*Jerzy Nowak**

An Analysis of Designing a Superachromatic Objective with a Flat Field

Optical systems with corrected longitudinal chromatic aberration within the range $0.365 \leqslant \lambda \leqslant 1.014\mu$ are called superachromats. The problem of superachromatic correction was analyzed by a number of authors including Herzberger, Stephens, Schultz, Druck and Pulvermacher. It has been pointed out that the superachromatic correction in the paraxial region may be obtained in the two-lens or three-lens systems. The system exhibit, however, considerable remaining aberrations, e.g. a supcherachromatic aberration, which restricts their application. In the most cases the systems are part of more complex systems. A difference in partial dispersion of both the glasses is necessary to design a two-lens superachromat, i.e. —

$$P_{\lambda 1} = P_{\lambda 2}. \tag{1}$$

Practically it is sufficient to verify whether the relation (1) for two lines $\lambda_* = 1.014\mu$ and $\lambda_{**} = 0.365\mu$ is fulfilled and hence the condition (1) may be written as follows

$$P_{*1} = P_{*2},$$
$$P_{**1} = P_{**2}. \tag{2}$$

The fulfillment of equation (2) guarantees the superachromatic correction. To make the optical system one of practical importance the glasses should differ considerably by the value of the relative dispersion in addition to satisfying (2). For this reasons, fluorit ($\nu_F = 95.84$) or FK 50 ($\nu_F = 82.18$) would be among the usual glasses — the best material for the first lens. The second lens may be made of certain crowns or lanthan crowns. In the up to date works several types of superachromatic systems have been calculated including an already produced, three-lens system, consisting of two positive lenses made of fluorite and of one negative lens made of LaK 9.

The purpose of this paper is to design a superachromat with corrected Petzval field curvature. The simplest systems fulfilling the condition of superachromatic correction with the also corrected Petzval curvature is a two-lens system with an air spacing. An approximate dependence for the secondary spectrum of the longitudinal chromatic aberrations for a focal length normalized to unity was given by ČANŽEK [1, 2]

$$\delta s'_{F\lambda} = \frac{h(P_{\lambda 1} - P_{\lambda 2})}{h\nu_1 - \nu_2}, \tag{3}$$

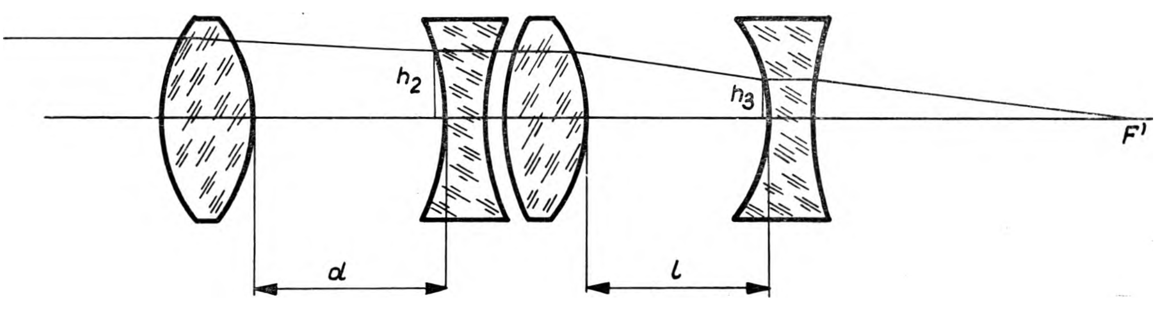
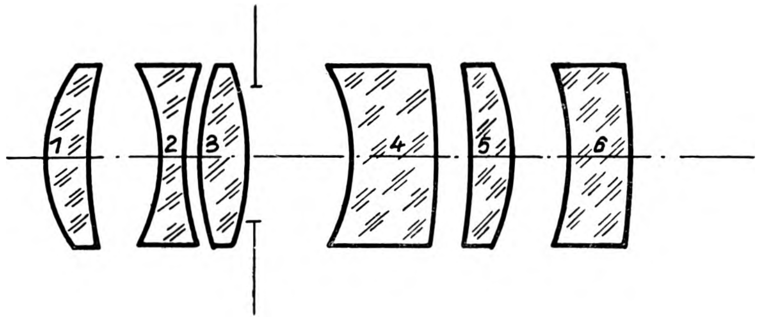


Fig. 1. Four-lens superachromat with the flat field

*) Institute of Technical Physics, Technical University of Wrocław, Wrocław, Wybrzeże Wyspiańskiego 27, Poland.



Lens No	Glass
1	Fluorite
2	LaK 11
3	Fluorite
4	LaK 9
5	FK 50
6	LaK 11

Fig. 2. Final superachromat system with the flat field

where

$\delta s'_{F\lambda}$ — difference between the image distance for the λ wavelength (assumed as basic) and the image distance for λ.

h — incidence height of the aperture ray on the second lens,

P — partial dispersion (index 1—denote the first lens, index 2—the second),

ν — Abbe's number.

As shown in the paper [3] this relation is correct only if the product of the focusing powers of the first lens and the distance between the lenses is relatively small. In this paper another relation—also approximative but more exact — has been derived namely

Table 1. Values of focusing power and the distance between the lenses for two chosen superachromatic systems with a flat field

No.	Glass	φ_1	φ_2	d
1	Fluoryt LaK 11	6.1581	−7.1404	0.045
2	FK 50 SK 7	9.1480	−9.9103	0.019

Table 2. The difference of image distances for the typical wave lengths

No.	$s'_\lambda - s'_F$ [mm]					
	*	A'	C	D	h	**
1	0.01	0.02	0	0.02	0.07	0.04
2	0.05	0.12	0.04	0.01	0.04	0.1
3	−0.04	0.03	0.02	0	−0.01	0.01
4	−0.06	−0.08	−0.07	−0.01	0.03	0.02

$$\delta s'_{F\lambda} = \frac{h(P_{\lambda 1}-P_{\lambda 2})}{h\nu_1-\nu_2} - \frac{(1-h)P_{\lambda 1}P_{\lambda 2}}{\nu_1(h\nu_1-\nu_2)}. \tag{4}$$

The two-lens system with an areal spacing exhibits also a lateral chromatic aberration. The aberration is determined by the equation

$$\delta l'_{FC} = \frac{l'd}{\nu_1(1-d)-\nu_2} \tag{5}$$

$\delta l'_{FC}$ — the difference of the paraxial image magnitudes for two colours, red and violet,

l' — image magnitude for the basic colour,

d — the distance between the lenses.

The secondary spectrum of the lateral chromatic aberration may be calculated by the formula

$$\delta l'_{\lambda F} = \bar{P}_\lambda l'_{FC} \tag{6}$$

as shown in the paper [4]; where $\bar{P}_\lambda$ the average value of the partial dispersion of the two glasses used. To obtain the values for the positive lenses and the distance between them so that a superachromat of corrected Petzval curvature can be obtained the following system of equations have to be solved

$$\frac{\varphi_1}{\nu_1} + (1-d\varphi_1)^2\frac{\varphi_2}{\nu_2} = 0,$$

$$\frac{\varphi_1}{n_1} + \frac{\varphi_2}{n_2} = 0, \tag{7}$$

$$1-\varphi_1-\varphi_2+d\varphi_1\varphi_2 = 0.$$

If we define

$$\frac{n_1}{n_2} = n \quad \frac{\nu_2}{\nu_1} = \nu, \tag{8}$$

then the solution for the system of practical importance takes the form

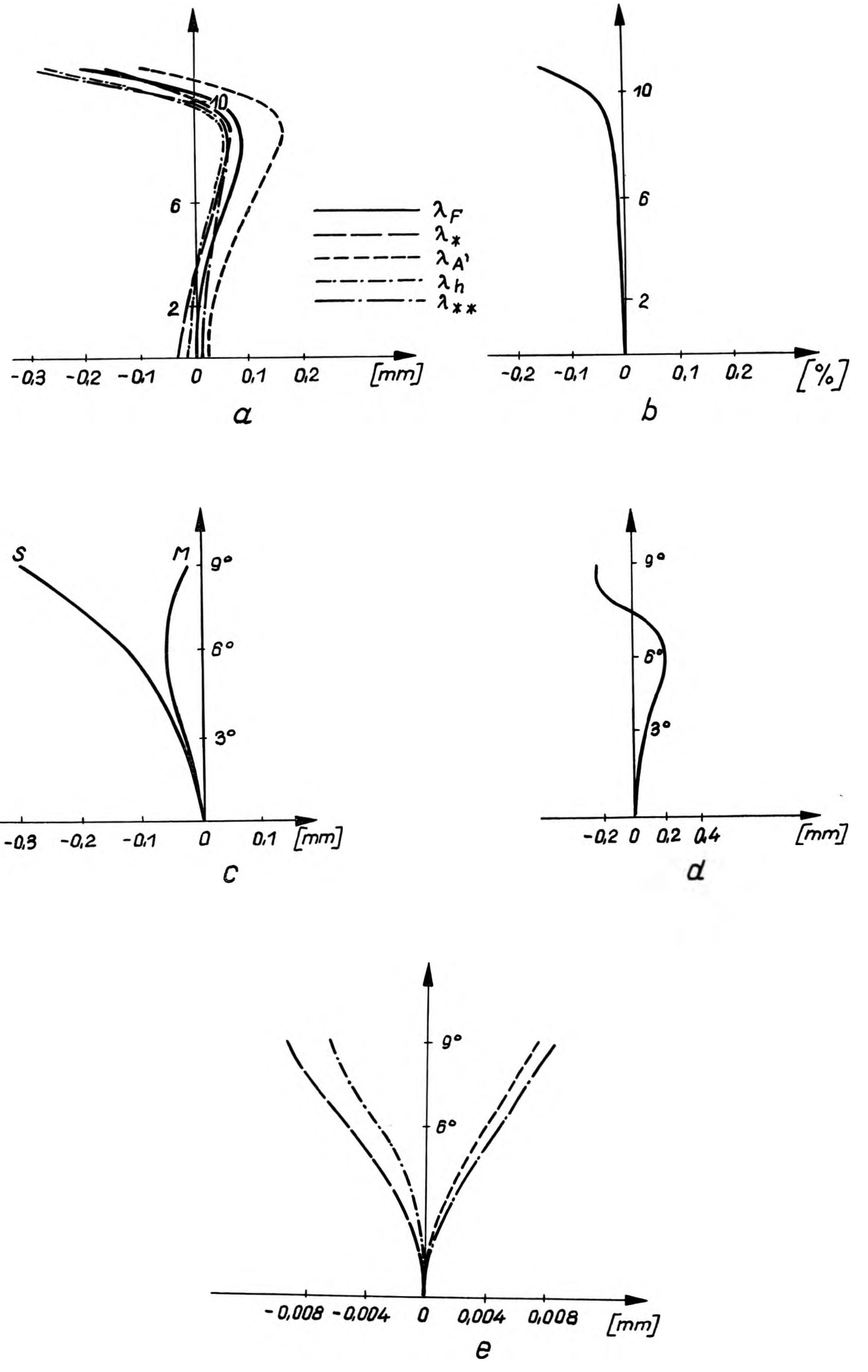


Fig. 3. Geometrical aberration of the superachromatic system with the flat field

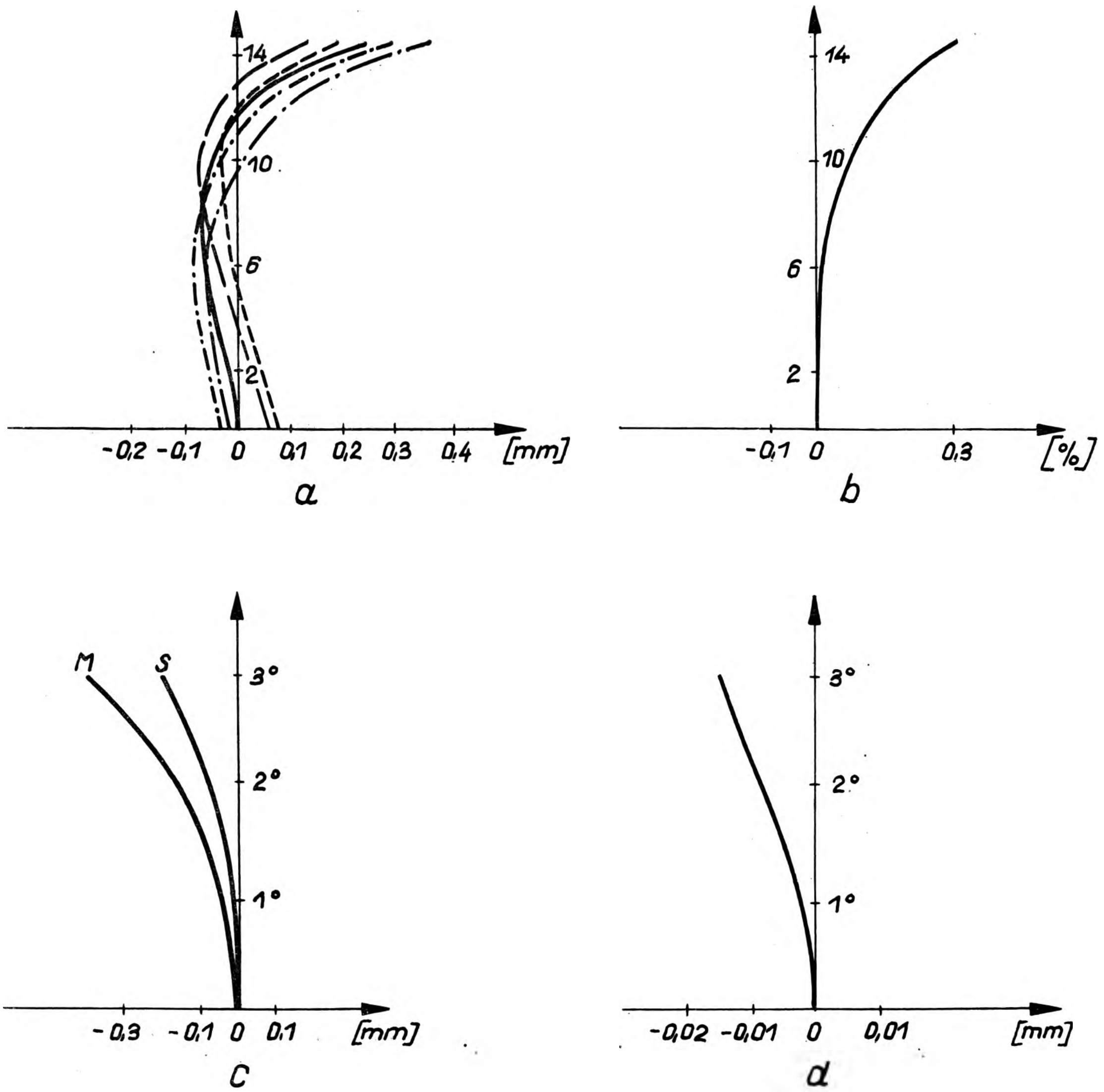


Fig. 4. Geometrical aberrations for the three-lens superachromat

$$\varphi_1 = \frac{n+\sqrt{n\nu}}{n-\nu} \quad d = \frac{1-n\nu}{\varphi_1(n+1)-n} \quad \varphi_2 = \frac{1-\varphi_1}{1-d\varphi_1}. \tag{9}$$

Two examples of systems complying with formula (9) are given in Table 1.

Two differences of the image distances of the two systems for the particular wavelengths ($f' = 100$ mm) are supplied in Table 2 in rows No. 1 and 2 respectively.

It may be seen that the longitudinal chromatic aberration is sufficiently small. Naturally, systems of that kind are of no practical importance considering the great power of particular lenses. Therefore, the system has been developed in such a way that it consists of four elements (to preserve the focal length of the system all the radia and thicknesses of lenses are multiplied by factor 2) (Fig. 1). If the two-lens system had a corrected lateral chromatic aberration the system developed in this manner would not meet the required condition. To fulfil this condition it is necessary that

$$h_2^2 = h_3. \tag{10}$$

The fulfillment of Eq. (10) requires, for instance, a slight change of the difference between the last but one and the last lenses. The difference may be determined by the formula

$$l = \frac{d\varphi_2(1-d\varphi_1)}{2\varphi_1+\varphi_2-d\varphi_1(\varphi_1+\varphi_2)}. \tag{11}$$

Next, the focal lens has to be normalized to a unity.

When examining the system with the help of trigonometrical tracing and Seidel's sums it has been found that spherochromatic aberration cannot be corrected for the relative aperture 1:4.5. Therefore, the system was developed so that it ultimately consisted

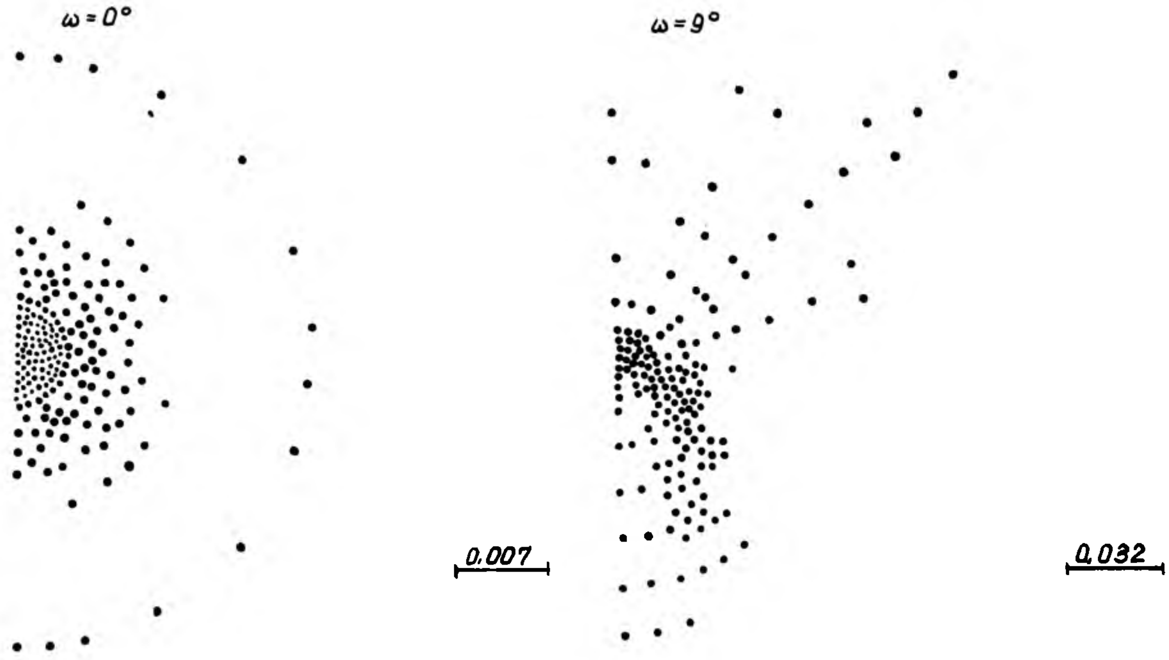


Fig. 5. Spot-diagram of the superachromatic system with the flat field

of six lenses. The system was further corrected to minimize the aberrations. Particular attention had to be paid to the lateral chromatic aberration, which have to be corrected within the whole spectrum range.

Finally a system has been achieved, which is diagrammatically presented in Fig. 2. Its focal length is $f' = 100$ mm, relative aperture 1:4.5, field angle $2\omega = 18°$. Its imaging quality was estimated by comparing the values of the chosen quality measures such as Seidel sums, geometrical aberrations, spot-diagrams with statistical analysis, geometric transfer function and the energy distribution in the aberration spot with those of the three-component superachromatic system without Petzval curvature correction; the latter system has been performed ($f' = 100$ mm, relative aperture 1:3.5, field angle $2\,\omega = 6°$).

As the wave aberrations for the field $\omega = 0$ is of order of wave-length the product of geometric transfer function and the MTF for the diffraction limited system [5] was assumed as a quality measure. For a maximum field angle the geometric transfer function alone is sufficient for the imaging quality evaluation. The statistical measures of imaging quality are defined as follows [6]:

$$KR1 = \frac{1}{N}\sum_{i=1}^{N} |r_i|,$$

$$KR2 = \frac{1}{N}\sum_{i=1}^{N} r_i^2,$$

$$KR3 = \frac{r_0}{N}\sum_{i=1}^{N} \frac{1}{|r_i|+r_0}, \tag{12}$$

$$KR4 = \frac{r_0^2}{N}\sum_{i=1}^{N} \frac{1}{r_i^2+r_0^2},$$

where

N — number of rays traced through the systems,

r_i — distance of the i-th ray intersection point with the image plane from the centre of gravity of the aberration spot,

r_0 — radius of the Airy spot.

In the third row of the Table 2 the image distance differences for the typical wavenlengths in the calculated system are given, while the corresponding values for the superachromate are presented in the fourth row. In an analogical sequence Seidel sums and the

Table 3. Seidel's sums for the superachromatic with flat field and those for the three-lens system

No.	S_1	S_2	S_3	S_4	S_5	S_{1CHR}	S_{2CHR}
1	0.1478	−0.1757	0.2376	0.4070	−0.2734	−0.0004	0.0080
2	0.2161	0.1688	0.3599	0.7642	−0.1793	0.0008	−0.0009

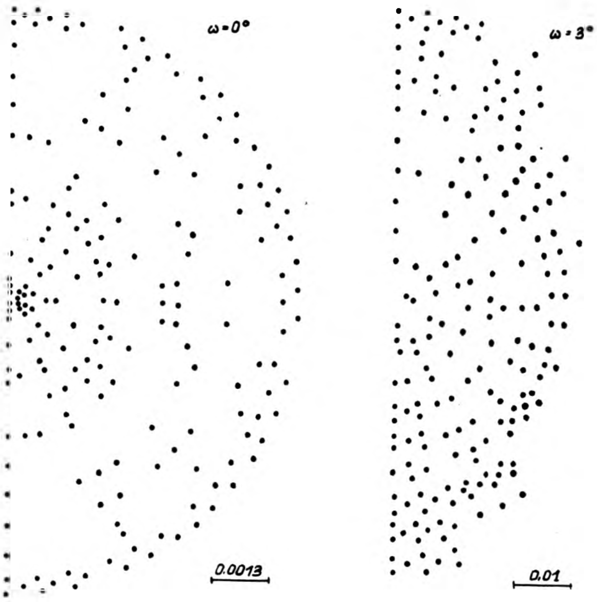


Fig. 6. Spot-diagram of the three-lens superachromat

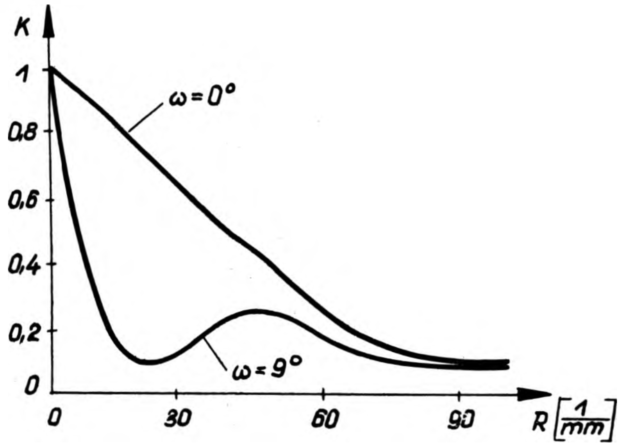


Fig. 7. Product of the geometric transfer function for the superachromatic with the flat field and the MTF of the corresponding diffraction limited system for $\omega = 0$ and the geometric transfer function for the same superachromat with $\omega = 9°$

values of the said statistic quality measures completed by the standard deviation value σ (all calculated for the maximum field angle as well as for $\omega = 0$) are presented in the Tables 3 and 4. Spherical aberration is calculated for $\lambda_F, \lambda_*, \lambda_{A'}, \lambda_h$ and λ_{**} (Figs. 3a and 4a). Deviation from the sine condition, field curvature and come are shown in the Figs. 3b, 4b, 3c, 4c, 3d

Table 4. Quality measure values for the superachromatic system with flat field and those for the three-lens system

No.	ω	*KR*1	*KR*2	*KR*3	*KR* 4	σ
1	0°	0.006	0	0.362	0.282	0.009
	9°	0.031	0.002	0.113	0.046	0.042
2	0°	0.008	0	0.218	0.129	0.013
	3°	0.027	0.001	0.048	0.0048	0.03

and 4d respectively. In the Fig. 3e chromatic aberration of magnification is given for the sand four wavenlengths $\lambda_*, \lambda_{A'}, \lambda_h, \lambda_{**}$ (for superachromatic correction the violet colour is assumed to be the basic one). The vignetting factor amounts to 0.7 for the maximum field $\omega = 9^0$.

The spot-diagram has been calculated for the field $\omega = 0$ and the maximum field angle for the basic

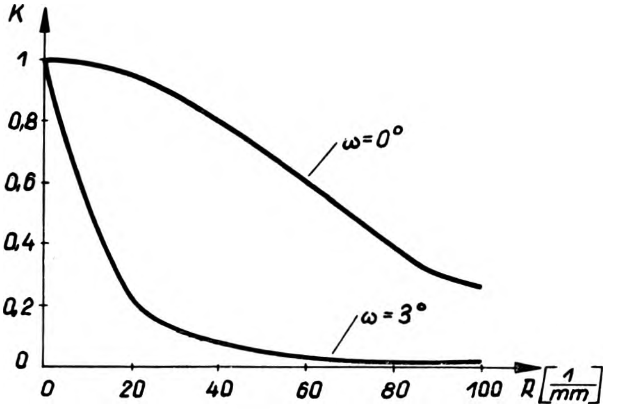


Fig. 8. Product of the geometric transfer function for the three-lens superachromat and the MTF of the corresponding diffraction limited system for $\omega = 0$ and the geometric transfer function for the same superachromat with $\omega = 3°$

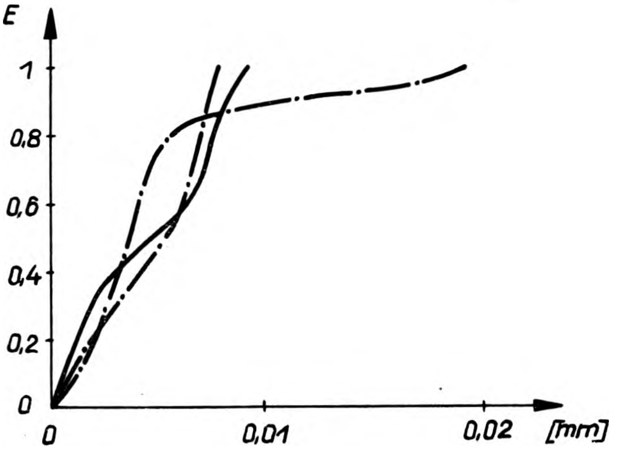


Fig. 9. Energy distribution in the aberration spot for the superachromatic system with the flat field

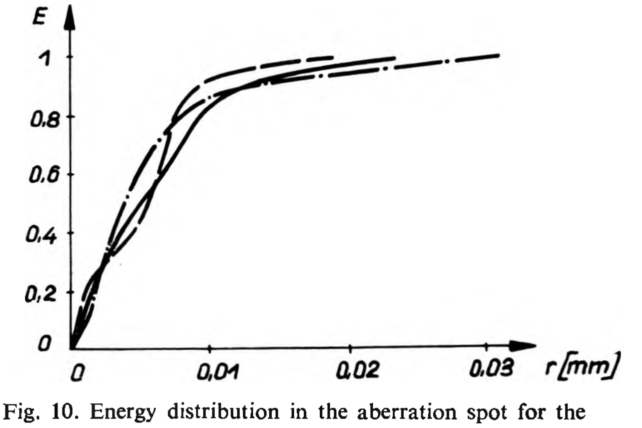


Fig. 10. Energy distribution in the aberration spot for the three-lens superachromat

colour (Fig. 5 and 6). Similarly the transfer function (Fig. 7 and 8) was computed for the some angles. Energy distribution in the aberration spot was estimated only for field angle $\omega = 0$ and for the three wavenlengths λ_F, λ_* and λ_{**} (Fig. 9 and 10) when comparing the respective values it is clear that it is possible to design the superachromatic optical system for the corrected field curvature. The imaging quality, which may be achieved in that kind of system, is comparable with that of the systems with uncorrected Petzval curvature for field angle $2\omega = 6°$. The problem of simoultaneous superachromatic correction together with the Petzval curvature correction require some further investigation, which would lead to further improvement of the imaging quality.

The author expresses his thanks to Prof. M. Gaj for the suggestion of the topic as well as many helpful discussions.

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