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On Stimulated Mandelstam-Brillouin Scattering

The article is a contribution to the discussion on the effects expected from the interaction of intensive laser radiation and partially ionized plasma, i.e. the spark discharge. The possibility of stimulated Mandelstam-Brillouin scattering (SMBS) in partially ionized plasma, and in absence of magnetic field has been discussed. It has been assumed that electromagnetic wave is scattered on the fluctuations of density in the neutral gas. This wave frequency has been compared with the electromagnetic wave frequency scattered on the fluctuations electron density, as well as with the wave frequency scattered on the ion sound.

1. Introduction

In any medium the light scattering is produced by the interaction between the light wave and the matter.

The Rayleigh scattering may be used to analyse the reciprocal effect between wave and molecule. The energy fraction transferred to molecules by light waves is so small, that no frequency change in the scattered light has been observed for a long time. High resolution spectral methods applied to scattered light allowed to state that beside the lines characterized by the same wave length, there are also some weak lines shifted on the left and right, with respect to the main line. This fine structure of Rayleigh-lines was described by Brillouin [1] and Mandelstam [2]. According to their explanations the light waves can interact not only with individual molecules of the medium, but also with the mechanical waves (e.g. ultrasound waves). In this way the energy exchange can be realized as a result of electrostriction effect.

In Raman stimulated scattering the shift $\lambda_{\text{incident}}-\lambda_{\text{scattered}}$ may approximate several tens Å, thus being different from the SMBS, where the corresponding values for liquid amounted to 0.05 Å and 0.1 Å, respectively [3].

The SMBS was observed mainly in liquids, crystals [4–6] and gases [7].

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For SMBS both the energy and momentum conservation hold, i.e.

$$(\hbar\omega) \qquad \omega_1 = \omega_2+\omega_S, \tag{1}$$

$$(h/\lambda = k\cdot\hbar) \qquad \boldsymbol{k}_1 = \boldsymbol{k}_2+\boldsymbol{k}_S, \tag{2}$$

where indices 1, 2, S correspond to the incident, and scattered electromagnetic wave and the sound wave, respectively.

The graphical illustration of the equation (2) is given in Fig. 1. By virtue of the equation (2) and Fig. 1

$$|\boldsymbol{k}_S| = |\boldsymbol{k}_1-\boldsymbol{k}_2| = k_1\sin\frac{\theta}{2}+k_2\sin\frac{\theta}{2}, \tag{3}$$

where θ is the scattering angle.

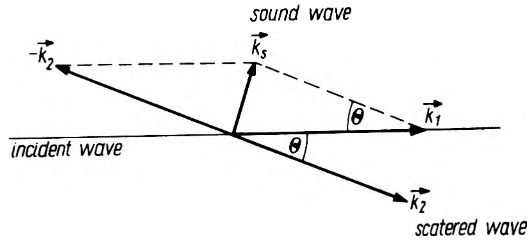

Fig. 1. The vector diagram $\boldsymbol{k}_1 = \boldsymbol{k}_2+\boldsymbol{k}_S$, which images the creation of photon (wave vector $\boldsymbol{k}_S$) and of scattered photon (wave vector $\boldsymbol{k}_2$) from primary photon (wave vector $\boldsymbol{k}_1$)

In our experiments the difference between ω_1 and ω_2 appeared to be very small and therefore $\boldsymbol{k}_1 = \boldsymbol{k}_2$ has been assumed.

The sound wave (SW) frequency $\omega_S = \omega_1-\omega_2$ is given by the equation (3) and by the definition of the wave vector $\boldsymbol{k} = \dfrac{\omega}{v}$:

$$\omega_S = \frac{2nv_S\omega_1}{c}\sin\frac{\theta}{2}, \tag{4}$$

where

n — index of refraction,

c — velocity of light in vacuum,

v_S — SW velocity.

From the classical theory point of view the SMBS can be represented as follows: The alternate electrical field induces the sinchronously changing deformation which initiates the emission of SW. On the other hand, SW modulates the dielectric permittivity of medium, which can lead to the exchange of the energy between electromagnetic waves characterized by different frequencies. This difference in the EM frequencies is equal to SW frequency.

2. Calculation of the laser power flux necessary for the generation of SMBS

According to YARIV [8] it is possible to obtain an equation for the acoustic field u_S of wave propagating in the $\boldsymbol{r}_S$ direction and for the field of scattered electromagnetic wave $\boldsymbol{E}_2$ propagating in the $\boldsymbol{r}_2$ direction. By using simplifying assumption $|\boldsymbol{E}_2|^2 \ll |\boldsymbol{E}_1|^2$ we can put $|\boldsymbol{E}_1|^2 = \text{const}$. Further, assuming the validity of $\omega_S = k_S \cdot v_S$ we have for the SW:

$$\frac{du_S}{dr_S} = -\frac{\imath}{2\varrho v_S} u_S - \frac{\gamma}{8\varrho v_S} E_1 E_2 \tag{5}$$

and for the scattered electromagnetic wave:

$$\frac{dE_2}{dr_2} = -\frac{\tau E_2}{2} - \frac{\gamma k_2 k_S}{4\varepsilon} E_1 u_S, \tag{6}$$

where ϱ — density of matter, $\imath$ — SW absorption constant, γ — electrostriction coefficient, τ — scattering term connected with the finite conductivity of medium σ by the relation: $\tau - \sigma\sqrt{\frac{\mu_0}{\varepsilon}}$ (where μ_0 — magnetic permeability of vacuum, ε — dielectric constant of medium).

The equations (5) and (6) can be modified by introducing a new coordinate ξ according to Fig. 2.

The equations discussed describe an increase (or absorption) of u_S and E_2, respectively, along two arbitrary directions, corresponding to q. Assuming the increase to be of exponential form we get:

$$u_S(q) = u_S^0 e^{lq}, \qquad E_2(q) = E_2^0 e^{lq}. \tag{7}$$

If $l > 0$, then, according to (7), the SW and electromagnetic wave will be simultaneously amplified in $\boldsymbol{k}_S$ and $\boldsymbol{k}_2$ directions, respectively.

By assuming $l > 0$ and in view of the relations (5), (6), (7) we get:

$$|\boldsymbol{E}_1|^2 \geqslant \frac{32T\varepsilon}{\gamma^2 k_2 L_2 k_S L_S}, \tag{8}$$

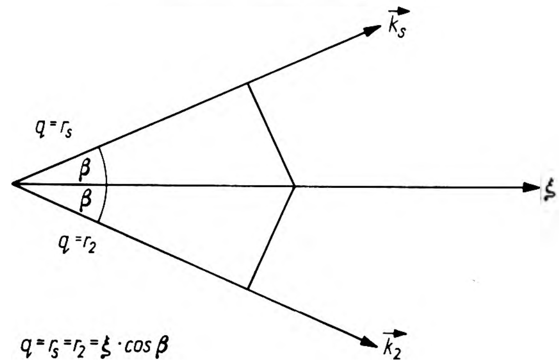

Fig. 2. The continuity between the distance r_S) in the direction normal to the sound wave front) and the distance r_2 (in the direction normal to the light wave front)

where T — modul elasticity, L_2, L_S — distances at which the field is being e-times reduced.

The formula for the threshold intensity is quoted also in papers [9] or [10].

Numerical value of E_1 can be calculated from the equation (8). This value is necessary for generation of SMBS. According to Yariv the threshold value of laser power flux for quartz is the following:

$$c\varepsilon|E_1|^2 \sim 10^{11}\ \mathrm{Wm^{-2}}.$$

For the spark discharge in air at atmospheric pressure and at the temperature $T = 15{,}000$ K we shall obtain:

$$c\varepsilon|E_1|^2 = 3\cdot 10^{15}\ \mathrm{Wm^{-2}}.$$

According to [10], by focusing the intensive laser beam with the laser power output $\sim 10^8$ W we get the laser power flux within the sample $\sim 10^{16}$ Wm^{-2}. Bearing in mind that power output of the most powerfull lasers is at most 10^9 W [11], (ISHCHENKO [12] gives the value $6\cdot 10^9$ W), we can suppose that SMBS is generated also in gases.

The possibility of SMBS generation is greater if the pressure of the gas used is higher than atmospherical one. Since the plasma may be obtained under extremely high pressure (up to $5\cdot 10^3$ atm) the conditions for SMBS generation are more advantagenous.

3. SMBS in partially ionized plasma

In the case of partially ionized plasma the situation is more complicated than in that of neutral gas, since the light scattering on the fluctuations density of electrons, ions and neutral gas must be considered individually. For the sake of simplicity, we assume additionally that plasma is not situated in magnetic field (in this case spectral lines forma more fine structure).

The light scattering on electrons is known as Thomson's scattering [13]. We are concerned with the frequency qualities of the scattered light. The shape of spectrum depends on parameters of plasma and on the factor

$$a = \frac{\lambda_1}{4\pi D \sin\frac{\theta}{2}},$$

where λ_1 — wavelength of incident light,

$$D \approx \sqrt{\frac{\varepsilon_0 k_B T_e}{N_e e^2}}$$

is so-called Debye length,

k_B — Boltzman constant,

T_e — temperature of electrons,

N_e — number of electrons in cm^3,

e — charge of electron.

If $a < 1$, the classical Thomson's scattering can be defined. From the Doppler's shift of lines we can find N_e and T_e.

If $a > 1$, the spectrum of scattering light is determined by collective effects, expressed by increased electrostatic oscillations in plasma, resulting in characteristic changes of the spectrum. The spectrum of the scattering radiation in the full ionized plasma consists of the central ion component and two electron satellites.

In warm plasma the amplitude of electron satellites is smaller, its difference from the basis frequency ω_1 is determined by the dispersion relation for the plasma oscillations. For the given temperature pressure the dispersion relation is determined according to [14]

$$\omega_e^2 = \omega_{ple}^2 + v_{S_e}^2 k^2, \tag{9}$$

where

$\omega_{ple} = \sqrt{\frac{N_e e^2}{m \varepsilon_0}}$ is the plasma frequency,

m — matter of electron,

v_{S_e} is sound velocity.

By the sound velocity we mean the velocity at which the pressure-perturbation is spread. In the case of an isothermic process for the electron plasma oscillations we can write

$$v_{S_e} = \sqrt{\frac{k_B T_e}{m}},$$

where v_{S_e} is sound velocity in electron gas.

The plasma oscillations can occure also under low density condition. Then the sound cannot spread within electron gas, because of the lack of the pressure-transmission mechanism. For the low density of plasma the dispersion equation has the following form:

$$\omega_e^2 = \omega_{ple}^2 + \frac{3k_B T_e}{m} k^2. \tag{9a}$$

Hence, the sound velocity takes the value

$$v'_{S_e} = \sqrt{\frac{3k_B T_e}{m}}.$$

Similar analysis can be also made for the ion component. The spectrum shape, produced by the scattering radiation on ions, depends on the qualities of the ion acoustic oscillations. If their damp is small, we can observe the ion component. The fine structure of the latter is determined by the dispersion relation for ion oscillations:

$$\omega_i^2 = \frac{k^2 \omega_i^2 \frac{\varkappa_e T_e + \varkappa_i T_i}{M} + k^2 \varkappa_e \varkappa_i \frac{T_e T_i}{mM}}{\omega_i^2 + k^2\left(\varkappa_e \frac{T_e}{m} + \varkappa_i \frac{T_i}{M}\right)}, \tag{10}$$

where

$\sqrt{\frac{\varkappa_i T_i}{M}} = v_{S_i}$ is the ion sound velocity,

$\varkappa_e$, $\varkappa_i$ —corresponding exponents of adiabatics,

T_i — temperature in energy units,

M — mass of ions.

From the scattered radiation spectrum (Fig. 3) we can get information on the temperature and density of electrons and ions. From the widths of lines the damping of plasma and ion oscillations, may be also estimated.

In the partially ionized plasma the scattering of light on the neutral component must be also taken into consideration. If the fine structure of the Rayleigh lines, i.e. SMBS is observed it is useful to compare the frequency of single scattered wavings.

For the spark discharge in the air at normal pressure, $N_e = 2\cdot 10^{17}$ cm^{-3}, $\theta = \frac{\pi}{2}$ $(a = 4.3)$, the numerical values of frequency have been determined from the relations (9) and (10) and calculated on differences of wavelengths.

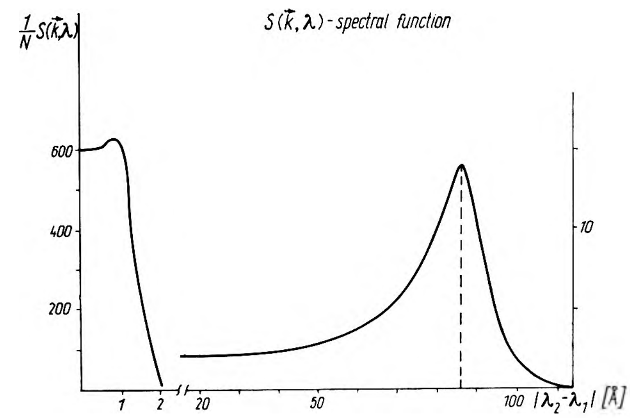

Fig. 3. The spectrum of scattered light in the completely ionized plasma in case $a = 2.17$; $N_e = 2\cdot 10^{17}$ cm^{-3}; $T_e = 5$ eV

The differences between the incidence and scattered light wavelengths are the following ones

electron sound $|(\lambda_2-\lambda_1)_e| = 67.23$ Å,
ion sound $|(\lambda_2-\lambda_1)_i| = 0.351$ Å.

For the sake of comparison these values were calculated from the relation (4), the velocity of sound being given for electron, ion and neutral gases. The values $(\lambda_2-\lambda_1)_e$ and $(\lambda_2-\lambda_1)_i$ are identical, and this means that the Thomson's scattering (considering the collective interactions in plasma and the temperature pressure) is the scattering of light on electron and ion sound.

From the relation (4) we have $|(\lambda_2-\lambda_1)_{\text{neutral gas}}| = 0.294$ Å.

The distance between the lines of light scattered on ion sound and on sound in neutral gas is:

$$|(\lambda_2-\lambda_1)_i| - |(\lambda_2-\lambda_1)_{\text{neutr}}| = 5.8\cdot 10^{-2} \text{ Å}.$$

4. Discussion and conclusion

In view of the performed experiment the lines of light scattered on sound waves of the neutral gas and on the ion sound are the only lines useful in the laser diagnostic of plasma, the difference $(\lambda_2-\lambda_1)_i-(\lambda_2-\lambda_1)_{\text{neutr}}$ discussed earlier being negligible. The present contribution points to the fact that it is the matter of two lines which can be distinguished by the high resolution methods.

The laser diagnosis can be applied to plasma only in case $N_e \geqslant 10^{13}$ cm^{-3}. There are also, moreover, the experimental difficulties associated with the synchronization of the laser pulse duration with the life-time of high-pressure plasma (or spark discharge).

Despite the limitations introduced and experimental difficulties the laser diagnosis of plasma is one of the best method because one spectrum of scattered radiation allows to make a number of statements on the qualities of plasma.

The article points to the fact that, by considering SMBS in plasma, more information on the partially ionized plasma may be obtained.

Contribution à la dispersion de Mandelstam-Brillouin

Ce travail est une contribution à la discussion concernant les effects de l'interaction d'un rayonnement laser intensif et du plasma ionisé; il s'agit notamment des décharges par étincelle. On a examiné la dispersion stimulée de Mandelstam-Brillouin (SMBS) possible dans un plasma ionisé en l'absence de champ magnétique. On supposait que l'onde électromagnetique était dispersée par des fluctuations de densité dans un gaz neutre. La fréquence de cette onde a été comparée avec fréquence de l'onde électromagnetique dispersée par des fluctuations d'électrons et avec celle de l'onde dispersée par l'onde acoustique ionique.

К вопросу о рассеянии Мандельштама-Бриллюэна

Работа связана с дискуссией об эффектах, которые возможны в результате взаимодействия интенсивного лазерного излучения и частично ионизированной плазмы, а именно изкровых разрядов. Обсуждена возможность стимулированного рассеяния Мандельштама-Бриллюэна (SMBS) в частично ионизированной плазме при отсутствии магнитного поля. Принято, что электромагнитная волна рассеивается на флуктуациях плотности в нейтральном газе. Частота этой волны сравнена с частотой электромагнитной волны, рассеянной на флуктуациях электронной плотности, а также с частотой волны, рассеянной на ионной звуковой волне.

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