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Influence of the Mirror Movement on the Image Blurring in the Ultra-High-Speed Cinematography

A formula for the kinematic aberration in the camera of Miller type (caused by the fact that the rotating mirror commuting the light beam does not pass through the axis of rotation) has been derived. The obtained formulae enable to determine the admissible distance of the rotating mirror plane from the axis of rotation and the optimal position of the intermediate image in the vicinity of the commuting mirror.

A general optical scheme of a camera with an image commutation is shown in Fig. 1 [1,2]. The primary lens *PL* images the object being photographed

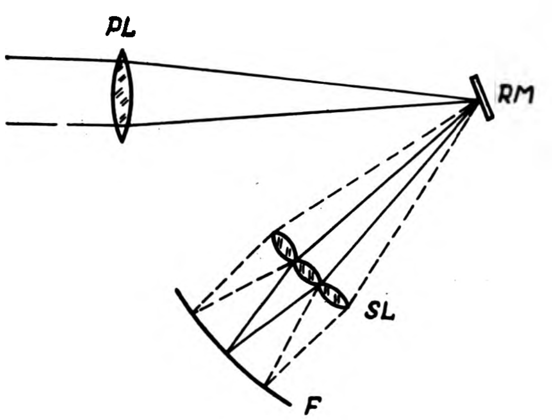

Fig. 1.

onto the plane of a rotating mirror *RM*. The beam reflected from *RM* hits succesively a series of secondary lenses mounted in the form of a gallery, each of which projects the final image onto the photographic film *F*. Possible variants of the setup are: a gallery of plane mirrors directing the light beam to a single secondary lens [3,4] or a gallery of concave spherical mirrors as shown in Fig. 2 [5,6].

The final image is stationary with respect to the film position (apart from a small rotation around an axis parallel to the rotation axis of the mirror) only if the axis of rotation lies in both the plane of the intermediate image and the reflecting surface of the rotating mirror, simultaneously. In the real setups the reflecting surface is usually shifted by a small distance r from the rotation axis because the mass distribution of the rotating set is highly recommended to be a symmetric one. Consequently, independently of the intermediate image position the final image is being displaced during the exposure time with respect to the film by certain value A_k. This displacement, which contributes (apart from the other aberrations) to the degradation of the resolution in the camera has been earlier called the kinematic aberration [5]. It has been proved [7] that the intermediate image B (see Fig. 3a) or this virtual image D produced by the rotating mirror (see Fig. 3b) moves along a trajectory known as the Pascal curve, if the rotating plane is distant by r from the axis of rotation, while the intermediate image position is defined by r'. The Pascal

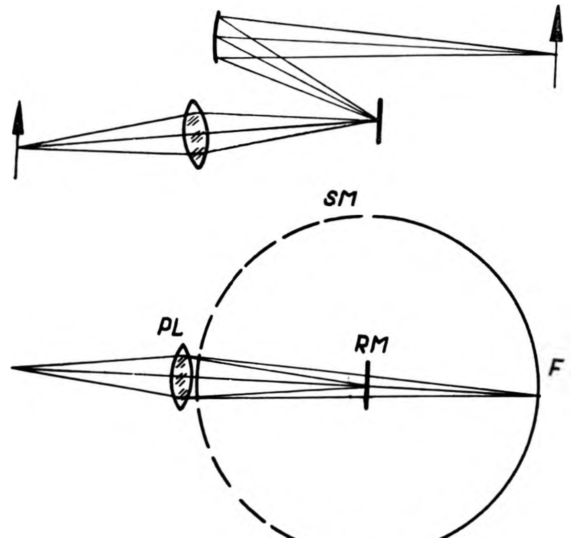

Fig. 2.

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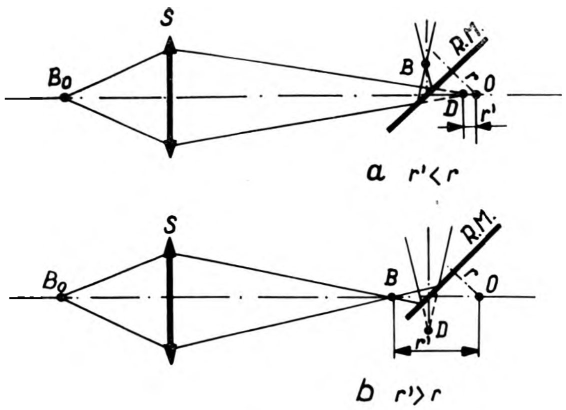

Fig. 3.

curve takes the form of a circle of $4r$ diameter for $r' = 0$ and of a cardioid for $r' = r$ (Fig. 4).

During the time needed for the principal ray of the rotating beam to pass through a single secondary

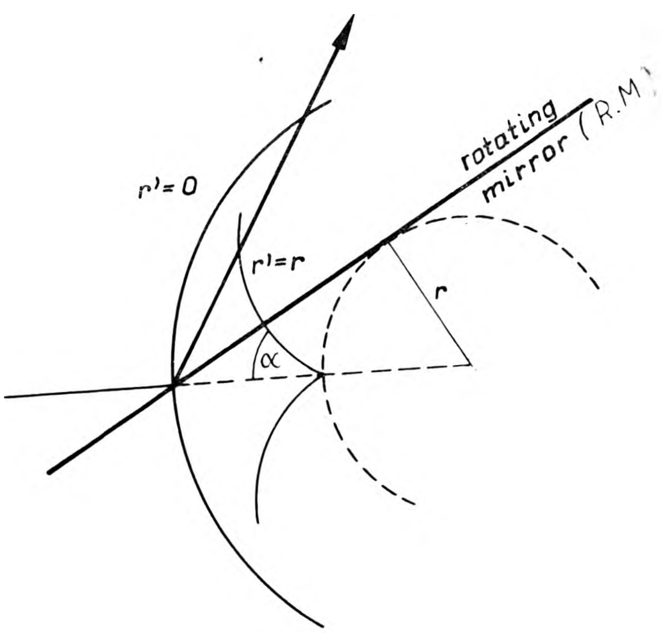

Fig. 4.

objective the intermediate image will be displaced along the Pascal curve from a point A to another point C (Fig. 5). The respective angle of rotation will be denoted by $2d\alpha$.

The arc ABC may be approximated by a straight line sector AC, with sufficient accuracy and it may be assumed that the point B associated with the principal ray, hiting the central part of the secondary lens, lies in the midpoint of the sector AC. This sector may be decomposed into two components, one parallel to the optical axis of SL and equal to $2g$, and the other one perpendicular to the axis and equal to $2e$. These two components may be expressed as follows:

$$2g = -4r\cos(\alpha-\gamma)\sin d\alpha + 2r'\cos\gamma\sin 2d\alpha,$$

$$2e = 4r\sin(\alpha-\gamma)\sin d\alpha + 2r'\sin\gamma\sin 2d\alpha,$$

where

α — angle between the normal to the rotating mirror and the principal ray of the incident beam,

γ — angle between the principal ray of the beam and the optical axis of the secondary mirror or lens objective at the moment, when the principal ray hits the central point of the secondary objective.

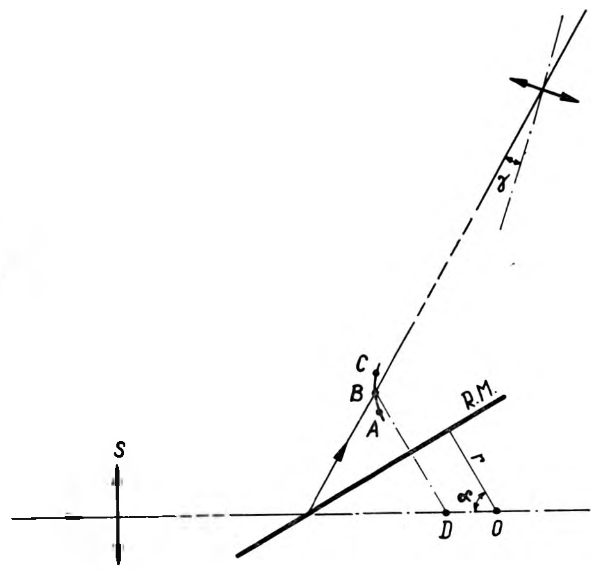

Fig. 5.

The Figure 6 shows the imaging of the sector ABC by a single secondary lens. The total kinematic aberration amounts to

$$A_k = |2g'| + |\sigma|$$

and its components may be calculated from the figure as being equal to

$$2g' = \beta(2g + 2e\tan\gamma),$$

$$\sigma = \frac{h}{\dfrac{b\cos\gamma}{\beta^2 e} - 1} \approx \beta^2 e \frac{h}{b\cos\gamma},$$

where h — is the diameter of the secondary lens and b — denotes a distance from the central point of the secondary objective to the image B' on the film corresponding to the point B on the Pascal curve.

Substituting the expressions for $2g$ and $2e$ into the above formulae and denoting by

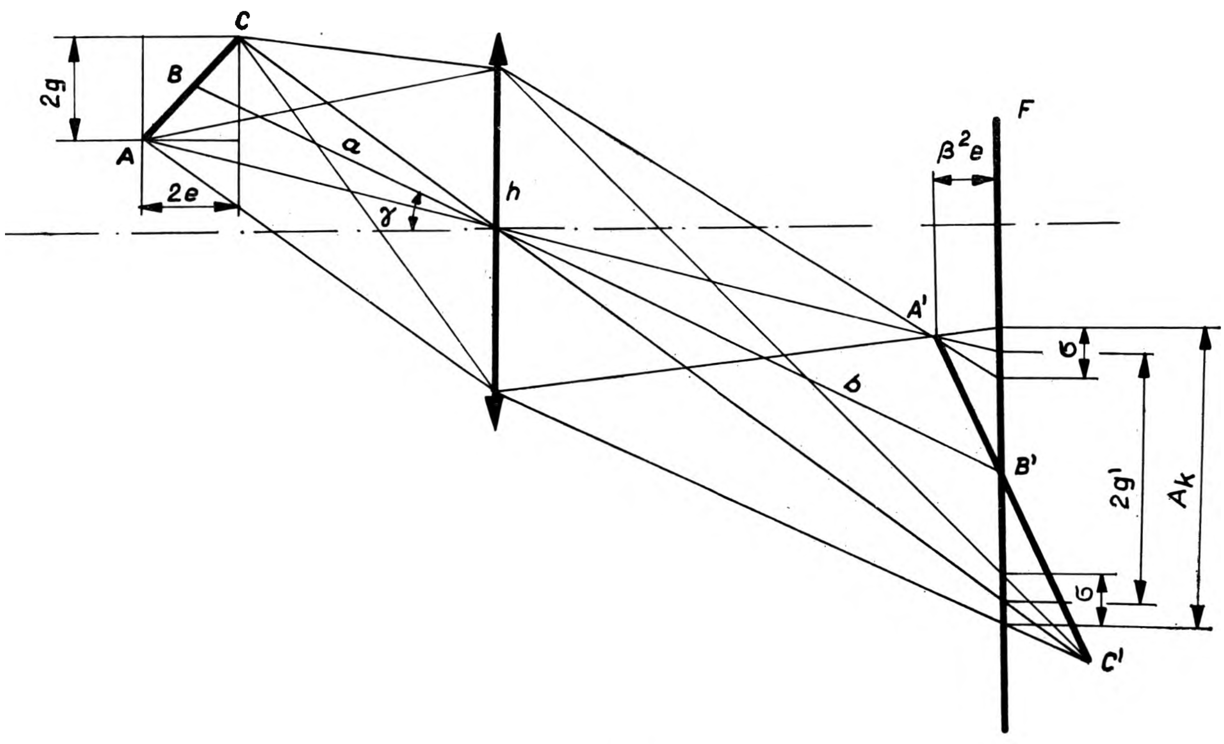

Fig. 6.

$$z = \frac{4\beta r \sin d\alpha}{\cos\gamma}$$

the term, which is almost constant within the whole operating sector (determined by the extreme values of α), we get

$$A_k = |2g'|\left(1+\frac{\beta h \sin|\gamma|}{2b}\right)+\frac{\beta h z \cos\gamma}{2b}\sin|\alpha|.$$

The fraction in the bracket is for all the real cameras very small in comparison to 1 and thus may be neglected. The expression for $2g'$ may be transformed to the form

$$2g' = z(\varrho \cos d\alpha - \cos\alpha),$$

where $\varrho = r'/r$.

Finally the kinematic aberration may be written as

$$A_k = z\left(|\varrho \cos d\alpha - \cos\alpha| + \frac{\beta h \cos\gamma}{2b}\sin|\alpha|\right).$$

A_k depends mainly on $\cos\alpha$, as the coefficient in front of the $\sin\alpha$ is very small for real devices. This means that the main contribution to the kinematic aberration comes from the component $2g$.

On the base of the above formula the angle α_m, which is responsible for the minimum aberration may be estimated. The optimal value of $\varrho(r')$, for which the aberration at the end point of the operating sector takes the minimum value, may be also calculated together with the aberration value. The last possibility is of some importance for the designer as it provides information about the admissible values of other camera parameters like r and the radius of curvature of the circle, along which the secondary lenses are set around the rotating mirror.

The theory of kinematic aberration presented above has been verified experimentally by help of a model with artificially increased values of r, h and $d\alpha$ to obtain easily measurable values of A . The measurement results proved to be in a good compliance with the respective theoretical estimations.

References

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