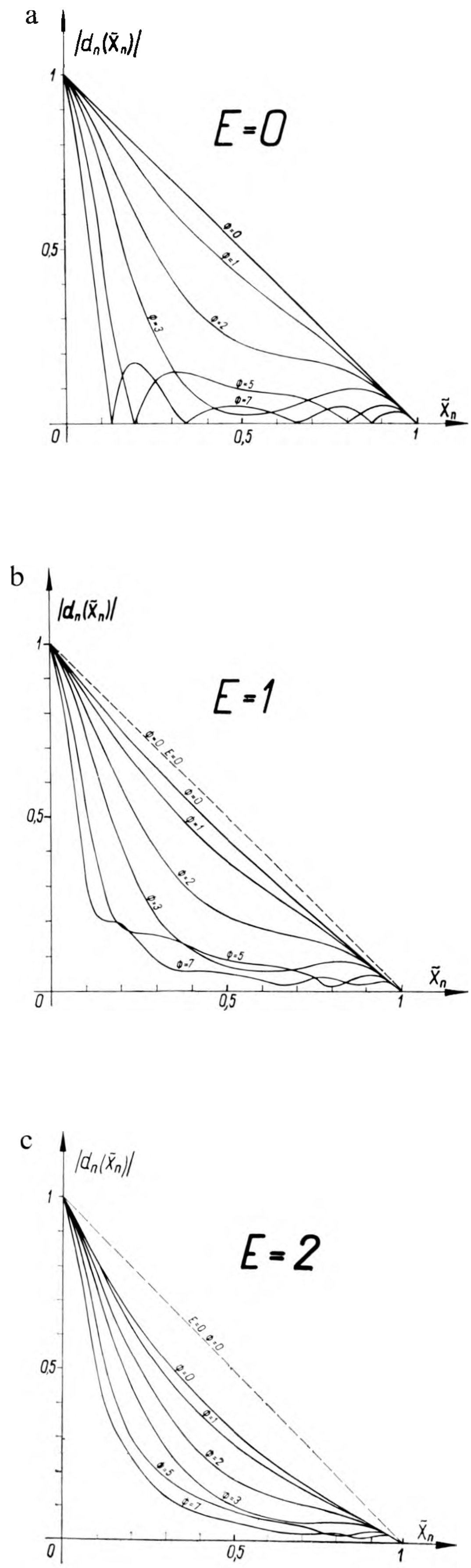


Fig. 4

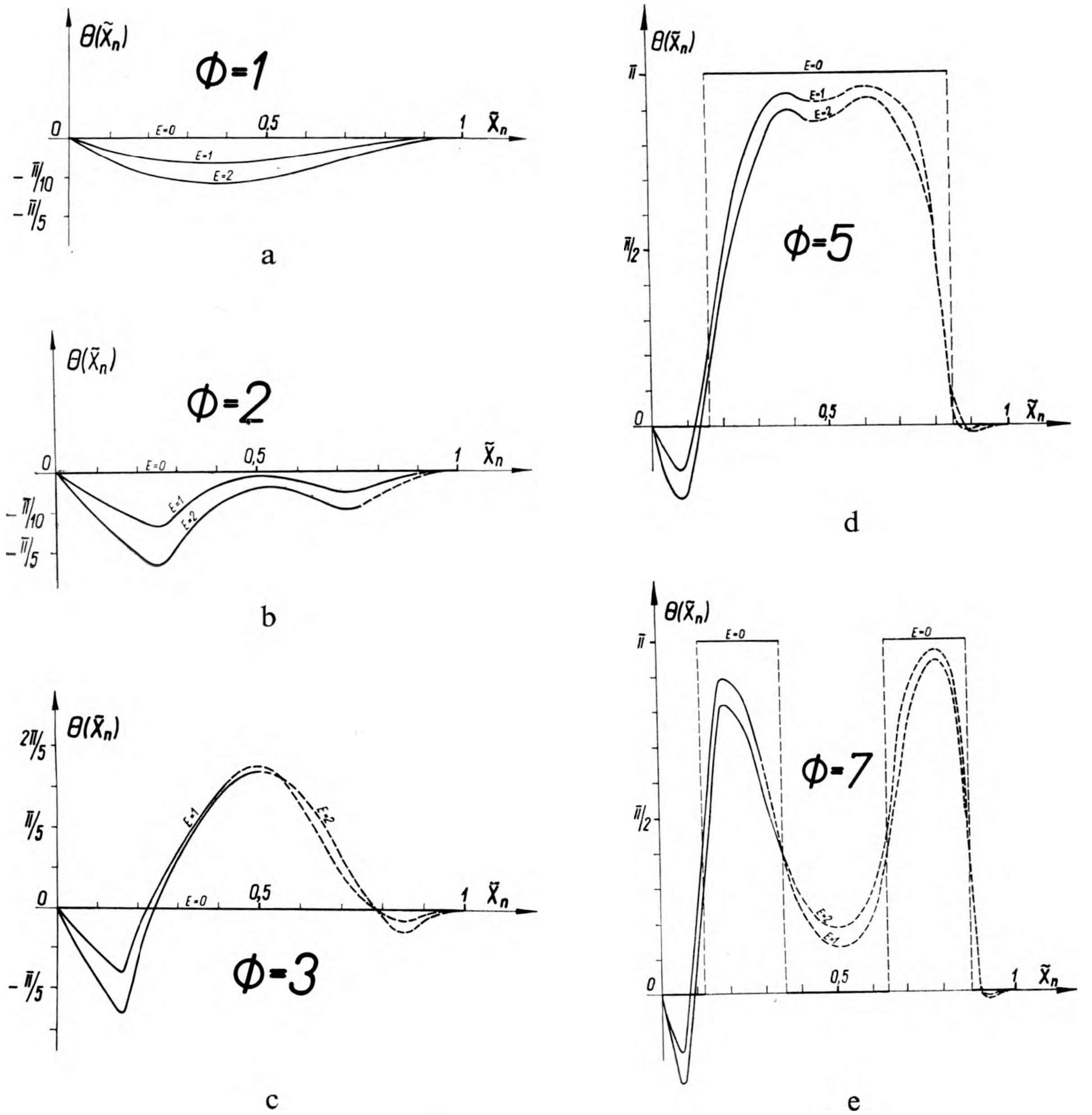


Fig. 5

But $f(-u_\varsigma) = f(u'_\varsigma)$, since $g(-s) - g(s) = 0$ and $\tan\Theta = 0$, then $\Theta = 0$ or $\Theta = \pi$ according to positive or negative sign of denominator of (15) respectively.

It follows that for a system free of aberrations with symmetrical function of apodization the defocusing may induce only a shift of phase on π like for a system without apodization.

For an asymmetrical apodization and $x_n \neq 0$ one can obtain the shift of phase different from 0 and π, moreover for the same parameters of apodization and defocusing the shift of phase may be different for different $\tilde{x}_n$.

To accentuate the changes of Θ with $\tilde{x}_n$ we expand the equation (14) in the power series at the point $\tilde{x}_n = 0$ and then

$$\tan\Theta = -4\phi\tilde{x}_n(T_0 + \tilde{x}_n T_1 + \tilde{x}_n^2 T_2 + \cdots), \tag{16}$$

where

$$T_0 = \frac{\int_{-1}^{1} f^2(u'_{o\varsigma}s)\, s\, ds}{\int_{-1}^{1} f^2(u'_{o\varsigma}s)\, ds}, \tag{17}$$

$$T_1 = \frac{T_0[f^2(u'_{o\varsigma}) + f^2(-u'_{o\varsigma})] - [f^2(u'_{o\varsigma}) - f^2(-u'_{o\varsigma})}{\int_{-1}^{1} f^2(u'_{o\varsigma}s)\, ds}. \tag{17a}$$

The next terms have not such a clear form but they keep finite values for a continuous function of apodization. It is interesting to note that the expression (17) has the form of a formula for the center of intensity area — this problem will be explained later.

A line shift of tintensity harmonic in the image plane for a fix $\tilde{x}$ can be calculated from the expression

$$l' = \frac{\Theta}{2\pi\tilde{x}} = \frac{1}{2\pi\tilde{x}_g}\cdot\frac{\Theta}{\tilde{x}_n}. \qquad (18)$$

If Θ is sufficiently small we can put $\tan\Theta = \Theta$ and according to (16)

$$l' = -\frac{2\phi}{\pi\tilde{x}_g}(T_0+\tilde{x}_nT_1+\tilde{x}_n^2T_2+\cdots). \qquad (19)$$

Hence the line shift, like the phase shift depends generally also on $\tilde{x}_n$.

For constant intensity distribution ($\tilde{x}_n = 0$) from (19)

$$l_0' = -\frac{2}{\pi\tilde{x}_g}T_0\phi. \qquad (20)$$

Although for $\tilde{x}_n = 0$ we have $\tan\Theta = 0$, but $l_0' \neq 0$. The line shift of image with constant intensity is proportional to the defocusing ϕ and constant T_0 depending on the function of apodization.

Expression (20) can also be deduced from the general equation (14). Taking (18), (10) and the fact that $\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$, then

$$l_0' = \lim_{\tilde{x}_n\to 0} l' = -\frac{1}{2\pi\tilde{x}_g}\lim_{\tilde{x}_n\to 0}\frac{\int\limits_{-m}^{m} g\,\frac{\sin(4\phi\tilde{x}_ns)}{\tilde{x}_n}\,ds}{\int\limits_{-m}^{m}\cos(4\phi\tilde{x}_ns)\,ds} =$$

$$= -\frac{2\phi}{\pi\tilde{x}_g}\,\frac{\int\limits_{-1}^{1} gsds}{\int\limits_{-0}^{1} gds}.$$

Hence for (9) and (17) putting $\tilde{x}_n = 0$ we have (20).

4. Geometrical consideration

Let π be the Gaussian plane for a fixed object plane of a optical system U (fig. 2). Z' — the exit pupil of the system, P'^* — the Gaussian image of a point P.

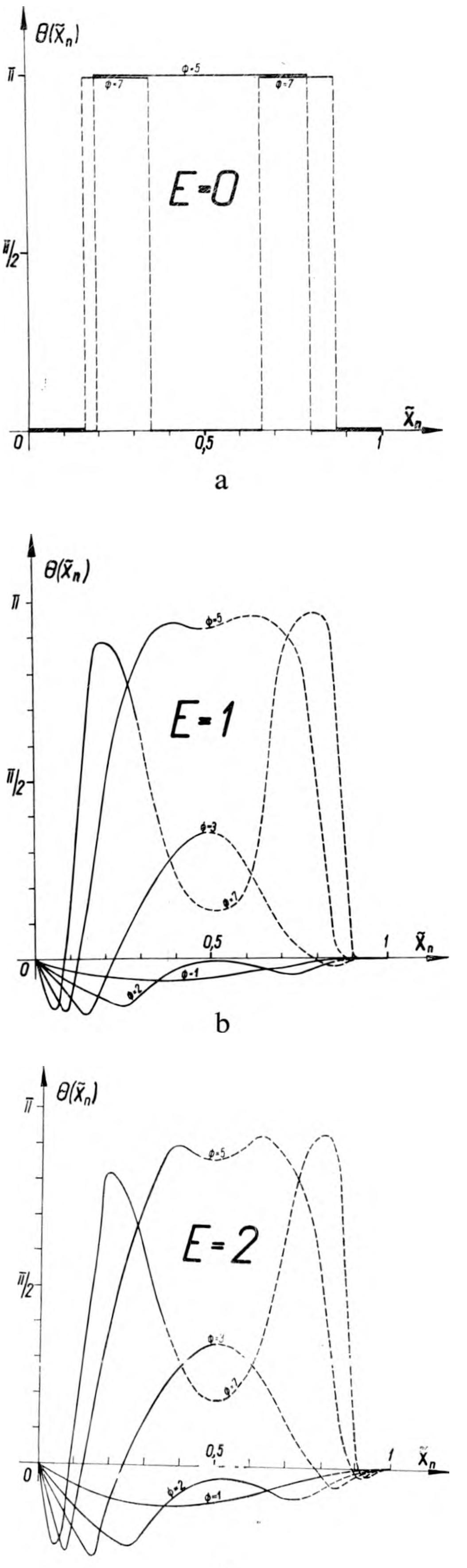


Fig. 6

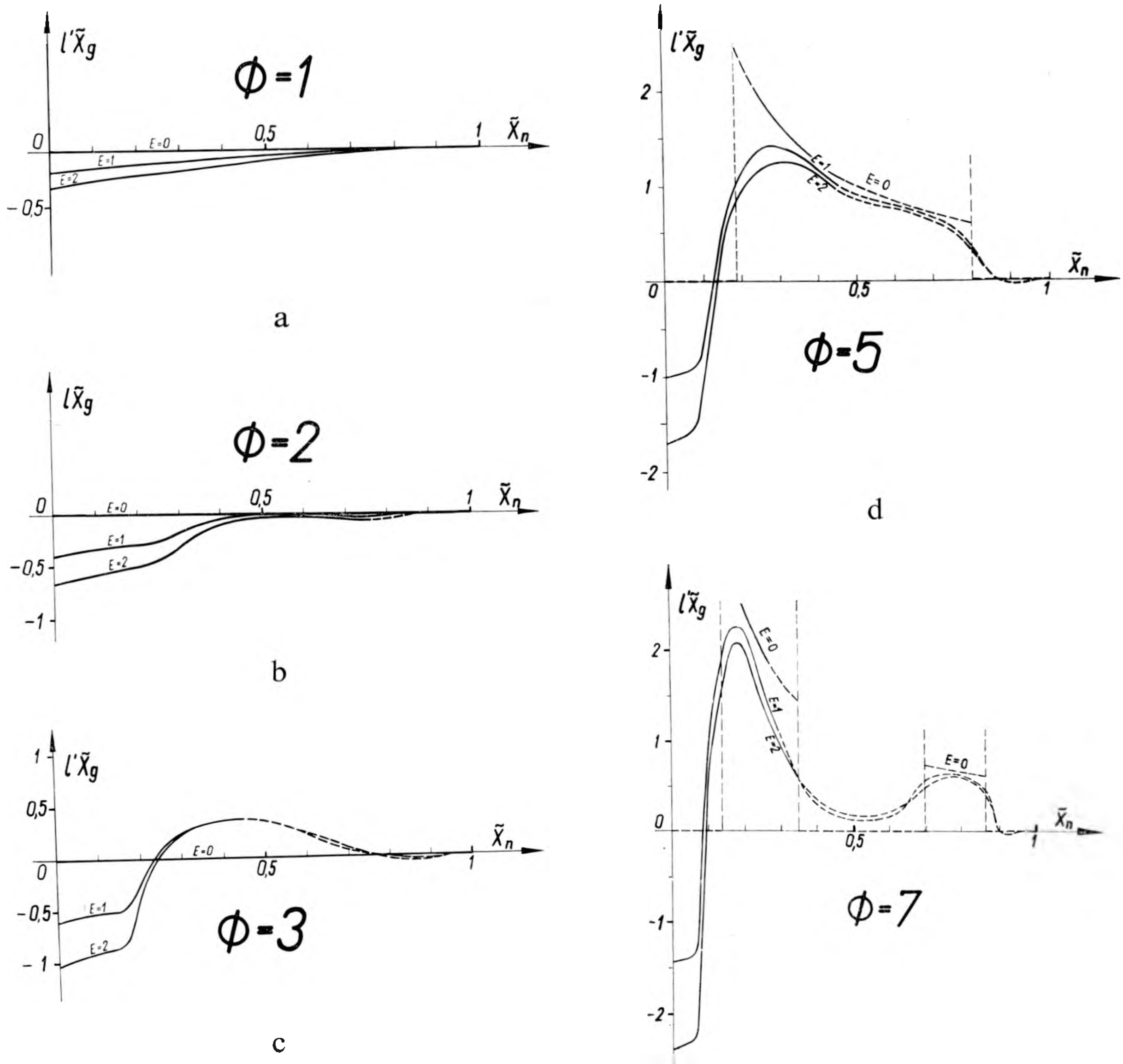


Fig. 7

e

For any system free of aberrations independent from the type of apodization all rays generating the image of P intersect at point P'^*. In the adjacent plane π_1 the image of P will be a spot with the intensity distribution depending on the intensity distribution in the pupil plane. Because all of rays intersect in P'^* the intensity distribution in π_1' will — from the geometrical point of view — be similar in the mathematical sense to the pupil intensity distribution, that was symbolically marked in fig. 2.

If the surface response of a detector is proportional to the illumination it may be accepted that the center of the point image is the center of area of the intensity distribution in this image. According to fig. 2, the centers of point images P in the different planes (for different values of z) lie on the intersections of planes π_1' with line $G_c'P'^*$, where G_c' is the center of the intensity area in the exit pupil. In this case point G_c is the center of perspective in the image space and line $G_c'P'^*$ — generalization of the principal ray for an optical system with asymmetrical apodization.

For a symmetrical pupil intensity distribution, the center of the intensity area coincides with the pupil center $\zeta' = 0$ and there is no image displacement for any defocusing. It is only partially in accordance with wave consideration, because, as regards geometrical optics, it is impossible to obtain any information about generating shift of phase on π for some interval of defocusing ϕ.

To find the geometrical image displacement l_g' caused by asymmetrical apodization we shall determine the position of the center of intensity area in the exit pupil.

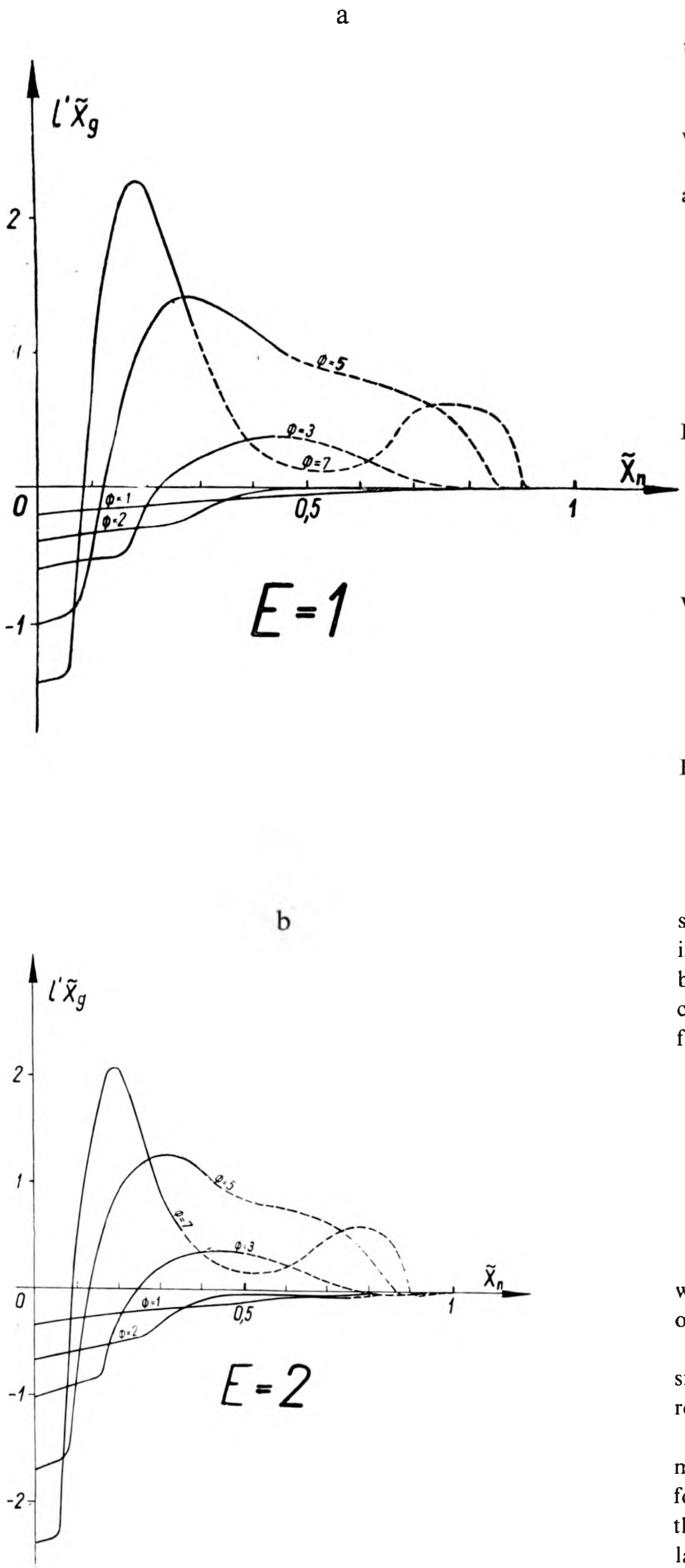


Fig. 8

According to (2) pupil amplitude distribution has the form $V = V_0 f(u'_\varsigma)$ and pupil intensity distribution

$$I = I_0 f^2(u'_\varsigma), \tag{21}$$

where $I_0 = V_0 V_0^*$ – intensity in the center of pupil.

The position of the center of intensity area in angular coordinates will be determined by

$$u'_{\varsigma c} = \frac{\int_{-u'_{o\varsigma}}^{u'_{o\varsigma}} I u'_\varsigma du'_\varsigma}{\int_{-u'_{o\varsigma}}^{u'_{o\varsigma}} I du'_\varsigma}.$$

Putting $u'_\varsigma = s u'_{o\varsigma}$ with (21) we have

$$u'_{o\varsigma} = \frac{\int_{-1}^{1} f^2(u'_{o\varsigma}s) s ds}{\int_{-1}^{1} f^2(u'_{o\varsigma}s) ds}. \tag{22}$$

With (17)

$$u'_{\varsigma c} = u'_{o\varsigma} T_0. \tag{23}$$

According to fig. 2

$$l'_g = -z u'_{\varsigma c}.$$

By (3) and (7), because $k = 2\pi/\lambda$

$$l'_g = -\frac{2}{\pi \tilde{x}_g} T_0 \phi. \tag{24}$$

It results from (24) and (20) that the image line shift determined on the ground of geometrical optics is equal the line shift of zero frequency harmonic, because the geometrical consideration supplies correct conclusions for the structures of sufficiently small frequencies.

5. Exponential function as example of asymmetrical function of apodization

Let the function of apodization be

$$f(u'_\varsigma) = \exp(b u'_\varsigma), \tag{25}$$

where b is the parameter depending on the degree of apodization.

The choice of such function was caused by the simplicity of expressions and facility of experimental realization.

The function of eye apodization [3] is nearly symmetrical with regard to the center of the eye. Yet for a sufficiently small diameter of the exit pupil of the instrument and sufficiently large transversal displacement of eye we can approximate the function of eye apodization by expression (25).

According to (9)

$$g = \exp(2bu'_{o\varsigma}s) = \exp(Es), \tag{26}$$

where

$$E = 2bu'_{o\varsigma} \tag{27}$$

is normalizing parameter of b.

After substituting (26) to (11) and solving the integrals we obtain

$$d_n(\tilde{x}_n) = \frac{1-\tilde{x}_n}{\frac{\sinh E}{E}} \frac{\exp[A+Bi]-\exp[-(A+Bi)]}{2(A+Bi)}, \tag{28}$$

where

$$A = E(1-\tilde{x}_n), \tag{29}$$

$$B = 4\phi\tilde{x}_n(1-\tilde{x}_n). \tag{29a}$$

For a given frequency the variable A characterizes the degree of apodization, and B — degree of defocusing.

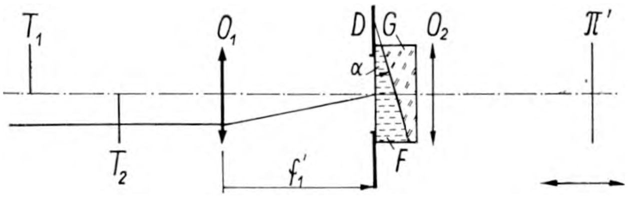


Fig. 9

From (28) and (12) the contrast transfer function has the form

$$|d_n(\tilde{x}_n)| = \frac{1-\tilde{x}_n}{\frac{\sinh E}{E}} \sqrt{\frac{\sinh^2 A+\sin^2 B}{A^2+B^2}}. \tag{30}$$

Phase of optical transfer function

$$\tan\Theta = \frac{B\cos B\sinh A - A\sin B\cosh A}{A\cos B\sinh A + B\sin B\cosh A}. \tag{31}$$

From (18) we can find l'.

For $\tilde{x}_n = 0$ from (17) we have

$$T_0 = \frac{\cosh E - \frac{\sinh E}{E}}{\sinh E} \tag{32}$$

and by (20)

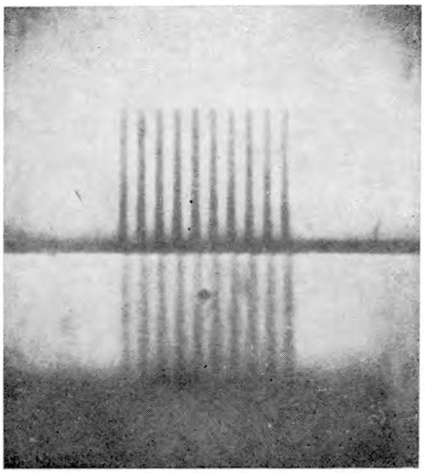
$$l'_0 = -\frac{2\phi}{\pi\tilde{x}_g} \frac{\cosh E - \frac{\sinh E}{E}}{\sinh E}. \tag{33}$$

For $E = 0$, 1 and 2 succesively $T_0 = 0$, 0.3130 and 0.5373.

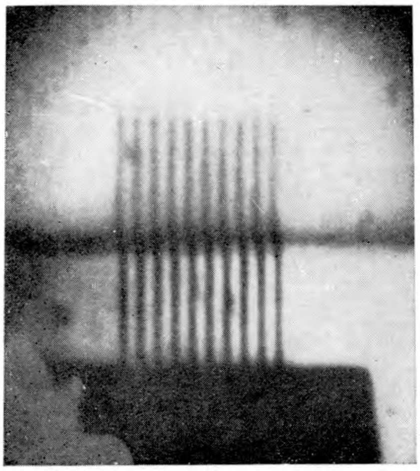
For sufficiently small E we can put $T_0 = E/3$.

By means of a computer we calculated the contrast transfer function $|d_n|$, the phase Θ of the optical transfer function and the line displacement l' as a function of defocusing ϕ, the degree of apodization E and normalized frequency $\tilde{x}_n$ according to expressions (29–33) and (18). The results of this calculation are represented on figs 3–8.

a



b



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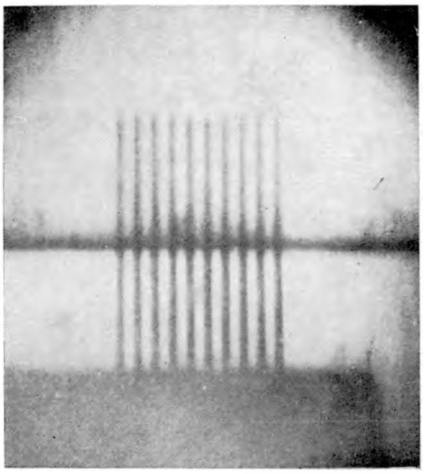

Fig. 10

Considered values of E (on fig. $E = 0$, 1 and 2) on the one hand could not be too small, because we could not verify them, and on the another hand could not be too great, because it practically signifies the reducing of pupil width (see fig. 3a for $E = 5$). The case $E = 0$ concerns a system without apodization.

Fig. 3a–e illustrates the influence of the degree of apodization E on the contrast transfer function $|d_n|$ for different values of defocusing ϕ. In case of small defocusing the increase in the degree of apodization E causes a decrease in the contrast transfer function analogical to the influence of small aberrations [6]. The greatest change of values $|d_n|$ appears in the intermediary frequencies between 0 and 1. Regarding the great degree of defocusing the change of the contrast transfer function is different for different $\tilde{x}_n$. It is of interest that in contradiction to $E = 0$, for $E \neq 0$ we have $|d_n| \neq 0$ in the interval $0 \leqslant \tilde{x}_n < 1$.

The fig. 4 contains the same results as fig. 3 but it emphasises the influence of defocusing ϕ for different degree of apodization E.

The phase Θ and the line shift l' are represented in the similar combinations in the figs 5,6 and 7,8, respectively. The intervals for which the value of the contrast transfer function is less than 0.1 are marked by broken lines. For convenience in the place l' $l'\tilde{x}_g$ is represented as a fraction of the limiting period $X_g (X_g = 1/\tilde{x}_g)$.

For $E = 0$ and $\phi > \pi$ we have a shift of phase by π only (fig. 5de and 6a). If $E \neq 0$, and $\phi \neq 0$ a continuous change of phase Θ occurs simoultaneously with a line shift l' for alternating $\tilde{x}_n$.

The sign of both the l' and Θ changes for sufficiently high values of ϕ, if $\tilde{x}_n$ increases, starting from 0. This means, that the different harmonics may move in different directions in the image plane for the given defocusing.

The change of phase Θ and line displacement l' with ϕ, E and $\tilde{x}_n$ is generally complicated and we can find simple systematics for sufficiently small $\tilde{x}_n$, when the phenomenon of paralax can be approximately described by the methods of geometrical optics.

6. Experimental verification

To verify our theoretical considerations we take the system showed in fig. 9.

Let T_1 and T_2 be the Foucault's tests with the same basic frequency, which are imaged by means of objectives O_1 and O_2 on the detector plane π'. Each test is illuminated by a incoherent light produced by an illuminator consisting of an incadescent lamp, a condenser and two ground glasses. The aperture of the condensor was several times greater than that of the optical system $O_1 + O_2$. The spectrum band of light was limited by absorbing filters ($\lambda_{max} = 545$ nm, $\Delta\lambda = 20$ nm). The illuminators are not marked in the figure. The longitudinal displacement of the detector gives corresponds to the respective changes of defocusing for two tests simultaneously. The aperture diaphragm D is placed in the image focus of the objective O_1, thus the principal rays in the object space of O_1 are parallel. Hence the test images on π' have the same frequencies independetly of the distance between T_1 and T_2. This fact facilitates the measurement of the relative line displacement of two tests.

The asymmetrical (exponential) function of apodization is produced by means of two colled wedge-shaped elements. One of them F is made of absorbing material (filter) and other one G of transparent glass. The section of the colled elements marked on figure is rotated by 90° round the optical axis. Since an absorbing material is used as the neutral filter NG3 (Schott — Jena, transmission $\tau = 0.1$ for thickness of 1 mm), what with $\alpha = 10°$, width of the aperture diaphragm equal 9 mm, $f_1' = 2$ m gives $E = 2$. The distance between tests was equal 250 mm, what corresponds $\phi = 7$. For the defocusing $\phi = 7$ the influence of the higher harmonics may be neglected because the contrast transfer function is then less than 0.1 according to fig. 3e.

If the detector plane would be displaced from the Gaussian plane of the test T_1 to the Gaussian plane of T_2, the relative shift of the image tests occurs in accordance with our considerations. The same relative positions apply to the extreme planes π'. The images of the tests for different detector planes are represented in fig. 10. The central image (fig. 10b) corresponds to a plane between the Gaussian images. For $E = 2$ and $\tilde{x}_n = 0.14$ the relative phase shift $\Delta\Theta \approx \pi/2$. The change of the relative test shift was continuous during the displacement of detector plane. The contrast of the image tests was different from 0 in every position of the detector plane, which is in accordance with theoretical consideration.

Conclusions

It was confirmed that systems with asymmetrical apodization as well as those free of aberration the line shift of harmonics during the defocusing introduce. The phase of the optical transfer function of such system changes continuously and its value depends on the frequency of the harmonic. The line image shift as determined by the geometrical optics corresponds to the line shift of zero frequency harmonic. Theoretical and experimental consideration indicate that the Stiles–Crawford's phenomenon influences the parallactic error of the visual instruments.

Фаза оптической функции переноса при перефокусировке и асимметрической аподизации в некогерентном свете

Определены теоретические свойства фазы функции переноса для систем с дефектом фокусировки и асимметрической аподизацией. Результаты сравнены с геометрическим описанием. Даны численные расчёты экспоненциальной функции аподизации и проведены сравнения с экспериментальными результатами. Теоретические рассуждения и экспериментальные данные указывают, что явление Стилеса-Крофорда влияет на погрешенность параллакса визуальных приборов.

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