Aleksandra Chojnacka, Tadeusz Kryszczyński**

Two-Component Pancratic Optical System with a Direct Image and Linear Movement of Elements

The system of variable magnification with the linear movement of elements allow considerably simplify the technology and assembling of optical systems. It has been proved [1] that in a system realizing linear movement of the elements the maximal number of lens positions of absolute stabilization of the image plane location is equal to the number of variable areal spacings occuring in the system between the image and object planes. There appears a deviation of the image plane location from the basic image position for all the remaining positions. To simplify the adjustment of changeable magnification system it is necessary to ensure the stabilization of the image plane for extreme magnifications. The simplest example of a system of variable magnification is a single movable lens [2].

If we have to obtain for such a system a definite variable magnification factor W and stabilization of the image plane position for the extreme positions then the lateral magnification introduced by the lens in one of its extreme positions amounts to $\beta_1 = \pm\sqrt{W}$, while in other locations

$$\beta_2 = 1/\beta_1 = \pm 1/\sqrt{W}.$$

The maximal deviation $\delta_{\max}$ of the image plane position within the said movement range occurs for that lens position, for which the lateral magnification is $\beta = \pm 1$ and is equal to

$$\delta_{\max} = \pm f\frac{(U+1)^2}{U}, \tag{1}$$

where

$U = -\sqrt{W}$,

W = assumed factor of magnification variation,

f = focal length of the lens.

The Table I illustrates an increase in the deviation of the image plane position for a single lens shifted accordingly to the total magnification variation factor (the focal length of the moved lens being assumed to be equal to -1).

As seen in the Table I the deviation of the image plane position for a single shifted lens increases rapidly together with the change of the total variation factor W, which definitely restricts the application of this solution exclusively for small W and small apertures.

Let us consider a case of a two-element system of variable magnification with the total magnification $\beta_c > 0$ (the system produces a direct image).

Two elements of focal lengths f_1 and f_2 is shifted in such a way that the lateral magnifications for boundary positions amount to β_1 and $1/\beta_1$ as well as β_2 and $1/\beta_2$, respectively. The component elements are positioned in such a way that for the total magnifications $\beta_c = \beta_1\cdot\beta_2$ and $\beta_c = 1/\beta_1\beta_2$ the deviations of the image plane position were equal to zero. If the focal lengths and the magnification of the component elements are so matched that the maximal deviations within the movement range of the elements are of the same absolute values differing only by the sign, then we get one more position within the movement range, for which the deviation of the image plane position is equal to zero.

The fulfillment of these requirements is possible only in the case of two elements: one of negative and the other of positive power (arranged in arbitrary sequence). A layout of the solution with the first component negative is presented in Fig. 1. Further solutions deal with the same case, because the variant concerning the first positive elements is practicaly identical with the first one rotated by 180°, its scale being changed accordingly. The requirement of equilizing absolute values of the maximum deviations leads to the following dependence

$$-f_1\frac{(\beta_1+1)^2}{\beta_1} = f_2\frac{(\beta_2+1)}{\beta_2}. \tag{2}$$

*) Central Optical Laboratory, Warsaw, ul. Kamionkowska 18, Poland.

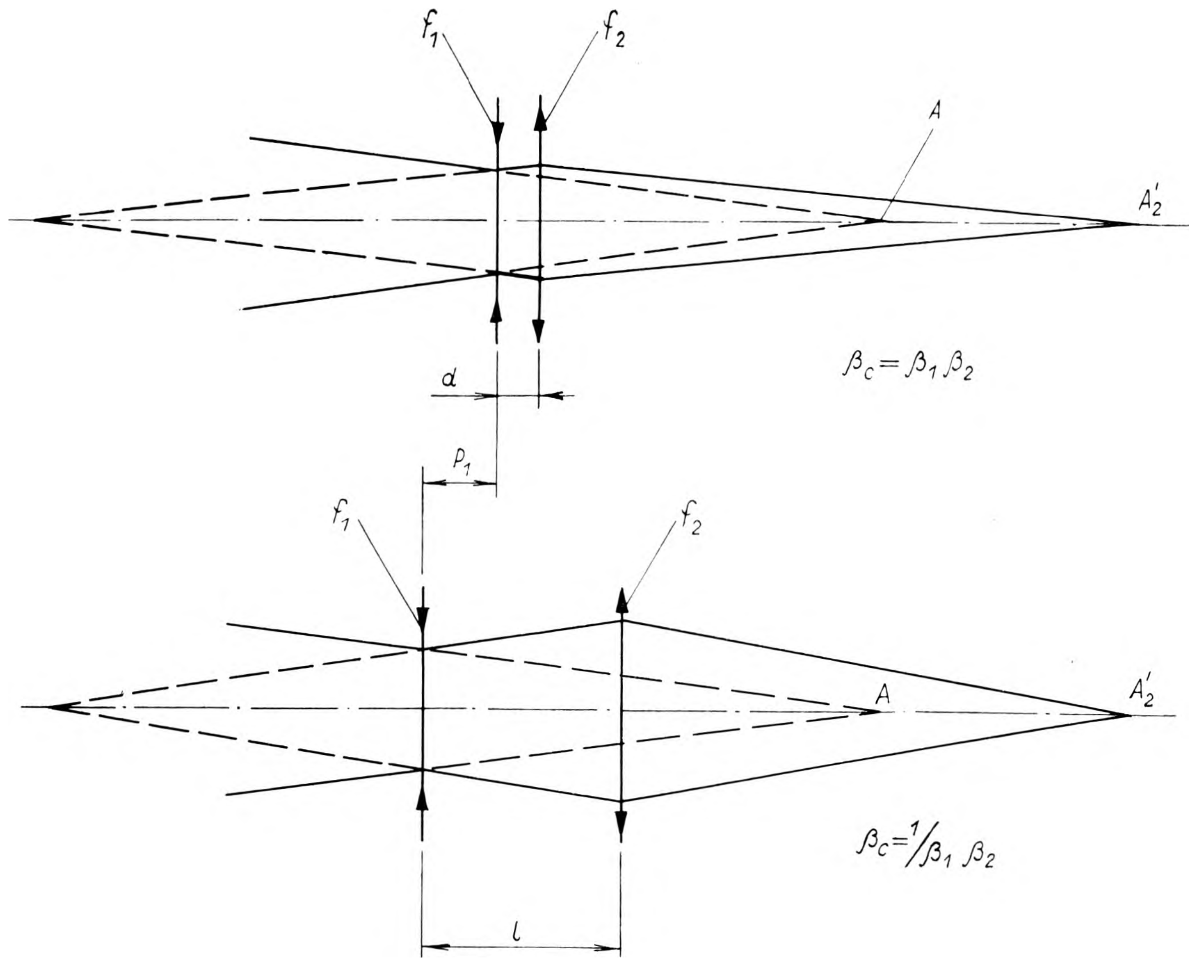

Fig. 1. An ideological scheme of a two-component solution for a pancratic optical system producing a direct image, supplied with the linear movement of elements

To make the total variation factor equal to W and the image plane position in the limiting positions equal to zero the following relationship should be fulfilled between the lateral magnifications of single elements and the variation factor W

$$\beta_1 \cdot \beta_2 = \sqrt{W}. \tag{3}$$

From the relations (2) and (3) β_1 may be determined depending on both the ratio $\varkappa = \frac{f_2}{f_1}$ and the total magnification variation factor W by corresponding rearrangements and the elimination of nonrealistic.

The lateral magnification is estimated from the following formula

$$\beta_1 = -\frac{\sqrt[4]{W}+\sqrt{-\varkappa W}}{\sqrt[4]{W}+\sqrt{-\varkappa}}. \tag{4}$$

The focal length and their magnifications must be so chosen that the elements do not touch each other in the critical position. Assuming that the least admissible distance between the lenses is equal to d the distance may be expressed as a difference of respective object and image distances depending on the focal lengths of component elements and the lateral magnifications at the position, at which there is a minimum distance between the elements. After performing the required transformations an additional condition is obtained for $\varkappa$ as related to the lateral magnification of the first element in the initial position, total magnification variation factor and the minimal relative distance d^*. This additional condition takes the form

$$\varkappa = \frac{(1-\beta_1-d^*)V}{\beta_1-V}, \tag{5}$$

where

$d^* = d/f_1$,

$V = \sqrt{W}$.

From the formulae (4) and (5) the equations, enabling to determine the magnification β_1 as a function of the total magnification variation factor W and

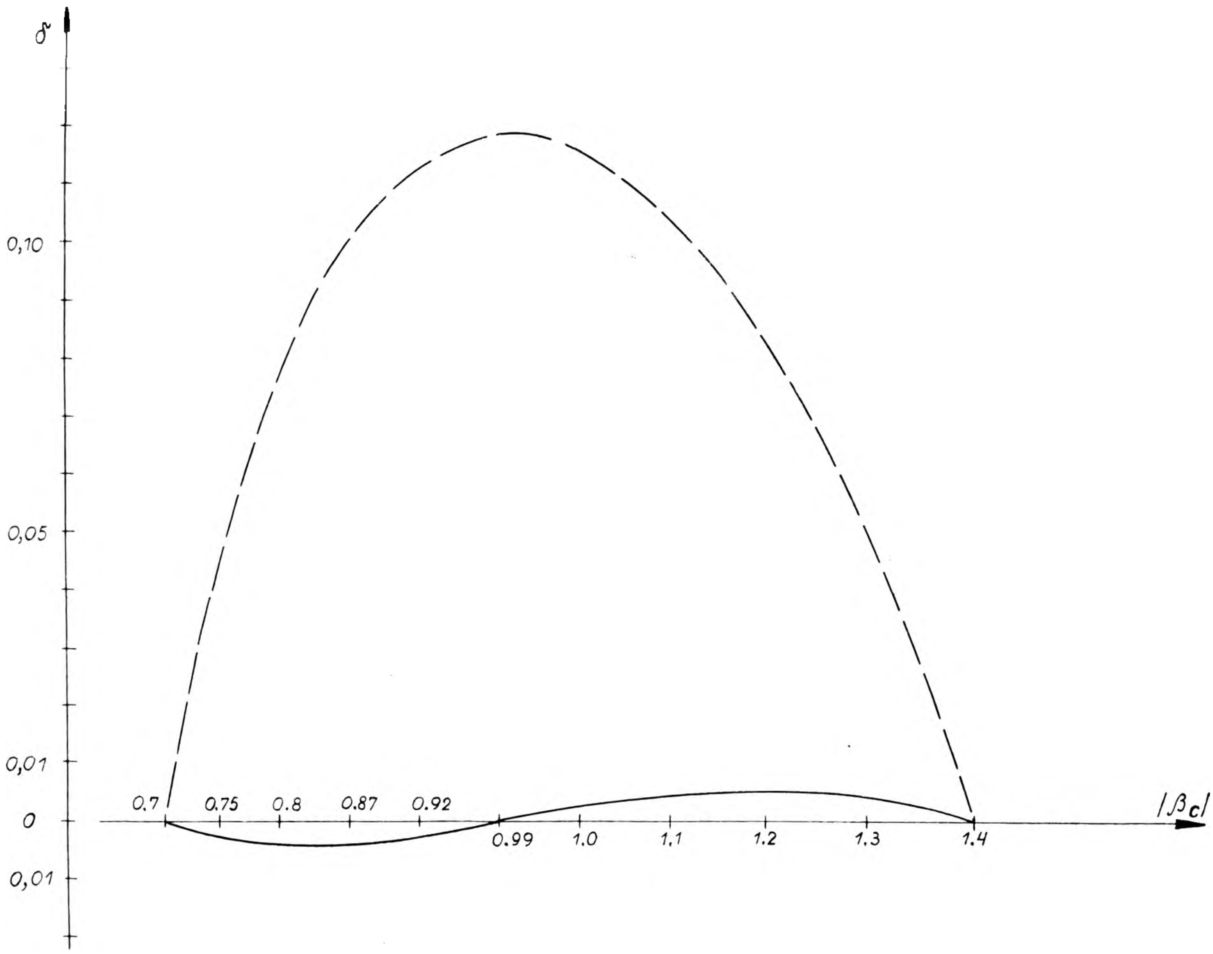

Fig. 2. Graph of a single and a two-element pancratic optical system
——— Deviation of the image plane position for the two-lens system of alterable magnification $W=2$
— — — Deviation of the image plane position for the single-element system of alterable magnification $W=2$

the minimum relative distance d^*, may be derived. The equation has the following form:

$$\beta_1^2(3V-3+d^*)+\beta_1(V^2+2Vd^*-1)-\\ -V(V-Vd^*-1)=0. \tag{6}$$

When determining β_1 from equation (6) and normalizing the power of the first element to a unity it is possible to determine on the basis of known formulae the remaining characteristic magnitudes of the two-element system of the variable magnification i.e. the focal length and the lateral magnification of the second element, the object and image distances as well the shift of each element.

A general formula for the deviation of the image plane position as a function of the first element shift z from the initial position (where the total lateral magnification of the system $\beta_c = \beta_1\beta_2$ has been determined for the discussed solution). The deviation of the image plane position is expressed by the formula:

$$\delta = \frac{1}{\beta_1(1-\beta_1)(V+\beta_1)}\;\frac{Az^3+Bz^2+Cz}{Dz^2+Ez+F}, \tag{7}$$

where

$$\begin{aligned}
A &= a\cdot b,\\
B &= f_1\beta_1[(1+\beta_1)(V-\beta_1)^2c-2V^2(1-V)g-\\
&\quad -(1-\beta_1^2)(V^2-\beta_1)e],\\
C &= f_1^2g[V^2(1-\beta_1)^2(V+\beta_1)-V^2c+b],\\
D &= a,\\
E &= f_1[(V-\beta_1)c-(1-\beta_1)(V+\beta_1)e],\\
F &= f_1^2g,\\
a &= 2\beta_1(1-V)(V-\beta_1),\\
b &= \beta_1(1+\beta_1)(V-\beta_1),\\
c &= (3-\beta_1)(V-\beta_1)-2(1-\beta_1)^2,\\
e &= V(1-\beta_1)-d^*\beta_1,\\
g &= (1-\beta_1)(V+\beta_1)(1-\beta_1-d^*).
\end{aligned}$$

The Figure 2 shows the image position deviation for the two-component system of variable magnification with the total magnification variation factor $W=2, f_1=-1$ and $d=-0.2$ as well as the deviation from the image plane position for a single lens with $f_1=-1$ and the same factor of magnification variation the assuming the factor $W=2$ the deviation of the image plane position in the two-element system is about 22x smaller than that for the single movable element.

Considering the problem of exit pupil stabilization in the system it has been assumed that the entrance pupil should be positioned at such a place that stabilization would occur at the extreme positions. Equation derived at allow to determine the distance x_1 of the entrance pupil from the first element at its initial position (for which the total lateral magnification of the system $\beta_c = \beta_1\beta_2$) as related to the known dimensions of the system. The equation has the following form:

$$x_1^2 G + x_1 H + J = 0, \tag{8}$$

where

$G = h(p_1 j - r) + f_1(d^{*2} f_1 - d^* f_1 - f_2) j,$

$H = h(-p_1^2 j - k) + f_1 jm + f_1(l - d^* f_1) n,$

$I = (d^* f_1 - f_2) f_1 (k + p_1^2 j) - d^{*2} f_1^3 [p_1 j + f_1 (l - f_2)],$

$h = f_1 - d^* f_1 + f_2,$

$j = f_1 - l + f_2,$

$k = p_1 l (2f_1 - l) + l^2 f_1,$

$m = d^{*2} f_1 (f_1 - p_1) + 2 f_2 p_1,$

$n = f_1 f_2 (2 - d^*) + l(d^* f_1 - f_2),$

$r = l^2 - f_1 (l + f_2),$

p_1 — shift range of the first element,

l — maximal length of the optical system.

From equation (8) we obtain two solutions, one of which corresponds to the position of the object, the other is employed to determine the deviation of the exit pupil position.

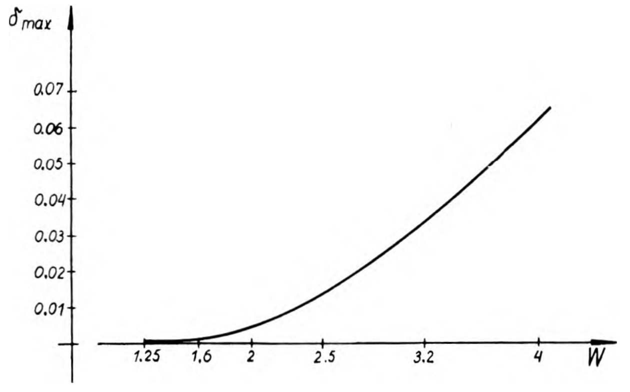

Fig. 3. Dependence between the extremal value of the image position deviation and the total magnification variation factor for the two-component pancratic optical system

A program for the Odra 1304 computer has been elaborated, to calculate the gabarites, the value of the image position deviation and the exit pupil deviation in the pancratic system with the two-element lens, to produce a direct image and a linear shift of the components.

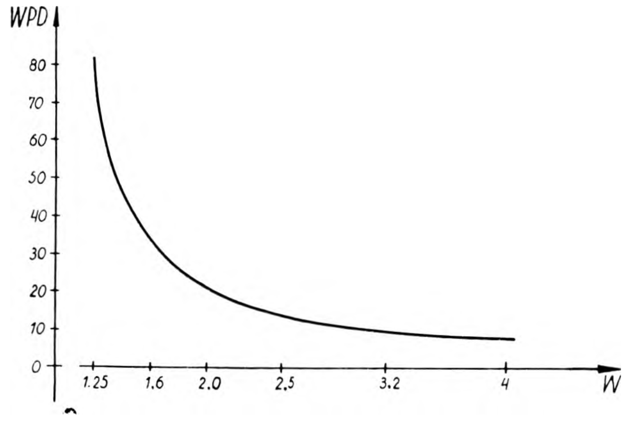

Fig. 4. Graph of the deviation improvement factor for the two-lens pancratic systems
WPD — coefficient of deviation improvement

A number of two-element systems of variable magnification and unit power of the first component have been examined for different factors of the total magnification variation and minimum relative distance $d^* = -0.2$.

The Figure 3 shows a relation of the extreme value of the image position deviation to the total magnification factor within the range $W = 1.25-4$. Fig. 3 shows the dependence between the maximal deviation of the image position and the entire coefficient of magnifying changeability within the range of $W = 1.25 - 4$. It may be seen that this deviation increases rapidly with a change in W. This restricts the application possibility of the two-component pancratic system for greater values of W the more, that the maximum position deviation of the image of the exit pupil considerably exceeds the maximal image position deviation. Accepting the system with a movable lens as a reference system an improvement factor for deviation has been determined for different W defined as a ratio of

Table 1. Deviation of the image plane position as a function of the assumed total magnification deviation factor (the focal length of the movable lens being assumed to be $f = -1$)

The total magnification variation factor W	The absolute value of the maximum image plane deviation δ_{max}
1.25	0.0125
1.6	0.0555
2	0.1213
2.5	0.2136
3.2	0.3479
4	0.5000

the maximum image position deviation in a single lens system to the maximum deviation in the two-component system for the same W. Obtained results are illustrated in Fig. 4.

As seen from the graph the superiority of the two-element pancratic system is particulary evident as regards small values of W. The numerical analysis indicates that the two-component pancratic system with the linear movement may be employed practically only within the region $1.25 < W < 2.5$. The application of the two-component pancrating system for $W > 2.5$ would require very small apertures so that the image deviation fulfills the Rayleigh condition for admissive focussing. It is also necessary to apply eyepieces with a great magnifying power to deminish the deviation of the exit pupil position in the instrument. The system described may be cascaded (for instance in the form of a succesion of two two-component lenses of variable magnification), which results in diminishing the maximal image position deviation as well as that for the pupil. It is thus possible to employ the said system for greater factors W of magnification variations.

References

[1] WOOTERS G., SILVERTOOTH E. W., J. Opt. Soc. Amer., vol. 55, No. 4, 1965, p. 347÷351.

[2] CHURILOVSKY W. N., Teoria opticheskykh priborov, Publishing House Mashinostroenye, Moskow-Leningrad 1966.