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Some Photometric Properties of the Systems of Imperfect Polarizers

It is shown that to describe its photometric properties a system of imperfect polarizers can be regarded as one single equivalent polarizer. One can define its photometric parameters being of an analogous meaning as in the case of one single polarizer. However, this analogy is incomplete, because it does not take into account the state of polarization of the transmitted light. Therefore the mentioned parameters of the system well describing the quantity of transmitted light are of little usefulness, if we wish to combine the polaroid system with further polarizing elements.

Polarizing sheets or systems of them are very often used in the applied and instrumental optics. They are not only useful as sources of polarized light, but their photometric properties in many cases serve to achieve an easy modulation of light intensity.

Polarizing filters which are used in the technical practice are never quite perfect. It is sometimes even advantageous to use imperfect polaroids [1]. Some good quality and commercially accessible filters may also in some cases be regarded as imperfect. As an example we can mention the device for optical indication of ultrasound fields [2] in which a polaroid system enables a considerable increase of modulation extent. The knowledge of the properties of imperfect polarizers is useful, too, for study of dichroic polarizers [3].

The photometric properties of a polarizing sheet are usually expressed by two parameters derived from its behaviour in the linearly polarized light. These parameters are the maximum T and the minimum T' value of the transmission factor for the linearly polarized light, determining at the same time the value of the transmission factor T_N for the non-polarized light by the relation

$$T_N = \frac{T+T'}{2}.$$

Another advantageous and frequently used parameters is the polarization degree K

$$K = \frac{T-T'}{T+T'}.$$

When determining the quantity of light passed through the system of several polarizers we have to add (besides the data of their mutual orientation) another parameter Δ for every polarizer. This parameter determines the phase difference of two components of the transmitted light disturbance with the directions of polarization corresponding to the maximum T and minimum T' transmittance.

The calculation of the emergent light is no doubt rather lengthy, if there is a greater number of polarizers involved, but it is quite elementary. It consists in the following up of the light components applying to the polarization directions of the maximum and minimum light transmission in the individual polarizers. After the transmission through n polarizers we get 2^n of components. Half of them apply to the direction of polarization corresponding to the maximum transmittance, half to the minimum transmittance of the last polarizer.

The calculation of amplitudes and of the phase constants of the different components can be mechanized. The parameters of the polarizers of the system T_i, T_i', Δ_i bear the indication of the index $i = 1,2, \ldots, n$ (n being the number of polarizers), signifying their sequence in the system. The components of the resultant light disturbance are marked by the index $p_1 p_2 \ldots p_n$ containing n ciphers $(123 \ldots i \ldots n)$. The index cipher i is marked by a comma i', if the component's direction of polarization, when passing through the polarizer in question, corresponds to the minimum transmittance T_i'. All components will be got as all possible combinations of all index ciphers with a comma and without a comma. The amplitude of the component equals the product of

$$a_{p_1 p_2 \ldots p_n} = A\sqrt{T_{p_1} T_{p_2} \ldots T_{p_n}}\, f_1(\alpha) f_2(\beta_2) \ldots f_n(\beta_n), \quad (1)$$

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where A is the amplitude of the incident light,

$$T_{p_i} = T_i, \quad \text{if} \quad p_i = i,$$

$$T_{p_i} = T_i', \quad \text{if} \quad p_i = i'.$$

The coefficient $f_1(\alpha)$ expresses the dependence on the polarization direction of the incident plane-polarized light destined by the angle α measured in connection with the first polarizer, being

$$f_1(\alpha) = \cos\alpha, \quad \text{if} \quad p_1 = 1,$$
$$f_1(\alpha) = \sin\alpha, \quad \text{if} \quad (p_1 = 1').$$

Other coefficients of (1) express the influence of the orientation of the different polarizers. They are goniometrical functions of the angle β, formed by the privileged polarization directions of the neighbouring polarizers, i.e. the filter i with the filter $i-1$, too.

$$f_i(\beta_i) = \cos\beta_i, \quad \text{if} \quad p_{i-1} = (i-1), \quad p_i = i,$$
$$f_i(\beta_i) = \sin\beta_i, \quad \text{if} \quad p_{i-1} = (i-1)', \quad p_i = i,$$
$$f_i(\beta_i) = -\sin\beta_i, \quad \text{if} \quad p_{i-1} = (i-1), \quad p_i = i',$$
$$f_i(\beta_i) = \cos\beta_i, \quad \text{if} \quad p_{i-1} = (i-1)', \quad p_i = i'.$$

Similarly for the phase constant $\phi_{p_1 p_2 \dots p_n}$ is valid

$$\phi_{p_1 p_2 \dots p_n} = \sum_{i=1}^{n} \delta_i, \tag{2}$$

where

$$\delta_i = 0, \quad \text{if} \quad p_i = i,$$
$$\delta_i = \Delta_i, \quad \text{if} \quad p_i = i'.$$

If we add (with regard to the phase relations) the transmitted components of the same direction of polarization, we get as a result for both directions a harmonic disturbance differing generally in the amplitude and in the phase, i.e. the elliptically polarized light. The resulting intensity is then obtained by squaring and adding the amplitudes of both components.

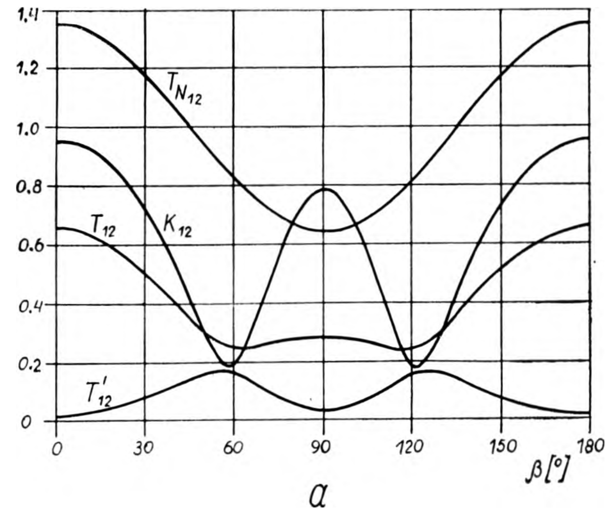
According to (1) the dependence of the amplitude of the single components on the direction of polarization of the incident light is expressed by the factor $f_1(\alpha)$, which is either $\sin\alpha$ or $\cos\alpha$. Every member of the resulting formula for calculating the transmitted light intensity contains therefore as factors either $\cos^2\alpha$ or $\sin^2\alpha$ or $\sin\alpha \cdot \cos\alpha$. The sum of such members may be — as known — always expressed in a form containing a simple harmonic function of the argument 2α. Thus, the transmittance of the system for plane-polarized light can be written as

$$T_{L12\dots n} = M + R\cos(2\alpha - \varphi).$$

This relation shows that from the view point of transmittance the system of imperfect polarizers can be regarded as one single imperfect polarizer. One can therefore define its photometric parameters as being of analogical significance as those of one single polarizer only. They are: the maximum and the minimum transmission factor for the linearly polarized light

$$T_{12\dots n} = M + R,$$
$$T'_{12\dots n} = M - R,$$

respectively the transmission factor for non-polarized light



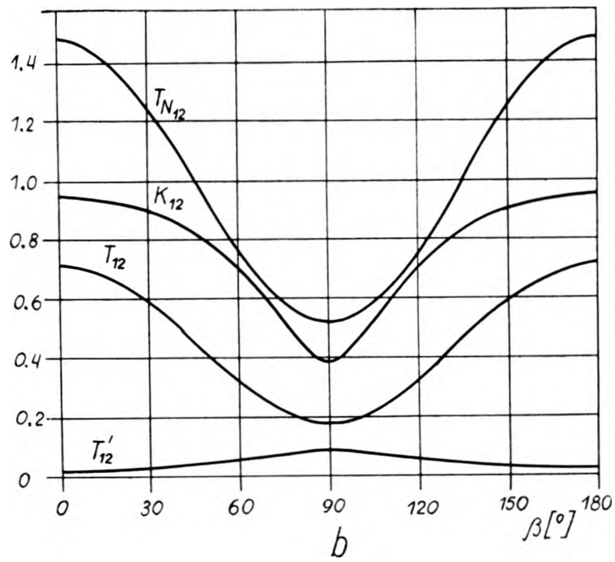


Fig. 1. Photometric parameters of a couple of imperfect polarizers as functions of their mutual orientation
a) $K_1 = 0.4$, $K_2 = 0.9$, $\cos\Delta_1 = 0.2$,
b) $K_1 = 0.8$, $K_2 = 0.6$, $\cos\Delta_2 = 0.6$.
The scale at the ordinate axis indicates the values of the parameters divided by the product of $4T_{N_1}T_{N_2}$

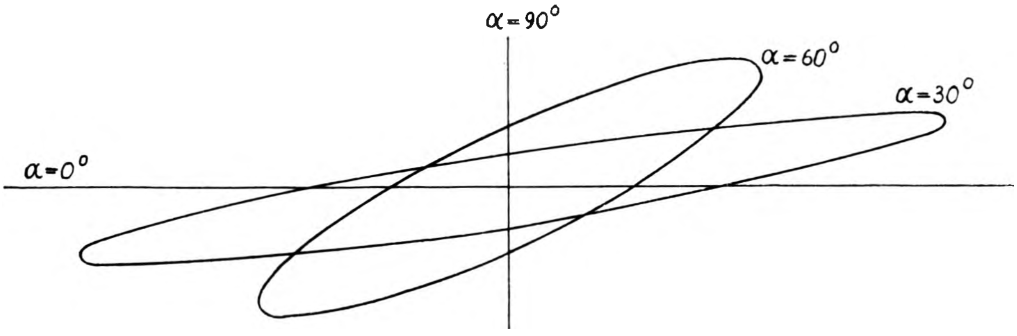


Fig. 2. Ellipticity of the light transmitted through a simple polarizer $T_1 = 1, T_1' = 0.09, \Delta_1 = 30°$

$$T_{N12...n} = M$$

and the degree of polarization

$$K_{12...n} = \frac{T_{12...n} - T'_{12...n}}{T_{12...n} + T'_{12...n}} = \frac{R}{M}.$$

As an example there have been derived parameters of a couple of polarizers. They are expressed on one hand with the help of parameters T_i, T_i', on the other with the help of T_{Ni}, K_i, $(i = 1,2)$.

$$M = T_{N12} = (T_1 + T_1')(T_2 + T_2') - (T_1 - T_1')(T_2 - T_2') \times \times \cos 2\beta = 4 \cdot T_{N1} T_{N2}(1 + K_1 K_2 \cos 2\beta),$$

$$R = \sqrt{[(T_1 - T_1')(T_2 + T_2') + (T_1 + T_1')(T_2 - T_2')\cos 2\beta]^2 + 4T_1 T_1'(T_2 - T_2')^2 \sin 2\beta \cos^2 \Delta_1} = 4T_{N1} T_{N2} \sqrt{(K_1 + K_2 \cos 2\beta)^2 + K_2^2(1 - K_1^2)\sin^2 2\beta \cos^2 \Delta_1}.$$

Photometric parameters of the couple are independent of the retardation in phase of the second polarizer. The transmittance for non-polarized light does not depend on the sequence of both polarizers. The remaining parameters are independent of the sequence of polarizers, only if $\Delta_1 = 0$.

The values of the parameters change at the mutual rotation of the elements of the couple. At T_{N12} this is a simple change (proportional to $\cos 2\beta$) from the maximum at parallel polarizers to the minimum at crossed polarizers.

The values of T_{12}, T'_{12}, K_{12} have asfunctions of the angle β extremes not only for $\beta = 0$ and $\beta = 90°$, but in some cases for a certain angle (greater than 45°), too. This angle is not identical for all three functions, but its values can be very close to each other, as numerical examples show. At this point T_{12} and K_{12} are at the msinimum, whereas T'_{12} reaches the maximum Figure 1 hows calculated examples of photometric parameters of a couple of polarizers as functions of the angle β.

Although from the photometric point of view the system of polarizers behaves like a single polarizer, there exist definitely a difference due to the polarization state of the transmitted light. In both cases the

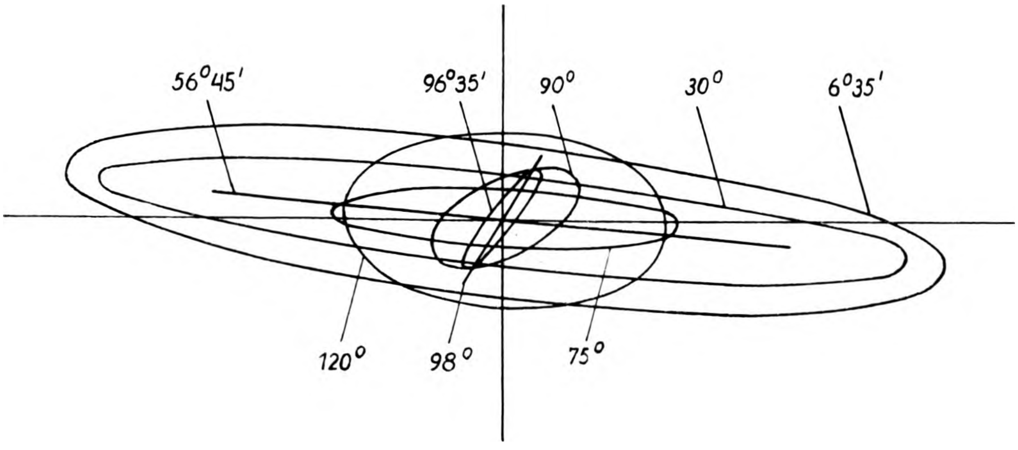


Fig. 3. Ellipticity of the light transmitted through a couple of polarizers $T_1 = 1, T_1' = 0.09, \Delta_1 = 30°, T_2 = 1, T_2' = 0.16, \Delta_2 = 60°, \beta = 30°$

linearly polarized light changes at the transmittance into an elliptically polarized light; only for two directions of polarization it stays a linearly polarized light. In the case of the single polarizer these polarization directions are perpendicular to each other and equal to the polarization directions of the maximum and minimum transmittance. In the case of the system of polarizers they are generally not perpendicular and do not correspond with the polarization directions of the maximum and minimum transmittance. In the case of a couple of polarizers they can be determined by solving the equation

$$T_1' \tan^2\alpha + \sqrt{T_1 T_1'} \times \frac{\sin^2\beta \sin(\Delta_1 - \Delta_2) + \cos^2\beta \sin(\Delta_1 + \Delta_2)}{\sin\beta \cos\beta \sin\Delta_2} \times \tan\alpha - T_1 = 0.$$

The angle α determined by this equation is independent of the photometric parameters T_2, T_2' of the second polarizer.

For the sake of better understanding Fig. 2 shows the ellipticity of the light transmitted through a simple polarizer calculated for $T_1 = 1$, $T_1' = 0.09$, $\Delta_1 = 30°$. Fig. 3 shows the same situation for a couple of polarizers composed of the same polarizer as in Fig. 2 and of the polarizer of $T_2 = 1$, $T_2' = 0.16$, $\Delta_2 = 60°$ and $\beta = 30°$.

These differences between the simple polarizer and the system of polarizers is the reason why the idea of an equivalent polarizer leads to parameters well describing the system from the point of view of the quantity of transmitted light, but is of little usefulness, if we wish to combine the system with further polarizing elements.

References

[1] PECHAR J., *Primenenye polyarizacyonnykh svetofiltrov v risovannykh filmakh*, Technika kino- i televideniya, **8** (1964), 17–20.

[2] KEŠNER Z., POLAŠEK J., *Modifikace Toeplerovy metody pro studium akustickich polý*, Jemná mechanika a optica, **13** (1968), 48–51.

[3] GARAJ J., VANÍK J., *On some Colour Properties of Dichroic filters*, Optica Acta, **19** (1972), 639-650.