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Light Polarization in Multiple Total Reflection

The behaviour of plane polarized light totally reflected in two rectangular prisms with co-planar or perpendicular planes of incidence have been analyzed. The resulting foımulae allow to establish the phase shift between componens of polarized light.

In instruments working with polarized light the knowledge of the way in which the polarized light entering the device is affected by its optical elements is a matter of major importance. This problem must be taken into consideration whenever such an instrument embodies, for example, a prism system which inverts the image and, at the same time, shortens the mechanical length of the device. If it is necessary to change plane polarized light to generally elliptically polarized light, this can be done through the use of a Fresnel parallelepiped producing double total reflection.

In case of total reflection, $\cos\varepsilon_1'$ (ε_1' — the angle of refraction) is a complex quantity and Fresnel's amplitudes are generally complex numbers [1], [2]. Between components p and s there exist phase shifts δ_p and δ_s. The phase difference $\varphi = \delta_p - \delta_s$ between the two components is determined by the formula ([1], equation 6.18)

$$\tan\frac{\varphi}{2} = \frac{\cos\varepsilon_1 \sqrt{n_1^2 \sin\varepsilon_1 - 1}}{n_1 \sin^2\varepsilon_1}, \qquad (1)$$

where ε_1 is incidence the angle of the ray on the glass-air boundary, and n_1 is the refractive index of the glass.

To achieve a shorter construction length of an instrument, one can use two right-angle prisms producing total internal reflection. There exist two arrangements of such prisms in which the planes of incidence in both prisms are either co-planar (fig. 1) or at a right angle to each other (fig. 2). In the latter case the system is the Porro inverting system of the first kind.

The phase difference for the first arrangement of the prisms (fig. 1) can be obtained by solving the amplitude condition of total reflection (see [1], equation 6.6) successively for all four boundaries.

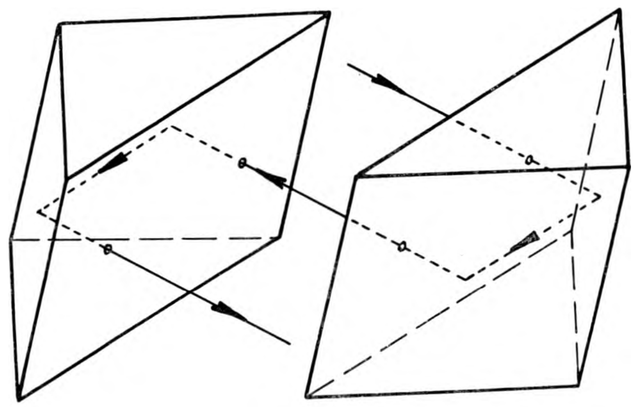

Fig. 1

Since the phenomenon here involved is total reflection, light incident on the next surface, it follows that

$$r_{1p}e^{i\delta_{1p}} = r_{2p}e^{i\delta_{2p}} = r_{3p}e^{i\delta_{3p}} = r_{4p}e^{i\delta_{4p}},$$

$$R_{1p} = A_{2p}; \quad R_{2p} = A_{3p}; \quad R_{3p} = A_{4p}; \qquad (2)$$

$$\delta_{1p} = \delta_{2p} = \delta_{3p} = \delta_{4p},$$

similar assumptions also apply to component s.

The amplitude conditions for component p yield for the Fresnel's amplitude of light in first total reflection

$$r_{1p}e^{i\delta_{1p}} = \frac{R_{1p}}{A_{1p}} = \frac{n_1\cos\varepsilon_1 + in^2\sqrt{n_1^2\sin^2\varepsilon_1 - 1}}{n_1\cos\varepsilon_1 - in_1^2\sqrt{n_1^2\sin^2\varepsilon_1 - 1}}. \qquad (3)$$

For light reflected on the second surface

$$r_{2p}e^{i\delta_{2p}} = \frac{R_{2p}}{A_{2p}} = \frac{n_1\cos\varepsilon_1 + in_1^2\sqrt{n_1^2\sin^2\varepsilon_1 - 1}}{n_1\cos\varepsilon_1 - in_1^2\sqrt{n_1^2\sin^2\varepsilon_1 - 1}}. \qquad (4)$$

With respect to (2), the amplitude of light twice totaly refleted turns out to be

$$R_{2p} = A_{1p}\left[\frac{n_1\cos\varepsilon_1 + in_1^2\sqrt{n_1^2\sin^2\varepsilon_1 - 1}}{n_1\cos\varepsilon_1 - in_1^2\sqrt{n_1^2\sin^2\varepsilon_1 - 1}}\right]^2. \qquad (5)$$

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and from there

$$r_{IIp}e^{i\delta_{IIp}} = \frac{R_{2p}}{A_{1p}} = r_{1p}e^{i\delta_{1p}}r_{2p}e^{i\delta_{2p}} = r_{1p}^2 e^{i\delta_{1p}}.$$

Accordingly, for $2m$ total reflections in m prisms it is

$$r_{2mp}e^{i\delta_{2m}} = r_{1p}^{2m}e^{i2m\delta_{1p}}. \tag{6}$$

In the case considered, there are four total reflections in two prisms. It is, therefore,

$$r_{IVp}e^{i\delta_{IVp}} = r_{1p}^4 e^{i4\delta_{1p}} \tag{7}$$

similarly, for component s

$$r_{IVs}e^{i\delta_{IVs}} = r_{1s}^4 e^{i4\delta_{1s}}. \tag{8}$$

The phase difference between components p and s is

$$\varphi = \delta_{IVp} - \delta_{IVs} = 4(\delta_{1p} - \delta_{1s}). \tag{9}$$

In systems with m co-planar prisms in which $2m$ total reflections take place, the phase difference is equal to $2m$-times the phase difference in one total reflection, i.e. to

$$\varphi_{2m} = 2m(\delta_{1p} - \delta_{1s}). \tag{10}$$

In the second case when the prisms with the planes of incidence at a right angle to each other form the Porro system of the first kind (fig. 2), the situation is very similar.

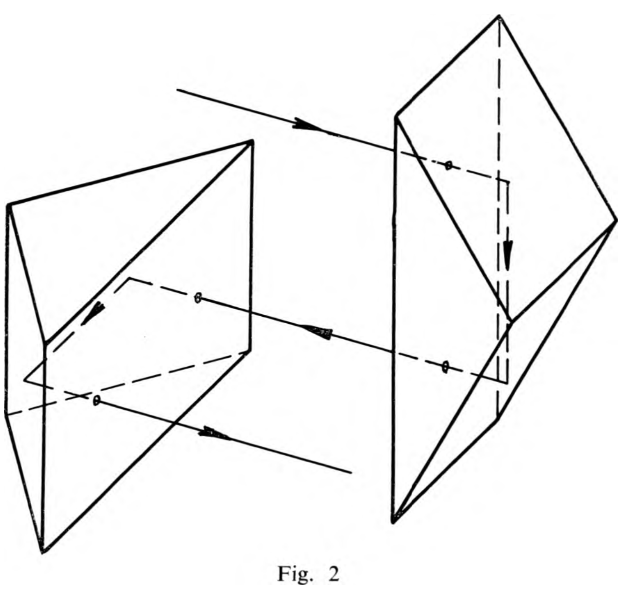

Fig. 2

Because of the perpendicularity of the planes of incidence an interchange of components p and components s occurs as the light passes from one prism to another.

It follows, therefore, that

$$\begin{aligned} r_{1p}e^{i\delta_{1p}} &= r_{2p}e^{i\delta_{2p}} = r_{3s}e^{i\delta_{3s}} = r_{4s}e^{i\delta_{4s}}, \\ r_{1s}e^{i\delta_{1s}} &= r_{2p}e^{i\delta_{2s}} = r_{3p}e^{i\delta_{3p}} = r_{4p}e^{i\delta_{4p}}. \end{aligned} \tag{11}$$

The appertaining Fresnel amplitudes are determined by a procedure identical with that used in the previous case. Since there occurs an interchange of the components, we use a different notation

$$ue^{i\xi} = r_{1p}e^{i\delta_{1p}} \cdot r_{2p}e^{i\delta_{2p}} \cdot r_{3s}e^{i\delta_{3s}} \cdot r_{4s}e^{i\delta_{4s}} = $$
$$= r_{1p}^2 e^{i2\delta_{1p}} \cdot r_{3s}^2 e^{i2\delta_{3s}}, \tag{12}$$

$$ve^{i\eta} = r_{1s}e^{i\delta_{1s}} \cdot r_{2s}e^{i\delta_{2s}} \cdot r_{3p}e^{i\delta_{3p}} \cdot r_{4p}e^{i\delta_{4p}} = $$
$$= r_{1s}^2 e^{i2\delta_{1s}} \cdot r_{3p}^2 e^{i2\delta_{3p}}. \tag{13}$$

The reflection being a total one, it follows that

$$r_{1p}^2 = r_{3s}^2 = 1; \quad r_{1s}^2 = r_{3p}^2 = 1, \tag{14}$$

for the components of the fourfold totaly reflected light we get

$$\begin{aligned} ue^{i\xi} &= r_{1p}^4 e^{i2(\delta_{1p}+\delta_{3s})}, \\ ve^{i\eta} &= r_{1s}^4 e^{i2(\delta_{1s}+\delta_{3p})} \end{aligned} \tag{15}$$

and consequently, for the phase difference

$$\varphi = (\xi - \eta) = 2[(\delta_{1p}+\delta_{3s}) - (\delta_{1s}+\delta_{3p})]. \tag{16}$$

With a view to relation (11) it is

$$\varphi = 0. \tag{17}$$

Relations (9) and (17) make it possible to determine the phase difference of components p and s in both cases considered, and thus also the resultant ellipticity of the light leaving the instrument. When a Porro system of the first kind is used, the resulting phase difference between the two components is zero and the plane polarized light will, therefore not be affected in any way.

If there are j prisms in one plane and k prisms in the plane at a right angle to it, the phase difference between the two components is established by the general relation

$$\varphi = 2(j-k)(\delta_p - \delta_s). \tag{18}$$

Relations (10) and (18) apply to the two special cases when the planes of incidence in the prisms are either co-planar or at a right angle to each other. The case of the general position of the two planes of incidence was not considered in the paper because its presentation is not as clear as that of the cases discussed and because complications are apt to arise when deriving the pertinent relations with the symbols used. It is more conveniently treated by some other method, for example, that of Jones.

In conclusion the author wishes to extend his thanks to Dr Z. Knittl and Dr J. Sládková for their critical comments and inciting discussions.

Поляризация света во время многократного полного отражения

Проанализировано было поведение плоско поляризованного света, полностью отражённого в двух прямоугольных призмах, с параллельными или перпендикулярными плоскостями падения. Полученные формулы дают нам возможность определить сдвиг по фазе между составляющими поляризованного света.

References

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