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A Laser Device for Displacement Measurement

A laser device for displacement measurement has been described taking account of the factors influencing the measurement accuracy. The problem of changes in the polarization state of the laser beams due to multiple reflection inside the prism has been considered.

1. Introduction

In contemporary technology a higher and higher accuracy is required for measuring distances, lengths and displacements. These requirements may be fulfilled by employing interference methods. Traditional light sources enable the application of the above mentioned methods — within the range measurements — not exceeding 30 cm. When a coherent beam is used the measurement range may be considerably increased. Great values of the coherent time available in laser radiation (the He—Ne laser for instance), being of an order of 10^{-5} s, assure the production of interference phenomena for the optical path differences up to 300 m. This makes possible measurements of distances up to 150 m with application of Michelson type interferometer. This is a range unattainable by other methods and other measuring devices.

2. Operation principles of the device

The operation of the device is based on counting succesive dark and light field of view which appear as a results of changes of the optical path difference in the Michelson type interferometer. A block diagram is shown in fig. 1.

By applying two photodetectors being the sources of two sinusoidal signals with a relative phase shift between them equal to $\pm 90°$ and counting them by application of a logical and electronic reversible counter the measurement error caused by the vibrations of the movable corner prism may be eliminated. In such the pass turns may be determined case.

The absolute accuracy of the measurement, as determined by the principle of the device operation, amouts to $\lambda/8$, where λ is the wavelength of the laser radiation in the medium in which the measurement

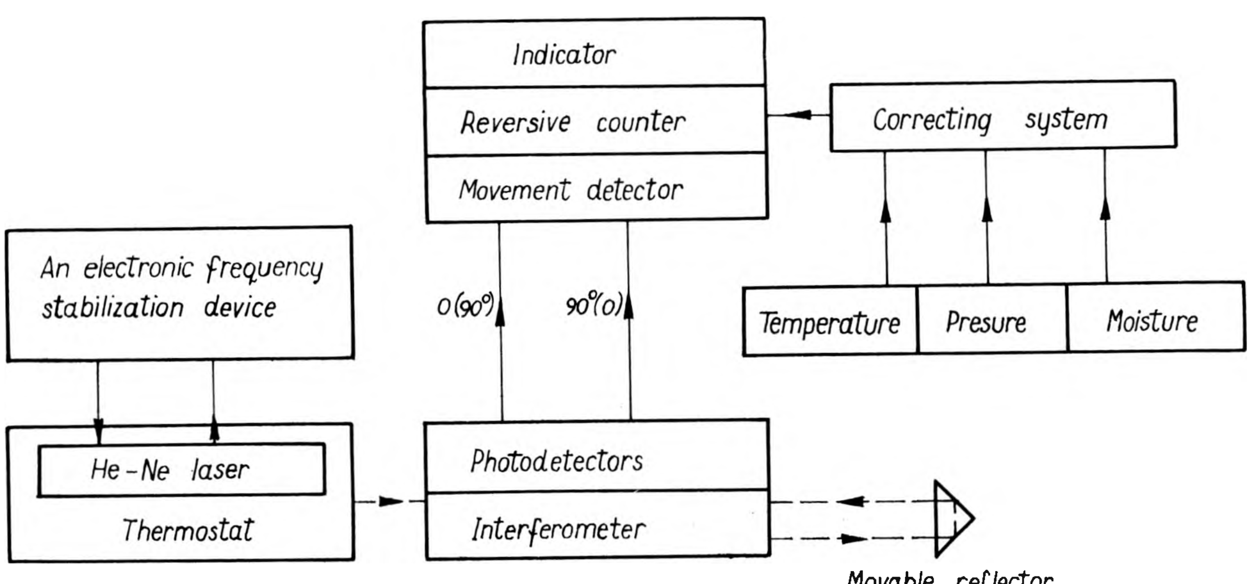

Fig. 1. A block scheme of the displacement measurement device

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has been made. The accuracy of the measurement is influenced by the stability of the laser radiation frequency as well as by medium parameters such as temperature, pressure and humidity i.e. those defining the index of refraction and consequently the wavelength of the propagating wave.

To produce an interference field for a great optical path difference the laser should operate in the lawest and basic mode TEM_{00q}, which may be achived by the proper choice of both the resonator and the discharge tube sizes. The hight degree of frequency stability in the He—Ne laser required in these experiments was ensured by a specific construction of the resonator, i.e., a coaxial quartz pipe immersed in a thermostatic head with the temperature stability equal to ± 0.01°C and application of electronic feedback system.

The measurement error caused by the influence of medium parameters on the wavelength may be avoided by keeping the values of these parameters fixed, Another way is to apply a special correcting unit introducing to the counter a correction term proportional to the momental indications of the gauge picks up for all these parameters.

If λ_0 is a wavelength in the vacuum and $n(t,p,w)$ denotes a refractive index of the air for given t[°C], p[Tr], w[Tr], then the wavelength for the same values of the parameters is expressed by the following relation

$$\lambda(t,p,w) = \frac{\lambda_0}{n(t,p,w)}.$$

Considering the wavelength λ as a function of the three variables and taking under consideration linear terms in the Taylor series in the expansion of $\lambda(t,p,w)$ around the point (t_0,p_0,w_0) we get

$$\lambda(t,p,w) = \lambda(t_0,p_0,w_0) - [n_t'(t-t_0)+n_p'(p-p_0+n_w'(w-w_0)] \times \frac{\lambda(t_0,p_0),w_0)}{n(t_0,p_0,w_0)}.$$

Substituting the numerical values of n_t', n_p' and n_w'

$$n_t' = -0.96\cdot 10^{-6}[°\mathrm{C}]^{-1},$$
$$n_p' = 0.36\cdot 10^{-6}[\mathrm{Tr}]^{-1},$$
$$n_w' = -0.054\cdot 10^{-6}[\mathrm{Tr}]^{-1},$$

we obtain

$$\lambda(t,p,w) = \lambda(t_0,p_0,w_0)+[16(t-t_0)+6(p-p_0)+ +(w-w_0)]\frac{6\cdot 10^{-8}\lambda(t_0,p_0,w_0)}{n(t_0,p_0,\phi_0)}.$$

The wavelength of the He—Ne laser for $t = 21$°C, $p = 725$ Tr and $w = 10$ Tr is equal

$$\lambda(21°\mathrm{C},\ 725\ \mathrm{Tr},\ 10\ \mathrm{Tr}) = 0.632828296\ \mu\mathrm{m}.$$

Information about the measurement is supplied to the electronic couting unit by the two sinusoidal signals generated by photodetectors and signals from gauges providing respective data of the medium parameters. One sinusoidal period from the detector corresponds to the change of the movable corner prism position in the interferometer equal to one half of the actual wavelength in the given medium i.e. $\frac{\lambda(t,p,w)}{2}$. For a constant relative phase shift between the both sinusoids equal to 90° any displacement is detectable with the accuracy $\frac{\lambda}{8}(t,p,w)$ with the simoultaneous determination of movement turns.

Thus the principle of operation of the counting system consists in:

1) determining medium parameters (t, p, w) at the moments τ_i,

2) determining momental values of the wavelength $\lambda(t_i, p_i, w_i)$,

3) detecting the movement, i.e., determining the moments τ_i, at which the change in position exceeds the succesive values $\frac{\lambda(t_i, p_i, w_i)}{8}$.

This is reduced, in practice, to generating the two types of signals

$$S^0(\tau_i)\begin{cases} 1, & \text{when the optical path difference increases by } \frac{\lambda}{8} \\ 0, & \text{when the optical path difference increases less than } \frac{\lambda}{8}. \end{cases}$$

$$S^1(\tau_i)\begin{cases} 1, & \text{when the optical path difference decreases by } \frac{\lambda}{8}, \\ 0, & \text{when the optical path difference decreases less than } \frac{\lambda}{8}. \end{cases}$$

The actual value of the total change in the movable corner prism position in the interferometer may by expressed as follows

$$P_n = \frac{1}{8}\sum_{i=0}^{n}\lambda(t_i, p_i, w_i)[S^0(\tau_i)-S^1(\tau_i)]$$

within the time interval $\tau_0 \leqslant \tau \leqslant \tau_n$, where τ_0 denotes the starting moment and τ_n is the actual moment of time.

The measurement result of the P_n is displayed on an optical indicatory device in the digital form recounted in μm (SI).

3. Description of the optical system of the interferometer

A scheme of the optical system of the measurement interferometer is presented in the fig. 2. It contains the following units and details:

1. A telescopic system for laser beam expanding,
2. A beam splitter unit,
3. The corner cube prism,
4. Elements enabling the distinguishing of shift turn of the corner prism,
5. Photodetectors.

After passing through telescope *1* the laser beam is expanded while its divergence decreases. Due to the partial reflectance from the internal *p* planes of element *2* two light bundles appear interfering with the beam reflected from the corner prism sides. There is a birefringent element *4* located in the path of the last beam, which introduces a phase difference between the vertical and the horizontal components of the laser beam needed to produce two sinusoidal sygnals shifted with respect to each other by a definite angle, for instance, 90° from the photodetectors *7* and *8*. This makes it possible to identify the movement turns of the corner prism *3* coupled with the object, the displacement of which is under study. Polarizers *5* and *6* are located in front of photoelements *7* and *8*.

Because of the total internal reflections which take place in our system some changes in the polarization state of the laser beam are likely to occur. We shall discuss this in details by using the Jones matrix formalism.

4. The influence of the corner prism on the polarization state in the laser beam

We want to find such an operator C, which would represent the operation of the corner prism i.e. to transform the components X, Y of the electric field vector of the beam entering the prism into components X', Y' of that leaving the corner prism. Let us write this down in the following form

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = C \begin{bmatrix} X \\ Y \end{bmatrix}.$$

It is easy to notice that

$$C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-\frac{i\varphi}{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \times$$

$$\times \begin{bmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-\frac{i\varphi}{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} e^{i\frac{\varphi}{2}} & 0 \\ 0 & e^{-\frac{i\varphi}{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} =$$

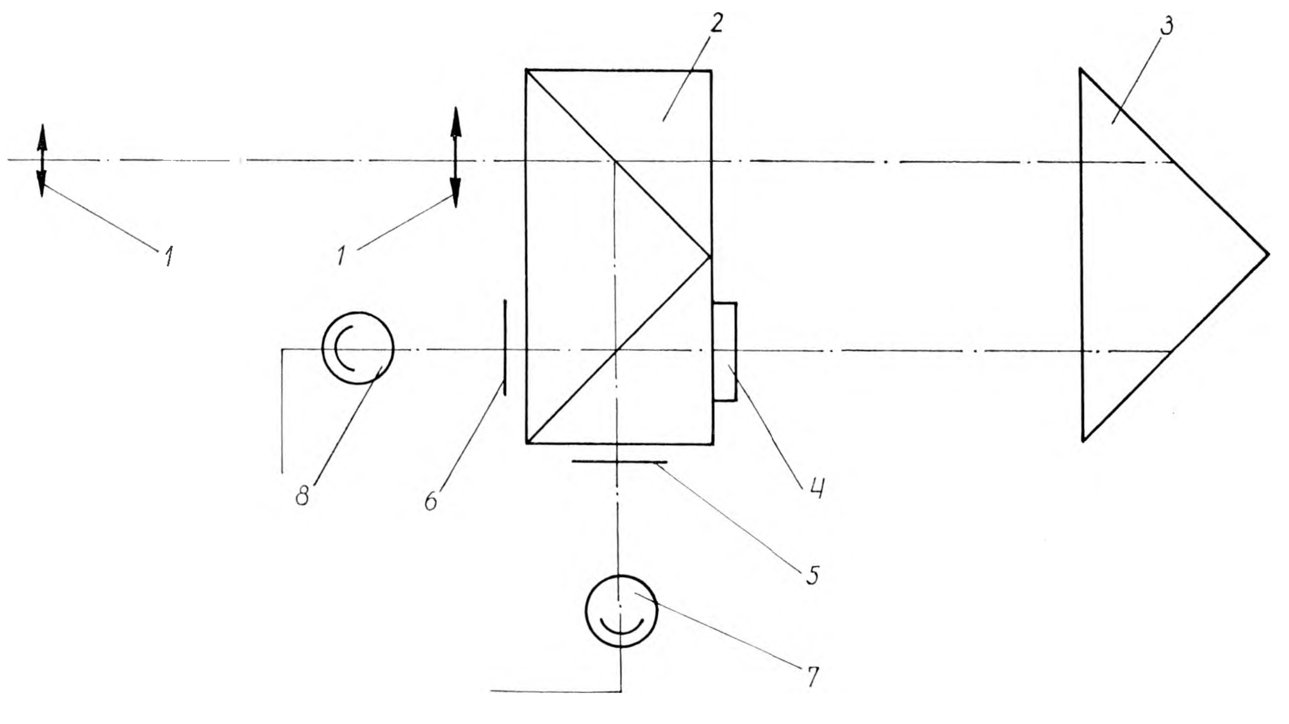

Fig. 2. A scheme of the interferometer for displacement measurements *1* — telescope, *2* — laser beam splitter unit, *3* — corner cube prism, *4* — phase shifter, *5* and *6* — polarizers, 7 and *8* — photodetectors

$$
= \frac{1}{8}\begin{bmatrix} -2\cos 3\alpha - 6\cos\alpha + i(\sin 3\alpha + 9\sin\alpha); \\ -i(\sin 3\alpha + \sin\alpha)\sqrt{3}\ ; \\ -i(\sin 3\ \alpha + \sin\alpha)\sqrt{3}\ ; \\ 2\cos 3\alpha + 6\cos\alpha + i(\sin 3\alpha + 9\sin\alpha). \end{bmatrix},
$$

where $\alpha = \frac{\varphi}{2}$ and φ denotes a phase difference between the vertical and the horizontal components of the electric field appearing as a result of the single internal reflection inside the corner prism. For the optical glass BK 7:

$$
\cos\alpha = 0.92321 \qquad \cos 3\alpha = 0.37782
$$
$$
\sin\alpha = 0.38431 \qquad \sin 3\alpha = 0.92588
$$

Then

$$
C = \begin{bmatrix} -0.78686 + i0.54808; & i0.28366 \\ i0.28366; & 0.78686 + i0.54808 \end{bmatrix}
$$

the matrix C has a general form

$$
C = \begin{bmatrix} -a + ib; & -ic \\ ic\ \ ; & a + ib \end{bmatrix}
$$

where

$$
a = 0.78686 \qquad b = 0.54808 \qquad c = 0.28366.
$$

As already mentioned

$$
\begin{bmatrix} X' \\ Y' \end{bmatrix} = C \begin{bmatrix} X \\ Y \end{bmatrix}
$$

Hence

$$
X' = -aX + i(bX - cY) \qquad Y' = aY + i(cX + bY).
$$

The linear polarization of the laser beam will be preserved after a threefold reflection from the planes of the corner prism, if the phases of magnitudes X' and Y' are equal to each other. We note it as follows

$$
\frac{bX - cY}{-aX} = \frac{cX + bY}{aY}.
$$

Hence

$$
\frac{b\frac{X}{Y} - c}{-a\frac{X}{Y}} = \frac{c\frac{X}{Y} + b}{a}.
$$

When substituting

$$
\frac{X}{Y} = Z = \tan\psi
$$

we obtain

$$
cz^2 + 2bz - c = 0,
$$

$$
z_{1,2} = \frac{b}{c}\left(-1 \pm \sqrt{1 + \left(\frac{c}{b}\right)^2}\right),
$$

$$
z_1 = 0.24254 \qquad \psi_1 = 13°37',
$$

$$
z_2 = -4.1068 \qquad \psi_2 = -(90° - 13°37').
$$

Thus we have reached the following two solutions

$$
\begin{bmatrix} X_1' \\ Y_1' \end{bmatrix} = C \begin{bmatrix} 0.24244 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2433\exp(i\ 38.46°) \\ 1.0015\exp(i\ 38.12°) \end{bmatrix}
$$

$$
\begin{bmatrix} X_2' \\ Y_2' \end{bmatrix} = C \begin{bmatrix} -4.1068 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.12\exp(i\ 38.16°) \\ 0.99975\exp(i\ 38.06°) \end{bmatrix}.
$$

It is clear from the above considerations that there are two orientations of the input beam polarization plane, for which the light beam leaving the corner prism remaines practically linearly polarized. If we place a birefringent element in the path of such a beam a decomposition of the electric field into two mutually perpendicular vector occurs, while the phase difference between them depends on the thickness of the element and the material dispersion. To determine the vibration plane of both components a linear polarizer (a polaroid film for instance) is placed in front of each detector. The required phase difference thus produced for the harmonic signals and a differentiation of the corner prism movement turns becomes thereby possible.

5. Conclusions

The described interferometer offers the following advantages important for the measurements:

1. There is no strong light beam reflected from the opical elements, which, when going back to the light source, would cause some changes in the generated frequency.

2. The possibility of unwanted derangement of the interferometer is considerably reduced by application of a corner prism, composed of three mutually perpendicular plane mirrors.

3. There is a possibility to distinguish the movement turns of the elements the displacement of which are to be measured.

All optical elements of the interferometer were made in the Institute of Quantum Electronics, Military Technical Academy, Warsaw, Poland. The authors are obliged to Mr. A. Tulibacki and Mr. R. Brunsz for taking part in the realization of the investigation.

Лазерное устройство для измерений перемещений

Описывается принцип действия лазерного устройства для измерений перемещений, учитывая факторы, влияющие на точность измерения. Подробно обсуждается вопрос изменений состояния поляризации лазерного пучка при его многократных отражениях внутри угловой призмы.

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