*Edward Maliszewski*, Renata Nowak**, Ireneusz Wilk***

# On Numerical Evaluation of the Direct Reconstruction Matrices for Incoherent Diffraction Limited Systems Operating without any a Priori Information

The goal of this paper was to give a numerical method of the reconstruction matrix calculation for the case when both the observing and imaging systems are diffraction limited and there is no a priori information about the object available.

The respective algorithm has been found for programming the computation. The method applied permits to considerably shorten the calculation time for each element $R_{i,k}$ of the reconstruction matrices. It is worth noting that the case under study is a natural come-out stage for investigating the optical aberration influence on the reconstruction procedure.

## 1. Introduction

The direct recovery problem for incoherent imaging without any a priori information about the object was discussed in the papers [1] — [3]. The basic idea consists in recovery of the image intensity distribution (obtained with a known optical system from the unknown optical object) from the corresponding sampled measurement representation; the representation being obtained by scanning discretely the image with an observing system***). If the partial coherence, which is introduced to the image by the imaging system, can be neglected we have to do with the so called incoherent approximation [3]. Within this approximation the image is considered to be absolutely incoherent, the assumption being particularly well fulfilled by the diffraction limited systems. An analysis of the latter case seems to be of considerable importance as the diffraction limited systems create a natural reference to what can be best achieved in practice. Therefore, the hereafter consideration will be devoted to some numerical aspects of the upper and lower bound reconstruction matrices evaluation for this reference situation. The reconstruction matrices have been defined in [3] but for the sake of convenience the definition will be remined below.

As shown in [2] the relationship between the sampled measurement representation $x(a_k b_k)$ and the image intensity distribution is of the form

$$x(a_k b_k) = \int\limits_{P} I_{\text{im}}(p,q)\Phi(p-a_k, q-b_k)dp\,dq, \qquad (1)$$

where $I_{\text{im}}(p, q)$ is the sought image intensity distribution and $x(a_k, b_k)$ denote the values of the observed image points taken at the $N$ positions $a_k, b_k$, $k = 1, \ldots, N$ of the observing system and $\Phi(p-a_k, q-b_k)$ is the so called instrumental function (see [1]). As it is clear that this relationship is by no means unique the problem is to recover the class of functions $I_{\text{im}}(p, q)$, which would be consistent with the given measurement representation generated by the given experiment. For the purpose of the reconstruction error estimation it is sufficient to find the two extreme intensity distributions consistent with the given sampled values $x(a_k, b_k)$, which would determine the maximum and minimum possible a posteriori values of $I_{\text{im}}(p, q)$. The procedure proposed in [3] reduces the problem of reconstruction to solving two sets of linear equations. These are

$$x(a_k\, b_k) = \sum_{i=1}^{N} R_{ik}\, c_i \qquad (2)$$

for the upper bound recovery, generating the respective maximal values $I_{\text{im}}^{\max}(a_k, b_k)$ by the formula

*) Institute of Mathematics, University of Wrocław, Wrocław, Plac Grunwaldzki 8, Poland.

**) Institute of Technical Physics, Technical University of Wrocław, Wrocław, Wybrzeże Wyspiańskiego 27, Poland.

***) The observing system is here meant as a setup consisting of another imaging system supplied with an element, which integrates the whole incident signal. As a particular case the observing system may be reduced to the integrating element, only. The observing system is being shifted step-wisely across the image plane generating a sampled measurement representation of the image.

$$I_{im}^{max}(a_k b_k) = \sum_{i=1}^{N} c_i \varphi_{i(\alpha_i\beta_i)}(a_k b_k) \tag{3}$$

and

$$x(a_k b_k) = \sum_{i=1}^{N} R'_{ik} c'_i \tag{4}$$

for the lower bound recovery, generating the respective minimal values $I_{im}^{min}(a_k, b_k)$ by the formula

$$I_{im}^{min}(a_k b_k) = \sum_{i=1}^{N} c'_{i\varphi(\alpha_i\beta_i)} (a_k b_k). \tag{5}$$

Here, $c_i$ and $c'_i$ are the respective solutions of (2) and (4). $\varphi_{i(\alpha_i\beta_i)}(a_k b_k)$ denotes the value of the intensity at the scanning points $(a_k, b_k)$ and coming from a point-sources located at $(\alpha_i, \beta_i)$ in the object plane with intensity normalized to unity.

The matrices

$$\left\{R_{ik} = \int_P \varphi_{(\alpha_i\beta_i)}(p, q)\Phi(p-a_k, q-b_k)dpdq\right\} \tag{6}$$

and

$$\left\{R'_{ik} = \int_P \varphi_{(\alpha'_i\beta'_i)}(p, q)\Phi(p-a_k, q-b_k)dpdq\right\} \tag{7}$$

are called the upper and lower bound reconstruction matrices, respectively, and will be the subject of our further analytical considerations.

## 2. Analytic Form of the Reconstruction Matrices

Hereafter, the analytical properties of the reconstruction matrices for the diffraction limited systems of rotational symmetry will be considered. From the mathematical viewpoint we have to estimate the convolution integrals (20) with the spread function $\varphi_{(\alpha_i\beta_i)}(p, q)$ and the instrumental function $\Phi(p-a_k, q-b_k)$ specified accordingly.

### 2.1. EVALUATION OF THE INTENSITY SPREAD FUNCTION $\varphi_{(\alpha\beta)}(p, q)$

From the assumption that the systems are diffraction limited we readily specify the spread function generated by an object-point located on the optical axis of the imaging system in the well known form

$$\varphi(p, q, K_1) = \left|2\frac{J_1(K_1\sqrt{p^2+q^2})}{K_1\sqrt{p^2+q^2}}\right|^2 = \varphi(\sqrt{p^2+q^2}, K_1) \tag{8}$$

(see [4]), where

$$K_1 = \frac{\Pi}{\lambda M_1}$$

$\lambda$ — wavelength of the light used,

$M_1$ — f-number of the imaging system,

$J_1$ — Bessel function of the first kind and first order.

Because of the rapid convergence of the spread function $\varphi(\sqrt{p^2+v^2}, K_1)$ to zero with increasing $\sqrt{p^2+q^2}$ it has been assumed that the function is practically equal to zero outside the second eventually third minimum; the location of the latter with respect to the middle point of the spread function will be denoted by $d_1 = d(K_1)$. Independently, it will be also assumed that the imaging system is space invariant, which means that the shape of the intensity spread function is identical all over the image plane with that generated by the axial object point. For the numerical purposes it is convenient to expand the Bessel function into the potential series of the form

$$J_1(p) = \frac{p}{2}\sum_{i=0}^{\infty} \frac{(-1)^i}{4^i i!(i+1)!}(p^2)^i. \tag{9}$$

Substituting the representation (9) of the Bessel function into the defining equation (8) results in

$$\varphi(\sqrt{p^2+q^2}, K_1)$$

$$= \left|2\frac{\frac{1}{2}K_1\sqrt{p^2+q^2}\sum_{i=0}^{\infty}\frac{(-1)^i}{4^i i!(i+1)!}[(K_1\sqrt{p^2+q^2})^2]^i}{K_1\sqrt{p^2+q^2}}\right|$$

$$= \left\{\sum_{i=0}^{\infty} a_i K_1^{2i}(p^2+q^2)^i\right\}^2$$

$$\approx \left\{\sum_{i=0}^{m} a_i K_1^{2i}(p^2+q^2)^i\right\}^2 = \sum_{i=0}^{m} a_i^2 K_1^{4i}(p^2+q^2)^{2i} +$$

$$+2\sum_{i=0}^{m-1}\sum_{j=i+1}^{m} a_i a_j K_1^{2(i+j)}(p^2+q^2)^{i+j}\Big\}. \tag{10}$$

Defining

$$c_i = \begin{cases} 2b_i + a_{i*5}^2 & \text{for even } i \\ 2b_i & \text{for add } i \end{cases} \tag{11}$$

where

$b_i = \sum_{j=\max(0, i-m)}^{\left[\frac{i-1}{2}\right]} a_j a_{i-j}$; (with $\left[\frac{i-1}{2}\right]$ denoting the entire function) we can put (11) in the form (10)

$$\varphi = \sum_{i=0}^{2m} c_i K_1^{2i}(p^2+q^2)^i \tag{10a}$$

after a respective rearrangement. This form appeared to be convenient for numerical estimation of $\Phi$.

### 2.2 EVALUATION OF THE INSTRUMENTAL FUNCTION $\Phi(p-a_k, q-b_k)$

The instrumental function is by definition equal to

$$\Phi(p-a_k, q-b_k) = \int\limits_E \varphi_{(p-a,q-b)}(u,v)\,du\,dv \qquad (12)$$

(see (1)) where $\varphi_{(p-a,q-b)}(u,v)$ is the intensity spread function of the imaging part of the observing function while $E$ denotes the integrating element. For the sake of simplicity we assume for the moment that $a_k = b_k = 0$, which is permissible, because $\Phi(p-a, q-b)$ is shift-invariant. (The last fact being the consequence of the spatial stationarity of the imaging part of the observing system assumed earlier.) If the intensity spread function in (12) is also of diffraction limited type we can use the representation (8) with $K_2$ replaced by

$$K_2 = \frac{\Pi}{\lambda M_2},$$

where $M_2$ — is the ratio of the aperture diameter to the image distances, and $(p, q)$ changed into $(uv)$. Thus

$$\varphi_{(p-a,q-b)}(u,v) = \varphi(u,v) = \sum_{i=0}^{2m} c_i K_2^{2i}(u^2+v^2)^i .$$

Consequently the instrumental function appears in the form

$$\Phi\left(\sqrt{(p-a)^2+(q-b)^2},\ e,\ K_2\right) = \iint\limits_{D(r,e)} \sum_{i=1}^{2m} c_i K_2^{2i}(u^2+v^2)^i\,du\,dv \qquad (13)$$

where $D$ denotes the common region of the integrating element of radius $e$ and the domain of the non-zero existence of the $\varphi(u,v)$ function (see Fig. 1), $r = \sqrt{(p-a)^2+(q-b)^2}$ — denotes the distance between the middle point of the integrating element $(a, b)$ and the point in the image plane, which generates the actual spread function $\varphi_{(p,q)}(u,v)$ of the observing system. After some rearrangements we get

$$\Phi\left(\sqrt{(p-a)^2+(q-b)^2};\ e, K_2\right) = \sum_{i=0}^{2m} c_i K_2^{2i} \sum_{j=0}^{i} \binom{i}{j} \iint\limits_{D(r,e)} u^{2(i-j)} v^{2j}\,du\,dv. \qquad (14)$$

The integral

$$\iint\limits_{D(r,e)} u^{2(i-j)} v^{2j}\,du\,dv$$

has to be analyzed separately. In the light of the said assumptions it is clear that the region $D$ may be either a circle or a common part of two circles shifted by $r$ with respect to each other (see Fig. 1).

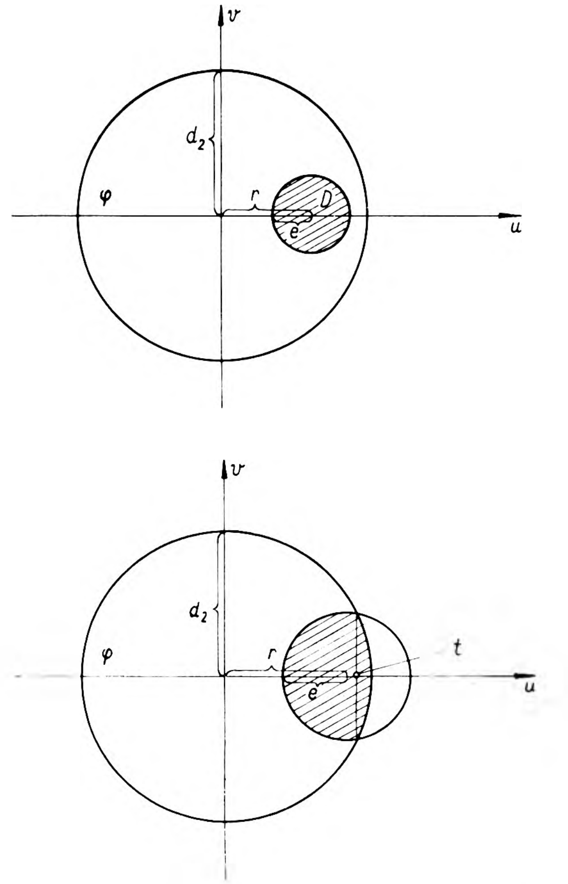


Fig. 1. Integration region $D(r, e)$ as determined by the position of the integrating element with respect to the $\varphi_{(p-a,q-b)}(u,v)$ function

Formally it can be written down as

$$D(r,e) = \begin{cases} \{(u,v) \mid (u-r)^2+v^2 \leqslant e^2\} & \text{for } r \leqslant d_2-e \\ \{(u,v) \mid r-e \leqslant u \leqslant t, \\ -\sqrt{e^2-(u-r)^2} \leqslant v \leqslant \sqrt{e^2-(u-r)^2}\} \cup \\ \{(u,v) \mid t \leqslant u \leqslant d_2, \\ -\sqrt{d_2^2-u^2} \leqslant v \leqslant \sqrt{d_2^2-u^2} \\ & \text{for } r > d_2-e \end{cases} \qquad (15)$$

where

$$t = \frac{r^2-e^2+d_2^2}{2r}, \quad d_2 = d(K_2)$$

is to be determined by solving the set of two equations

$$u^2+v^2 = d_2^2,$$
$$(u-r)^2+v^2 = e^2. \qquad (16)$$

In the first case i.e. for $r \leqslant d_2 - e$ we have readily

$$\iint\limits_{D(r,e)} u^{2(i-j)} v^{2j} du dv = \int\limits_{r-e}^{r+e} u^{2(i-j)} \int\limits_{-\sqrt{e^2-(u-r)^2}}^{\sqrt{e^2-(u-r)^2}} v^{2j} du dv$$

$$= \frac{2}{2j+1} \int\limits_{r-e}^{r+e} u^{2(i-j)} \sqrt{(-u^2+2ur+(e^2-r^2))^{2j+1}} \, du. \tag{17}$$

In the second case i.e. for $r > d_2 - e$ we obtain

$$\iint u^{2(i-j)} v^{2j} \, du dv = \int\limits_{r-e}^{t} u^{2(i-j)} \int\limits_{-\sqrt{e^2-(u-r)^2}}^{\sqrt{e^2-(u-r)^2}} v^{2j} du dv + \int\limits_{t}^{d_2} u^{2(i-j)} \int\limits_{-\sqrt{d_2^2-u^2}}^{\sqrt{d_2^2-u^2}} v^{2j} du dv$$

$$= \frac{2}{2j+1} \left| \int\limits_{r-e}^{t} u^{2(i-j)} \sqrt{(-u^2+2ur+(e^2-r^2))^{2j+1}} \, du + \int\limits_{t}^{d_2} u^{2(i-j)} \sqrt{(d_2{}^2-u^2)^{2j+1}} \, du \right|. \tag{18}$$

The last integrals (17) and (18) may be calculated exactly on the base of the following recurrence formulae. Let $R(u) = cu^2 + bu + a$, $\Delta = 4ac - b^2$, than:

1.

$$\int u^w \sqrt{R(u)^{2j+1}} \, du = \frac{u^{w-1} \sqrt{R(u)^{2j+3}}}{(w+2j+2)c} - \frac{(2w+2j+1)b}{2(w+2j+2)c} \int u^{w-1} \sqrt{R(u)^{2j+1}} \, du - \frac{(w-1)a}{(w+2j+2)c} \int u^{w-2} \sqrt{R(u)^{2j+1}} du,$$

2.

$$\int \sqrt{R(u)^{2j+1}} \, du = \frac{2cu+b}{4(j+1)c} \sqrt{R(u)^{2j+1}} + \frac{2j+1 \, \Delta}{8(j+1)c} \int \sqrt{R(u)^{2j-1}} \, du, \tag{19}$$

3.

$$\int \frac{du}{\sqrt{R(u)}} = -\frac{1}{\sqrt{-c}} \arcsin \frac{2cu+b}{\sqrt{-\Delta}} (\text{for } c < 0, \Delta < 0),$$

4.

$$\int u \sqrt{R(u)^{2j+1}} \, du = \frac{\sqrt{R(u)^{2j+3}}}{(2j+3)c} - \frac{b}{2c} \int \sqrt{R(u)^{2j+1}} \, du.$$

When applying these formulae to (17) and (18) the following relationships may be obtained

1. For the integral (17) $R(u) = -u^2 + 2ru + (e^2 - r^2)$, $\Delta = -4e^2$ and consequently

$$\int\limits_{r-e}^{r+e} u^w \sqrt{R(u)^{2j+1}} \, du = \frac{2w+2j+1}{w+2j+2} r \int\limits_{r-e}^{r+e} u^{w-1} \sqrt{R(u)^{2j+1}} \, du + \frac{(w-1)(e^2-r^2)}{(w+2j+2)} \int\limits_{r-e}^{r+e} u^{w-2} \sqrt{R(u)^{2j+1}} \, du,$$

$$\int\limits_{r-e}^{r+e} \sqrt{R(u)^{2j+1}} \, du = \frac{2j+1}{2(j+1)} e^2 \int\limits_{r-e}^{r+e} \sqrt{R(u)^{2j-1}} \, du,$$

$$\int\limits_{r-e}^{r+e} \frac{du}{\sqrt{R(u)}} = \Pi,$$

$$\int\limits_{r-e}^{r+e} u \sqrt{R(u)^{2j+1}} \, du = r \int\limits_{r-e}^{r+e} \sqrt{R(u)^{2j+1}} \, du.$$

2. The first term of the integrals (18) may be put in the form

$$\int\limits_{r-e}^{t} u^w \sqrt{R(u)^{2j+1}} \, du = \frac{2w+2j+1}{w+2j+2} r \int\limits_{r-e}^{t} u^{w-1} \sqrt{R(u)^{2j+1}} \, du + \frac{(w-1)(e^2-r^2)}{w+2j+2} \int\limits_{r-e}^{t} u^{w-2} \sqrt{R(u)^{2j+1}} \, du -$$

$$-\frac{t^{w-1}\sqrt{R(t)^{2j+3}}}{w+2j+2},$$

$$\int_{r-e}^{t}\sqrt{R(u)^{2j+1}}\,du$$

$$=\frac{t-r}{2(j+1)}\sqrt{R(t)^{2j+1}}+$$

$$+\frac{2j+1}{2(j+1)}e^2\int_{r-e}^{t}\sqrt{R(u)^{2j-1}}\,du,$$

$$\int_{r-e}^{t}\frac{du}{\sqrt{R(u)}}=\frac{\Pi}{2}-\arcsin\frac{r-t}{e},$$

$$\int_{r-e}^{t}u\sqrt{R(u)^{2j+1}}\,du$$

$$=r\int_{r-e}^{t}\sqrt{R(u)^{2j+1}}-\frac{\sqrt{R(t)^{2j+3}}}{2j+3},$$

while the second term in (18) may be further developed basing on the fact that now $R_1(u)=-u^2+d_2^2$, $\Delta=-4d_2^2$, which results in

$$\int_{t}^{d_2}u^w\sqrt{R_1(u)^{2j+1}}\,du$$

$$=\frac{w-1}{w+2j+2}d_2^2\int_{t}^{d_2}u^{w-2}\sqrt{R_1(u)^{2j+1}}\,du+$$

$$+\frac{t^{w-1}\sqrt{R_1(t)^{2j+3}}}{w+2j+2},$$

$$\int_{t}^{d_2}\sqrt{R_1(u)^{2j+1}}\,du$$

$$=\frac{2j+1}{2(j+1)}d_2^2\int_{t}^{d_2}\sqrt{R_1(u)^{2j-1}}\,du-$$

$$-\frac{t}{2(j+1)}\sqrt{R_1(t)^{2j+1}},$$

$$\int_{t}^{d_2}\frac{du}{R_1(u)}=\frac{\Pi}{2}+\arcsin\left(-\frac{t}{d_2}\right).$$

### 2.3. ESTIMATION OF THE MATRIX ELEMENTS $R_{i,k}$

Now, as a final step of our analysis we show the way of evaluating the reconstruction matrix elements for the band limited systems of rotational symmetry under the assumption of space stationarity of both the imaging and observing systems. Under these circumstances it is evident that both the integrand functions $\varphi$ and $\Phi$ appearing in the convolution integrals [6, 7] exhibit the rotational symmetry, which allows to simplify the general expression for $R_{i,k}$ to the form

$$R_{ik}=2\iint_{\pi'}\varphi(\sqrt{p^2+q^2},K_1)\cdot\Phi(\sqrt{(p-A)^2+q^2},e,K_2)\,dp\,dq,$$

where

$$\pi'=\{(p,q)\,|\sqrt{p^2+q^2}\leqslant d_1\cap q\geqslant 0\}\wedge\{(p,q)\sqrt{(A-p)^2+q^2}\leqslant dq\;\wedge\;q\geqslant 0\}\tag{20}$$

and the mutual displacement of both the convolved functions is along the $p$-axis only.

The said rotational symmetry of both the integrals implies a transformation of the coordinate system

$$p=A+\varrho\cos\alpha,$$
$$q=\varrho\sin\alpha.\tag{21}$$

to get a more convenient representation for further calculations.
Hence

$$\begin{vmatrix}\cos\alpha & -\varrho\sin\alpha\\ \sin\alpha & \varrho\cos\alpha\end{vmatrix}=\varrho\tag{21a}$$

and then

$$R_{i,k}=2\iint_{\pi'}\varrho\Phi(\varrho,e,K_2)\times$$

$$\times\varphi\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2},K_1\right)d\alpha\,d\varrho=\int_{\varrho'}^{\varrho''}\varrho\Phi(\varrho,e,K_2)\times$$

$$\times\int_{f(\varrho)}^{\pi}\varphi\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2},K_1\right)d\alpha\,d\varrho.\tag{22}$$

Now, consider the case, when the domains of $\varphi$ and $\Phi$ are positioned with respect to each other as shown in Fig. 2.

$$R_{ik}=\int_{0}^{d_1-A}\varrho\Phi(\varrho,e,K_2)\times$$

$$\times\int_{0}^{\pi}\varphi\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2},K_1\right)d\alpha\,d\varrho+$$

$$+\int_{d_1-A}^{d_2+e}\varrho\Phi(\varrho,e,K_2)\int_{f(\varrho)}^{\pi}\varphi\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2},K_1\right)d\alpha\,d\varrho.\tag{23}$$

The function $f(\varrho)$ may be expressed in the following form (see Fig. 2): By solving the set of two equations

$$(p^2+q^2)=\varrho^2,$$

$$(p^2+A)^2+q^2=d_1^2$$

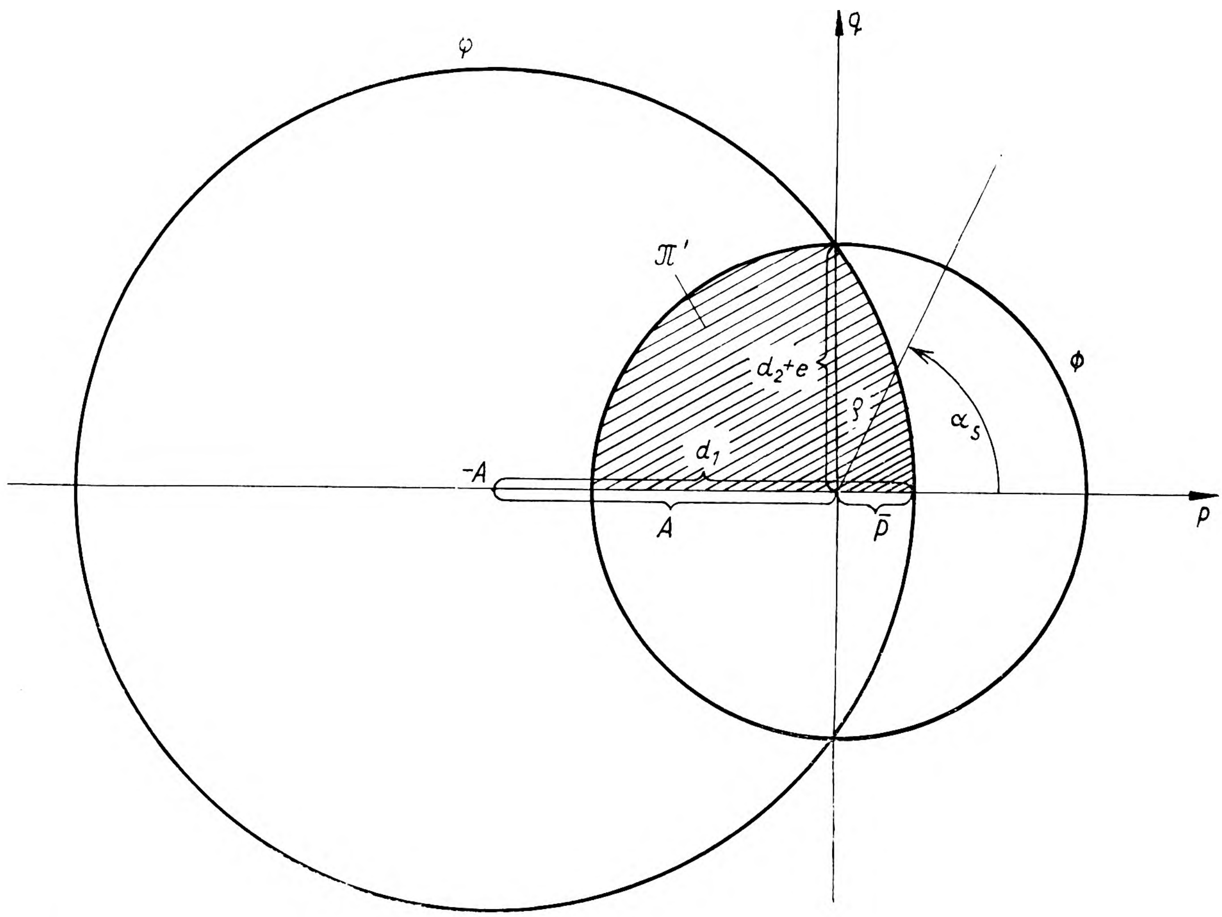


Fig. 2. Mutual position of the domains of both the $\Phi$ and functions defining the common integration region $\Pi'$.

we have

$$\bar{p} = \frac{d_1^2-\varrho^2-A^2}{2A}$$

then

$$\cos\alpha_s = \frac{\bar{p}}{\varrho}$$

and

$$f'(\varrho) = \arccos\frac{d_1^2-\varrho^2-A^2}{2A\varrho}. \tag{24}$$

In the general case for any admissible region we have

$$R_{i,k} = \int\limits_0^{\min\{d_2+e,\, d_1-A\}} \varrho\Phi(\varrho, e, K_2) \times$$

$$\times \int\limits_0^{\pi} \varphi\left(\sqrt{(\varrho^2+2\varrho\cos\alpha+A^2}, K_1\right) d\alpha d\varrho +$$

$$+ \int\limits_{|A-d_1|}^{\min\{d_2+e,\, d_1+A\}} \varrho\Phi(\varrho, e, K_2) \times$$

$$\times \int\limits_{f'(\varrho)}^{\pi} \varphi\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2}, K_1\right) d\alpha d\varrho \tag{25}$$

where

$$f'(\varrho) = \begin{cases} f(\varrho) & \text{for} \quad \varrho \neq 0 \\ \pi & \text{for} \quad \varrho = 0 \end{cases} \tag{26}$$

Let us consider the integral

$$\int\limits_{f'(\varrho)}^{\pi} \varrho\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2}, K_1\right) d\alpha. \tag{27}$$

$$f'(\varrho)$$

From (10a) we have

$$\int\limits_{f'(\varrho)}^{\pi} \varrho\left(\sqrt{\varrho^2+2\varrho\cos\alpha+A^2},\ K_1\right) d\alpha$$

$$= \sum_{i=0}^{2m} c_i K_1^{2i} \sum_{j=0}^{i} \binom{i}{j} (\varrho^2+A^2)^{i-j} (2\varrho)^j \int\limits_{f'(\varrho)}^{\pi} \cos^j\alpha d\alpha. \tag{28}$$

But it is easy to notice that

$$\int \cos^i\alpha d\alpha$$

$$= \frac{\cos^{i-1}\alpha\sin\alpha}{i} + \frac{i-1}{i}\int\cos^{i-2}\alpha d\alpha,$$

$$\int\cos\alpha d\alpha = \sin\alpha, \quad \int 1 d\alpha = \alpha.$$

Therefore, if $I_i$ denotes $\int\limits_{f'(\varrho)}^{\pi} \cos^i\alpha d\alpha$

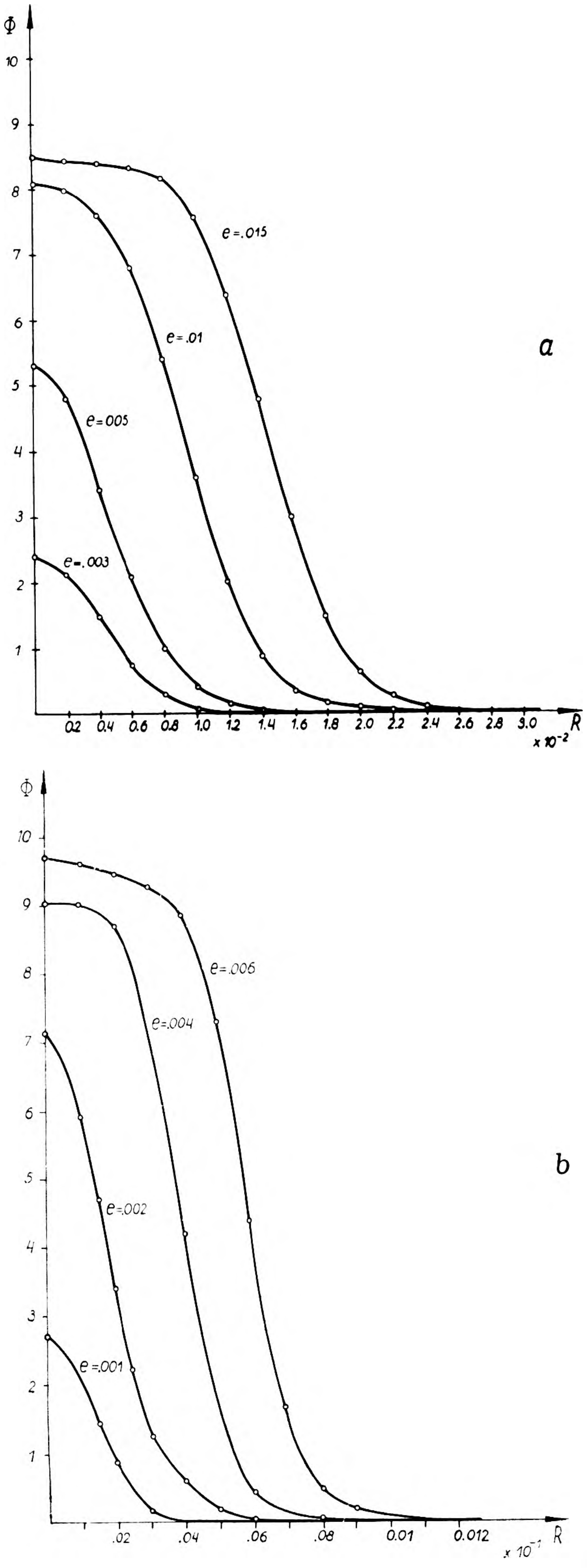


Fig. 3. Plot of the instrumental function Φ versus the distance $\sqrt{p^2+q^2}$ with the integrating element radius as a parameter. Part "a" represents the case of relative aperture $d/f = 1:5$, while part "b" shows the case of $d/f = 1:15$.

Example computation of matrices $R_{ik}$

| $M_\varphi$ | $M_\Phi$ | $e$ | $A$ | $R_{ik}$ |
|---|---|---|---|---|
| 1:5 | 1:15 | 0.005 | 0 | $4.7131\cdot 10^{-10}$ |
| 1:5 | 1:16 | 0.005 | 0.005 | $2.6997\cdot 10^{-10}$ |
| 1:5 | 1:15 | 0.01 | 0 | $7.7793\cdot 10^{-10}$ |
| 1:5 | 1:15 | 0.01 | 0.005 | $7.1050\cdot 10^{-10}$ |
| 1:5 | 1:15 | 0.01 | 0.02 | $2.6101\cdot 10^{-11}$ |

we have

$$I_i = [(i-1)I_{i-2} - \cos^{i-2} f'(\varrho)\sin f'(\varrho)] : i.$$

Because of

$$\cos f'(\varrho) = \begin{cases} \dfrac{d_1^2 - q^2 - A^2}{2A\varrho} & \text{for } q \neq 0 \\ -1 & \text{for } q = 0 \end{cases}$$

and

$$\sin f'(q) = \sqrt{1 - \cos^2 f'(q)}$$

while

$$\sin \alpha \geqslant 0 \qquad \alpha \in [0, \Pi]$$

then

$$I_1 = -\sin f'(q), \qquad I_0 = \Pi - f'(\varrho).$$

Finally, the elements $R_{i,k}$ are calculated according to the formula (25) in such a way that the external integrals are calculated from (28) while the internal integrals are evaluated numerically.

## Sur l'estimation numérique de la matrice de la reconstruction immédiate pour des systèmes incohérents limités par la diffraction et fonctionnant sans aucune information a priori

Dans ce travail on a présenté une méthode numérique de calcul de la matrice de la reconstruction immédiate dans le cas où les deux systèmes, l'un qui sert à l'observation et l'autre qui forme les images, sont limités par la diffraction et quand il n'y a aucune information a priori concernant l'objet.

La méthode appliquée permet de diminuer considérablement le temps de calcul d'un élément $R_{ik}$ de la matrice de reconstruction. Le cas discuté est une étape naturelle de départ pour l'examen de l'influence des aberrations optiques sur le procédé de la reconstruction.

## О численном определении матрицы прямой реконструкции для некогерентных дифракционно-ограниченных систем, действующих без какой-либо априорной информации

В работе изложен метод численного расчета матрицы восстановления в случае, когда наблюдательная и отображающая системы дифракционно ограничены при одновре-

менном отсутствии какой-либо априорной информации о предмете.

Примененый метод позволяет значительно сократить время расчета отдельного элемента $R_{ik}$ матрицы восстановления. Обсуждаемый случай представляет натуральный исходный этап для исследования влияния оптических аберраций на восстановительную процедуру.

## References


[1] WILK I., Zesz. Nauk. Pol. Wrocł. No. 214, Fizyka XV, 1969, p. 11.

[2] WILK I., Prace Nauk. Inst. Fiz. Techn. Pol. Wrocł. No. 2, Studia i Materiały No. 2, 1969, p. 23.

[3] WILK I., Prace Nauk. Inst. Fiz. Techn. Pol. Wrocł. No. 4, Studia i Materiały No. 4, 1970, p. 3.

[4] GOODMAN J. W., Fourier Optics, Mc Graw-Hill 1968, p. 64.

[5] RYŽYK J. M., GRADSTEJN J. S., Tablitsy integralov, sum, ryadov i proizvedeniy, 3rd issue, Moskow 1951.



*Received, April 17, 1973*