

Changes of the Refractive Index in Thermally Stressed Glass Disks

By measuring the stress-induced birefringence in glass disks it is possible to determine the difference of refractive indices n' – n'' of the rays polarized in mutually perpendicular directions. However, real changes of the refractive index, as related to the refractive index n corresponding to the zero isochromatic line, are greater than $n' - n''$. In particular, for the case of an uniaxial mechanical stress the respective changes in the refractive index were given in paper [1]. The present paper deals with similar investigations generalized to the case of two-dimensional stress induced thermally by freely cooling glass disks.

1. Introduction

The theory of thermal stress distribution in glass disks was formulated in paper [2]. Local stresses σ_r and σ_φ associated with an elementary area ΔS of the disk surface (Fig. 1) can be represented as follows

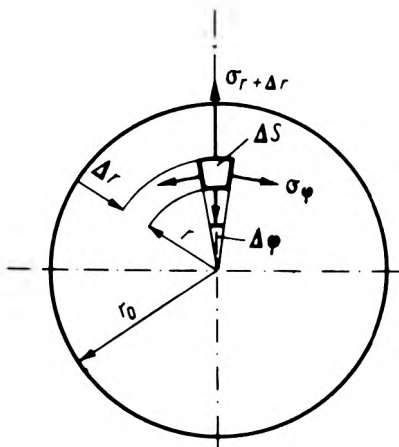


Fig. 1. Glass disk. r_0 – radius of the disk, $\sigma_r, \sigma_r + \Delta r$ – local stresses within the elementary area

$$\sigma_r = \frac{\lambda}{Gd} \sum_{i=1}^{\infty} \frac{l_i}{i} (1 - \varrho^i),$$

$$\sigma_\varphi = \frac{\lambda}{Gd} \sum_{i=1}^{\infty} \frac{l_i}{i} [1 - \varrho^i(i+1)]$$

(1)

(see [2]) where

- λ – wavelength of the light used,
- G – photoelasticity constant,
- d – disk thickness,
- l_i – expansion coefficient,
- $\varrho = r/r_0$.

The refractive indices n_r and n_φ corresponding to the rays polarized radially and tangentially may be estimated from the Maxwell-Neumann equations

$$n_\varphi - n_0 = C_\varphi \sigma_r + C_r \sigma_\varphi, \tag{2}$$

$$n_r - n_0 = C_r \sigma_\varphi + C_\varphi \sigma_r,$$

where

- C_r, C_φ – material constants,
- n_0 – refractive index in the glass free from stress.

Substituting (1) into (2) we obtain

$$\Delta n_r = n_r - n = -\frac{\lambda}{Gd} \sum_{i=1}^{\infty} \frac{l_i \varrho^i}{i} [(1+i)C_r + C_\varphi], \tag{3a}$$

$$\Delta n_\varphi = n_\varphi - n = -\frac{\lambda}{Gd} \sum_{i=1}^{\infty} \frac{l_i \varrho^i}{i} [(1+i)C_\varphi + C_r], \tag{3b}$$

$$w(\varrho) = n_r - n_\varphi = -\frac{\lambda}{Gd} \sum_{i=1}^{\infty} l_i \varrho^i, \tag{4}$$

where

- n – refractive index in the middle point the stressed glass disk.

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It is worth noting that if only one term in (3,4) is non-zero we obtain:

$$\frac{\Delta n_r}{w(\varrho)} = \frac{(1+i)C_r + C_\varphi}{i} = C_1, \quad (5)$$

$$\frac{\Delta n_\varphi}{w(\varrho)} = \frac{(1+i)C_\varphi + C_r}{i} = C_2.$$

C_1, C_2 – constants

In this case the determination of Δn_r and Δn_φ is reduced to the measurement of $w(\varrho)$, which is a relatively simple procedure.

2. Measurement of the Birefringence along the Disk Radius

In Fig. 2 the birefringence distribution along the disk radius normalized by birefringence value

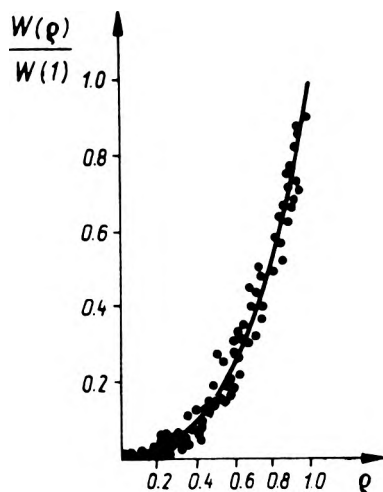


Fig. 2. Graph of the measured birefringence in glass disks

at the disk rim is presented. The points on the curve refer to both the moulded and machined disks. The experimental points may be approximated with the help of the formula (6):

$$w(\varrho) = w(1)\varrho^3. \quad (6)$$

3. Measurement of Δn_φ and Δn_r the Disk Radius

Sizes of the sample to be measured were: 60 mm in diameter and 15 mm in thickness. The measurements were carried out on a Michelson interfe-

rometer shown in Fig. 3. After having determined the position of the singular point as well as the location of the isochromatic lines, the polarizer P_2 was removed from the beam. Then interference fringes appeared within the disk area. Let us denote by $m_{i\varphi}$ and m_{ir} the numbers of fringes along the vertical and horizontal axes, respectively, for the case when the light is polarized along the vertical axis (Fig. 4).

The change of the refractive index may be calculated from (7):

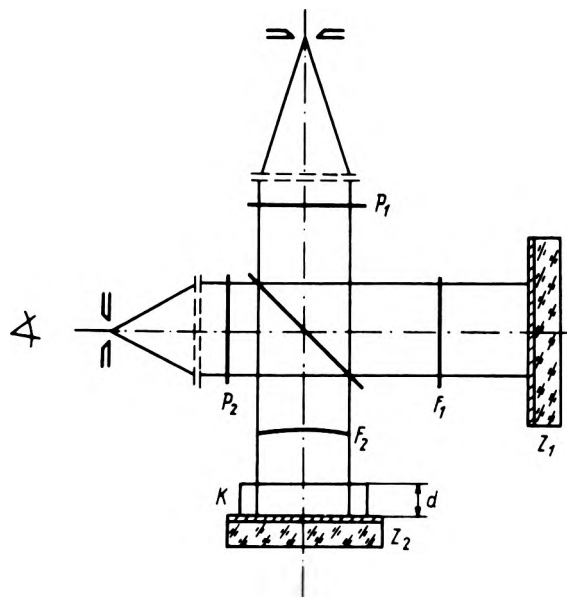


Fig. 3. Scheme of the Michelson interferometer used for measurements K – glass disk, z_1, z_2 – mirrors, P_1, P_2 – polarizers, F_1, F_2 – wavefronts

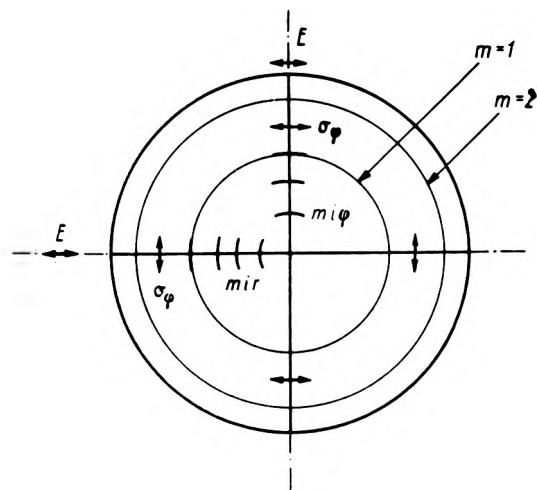


Fig. 4. Pattern of interference fringes in a disk m – order of isochromatic line

$$\Delta n_{\varphi} = \frac{m_{i\varphi}\lambda}{2d}, \quad (7)$$

$$\Delta n_r = \frac{m_{ir}\lambda}{2d}.$$

Let us denote

$$m_{\varphi} = \frac{m_{i\varphi}}{m_{ir} - m_{i\varphi}} = \frac{m_{i\varphi}}{\Delta m}, \quad (8)$$

$$m_r = \frac{m_{ir}}{m_{ir} - m_{i\varphi}} = \frac{m_{ir}}{\Delta m}.$$

Fig. 5 shows a typical relations of $m_{i\varphi}$ and m_{ir} to Δm for the BK 516-64 glass; the values of $m_{i\varphi}$ and m_{ir} being determined from the interferogram

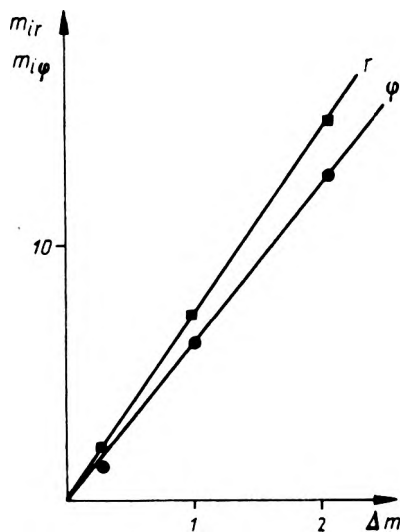


Fig. 5. Experimentally determined relations of $m_{i\varphi}$ and m_{ir} to Δm for BK 516-64 glass.

for the same value of ϱ (for instance, for corresponding to the first isochromatic $m = 1$, see Fig. 4).

From (7) and (8) we obtain the following relation between the refractive index variation and the birefringence:

$$\Delta n_{\varphi} = m_{\varphi} w(\varrho), \quad (9)$$

$$\Delta n_r = m_r w(\varrho).$$

It is worth noting that an identical relation was obtained earlier on the base of theoretical considerations resulting in (5).

In Table the results obtained for selected glass sorts of Polish made (Jeleniogórskie Zakłady Optyczne — Jelenia Góra) are presented.

As may be seen the refractive index variation Δn_r and Δn_{φ} are greater than birefringence $w(\varrho)$. Moreover, the coefficients m_r and m_{φ} are greater than those given by SCHUSTER and REITMAYER in paper [1]. For instance, these authors obtained estimations* $m_y = 3.5$ and $m_x = 4.5$ for the SF4 glass (which is a German equivalent of Polish CF 755-28 glass) and $m_y = 0.125$ and $m_x = 1.125$ for

Sort of the glass*	m_{φ}	m_r
BaCK 620-60	5	6
BaCK 623-53	6	7
F 620-36	5.5	6.5
CF 647-32	7	8
BK 516-64	7.3	8.3

* Each type of optical glass produced by J. Z. O. is indicated by letters and by two numbers. The first one, consisting of three digits, indicates the refractive index n_d within three decimals; the second one, consisting of two digits, the value of Abbes number ν_d , i. e. the reciprocal dispersion value. Example: BaCK 620-60 indicates the dense barium crown glass of $n_d = 1.620$ (1) and $\nu_d = 60$ (1).

the BK-7 (which is a German equivalent of Polish BK 516-64 glass).

It is supposed that the discrepancy between our results and those given in [1] are due to differences in glass treatment as our samples were highly stressed (the optical path differences ranged between 100 nm up to 600 nm) while glass used in [1] was free from thermal stress.

4. Conclusions

Both the theoretical and experimental results speak for linear dependence between the birefringence and the refractive index variation in the glass disks. Thus, knowing the proportionality constant m_r and m_{φ} , we are able to estimate the refractive index variation in the glass disks immediately from the birefringence measurement.

Variations des indices réfraction dans les lames de verre coupées en rond de tensions thermiques

La mesure de la biréfringence des verres optiques à tensions internes permet de déterminer la différence des indices de réfraction $n' - n''$ pour des rayons polarisés dans les deux directions perpendiculaires. Les variations réelles des indices

* The coefficients m_x and m_y correspond to our m_{φ} and m_r .

In both cases they determine the factors by which the changes of refractive index calculated with respect to zero-order isochromatic line for rays polarized in mutually perpendicular directions are greater than the birefringence.

de réfraction par rapport à n pour la isochrome nulle sont plus grandes que $n' - n''$. Les variations des indices de réfraction produites par des tensions mécaniques uniaxes sont présentées dans la travail de Schuster et Reitmayer. Les auteurs de ce travail ont présenté les résultats des recherches pour des lames de verre coupées en rond refroidies librement et de tensions thermiques biaxes.

Изменения коэффициентов преломления в термически напряженных стеклянных дисках

Измерение двупреломления в напряженных оптических стеклах позволяет определить разность коэффициентов преломления $n' - n''$ для лучей, поляризованных по перпендикулярным друг к другу направлениям. Действительные изменения коэффициентов преломления по отношению

к n для нулевой изохромы больше, чем $n' - n''$. Изменения коэффициентов преломления, вызванные одноосными механическими напряжениями, изложены в работе Э. Шустера и Ф. Рейтмера. В этой работе представлены результаты аналогичных исследований для охлажденных стеклянных дисков с двухосными термическими напряжениями.

References

- [1] SCHUSTER E., REITMAYER F., *Glastechnische Berichte* **34**, 130 (1961).
- [2] RALF H., SCHWESINGER C., *Glastechnische Berichte*, **34**, 133 (1961).

Received, December 18, 1973