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Statistics of Radiant Power Distribution of Electromagnetic Field Scattered by Randomly Diffusing Objects**

When a diffusing object is illuminated by a coherent monochromatic light, the speckle pattern appears. In many cases the knowledge of such values as the mean-square value of radiant power density, autocorrelation function and radiant power density distribution is required. The knowledge of the radiant power density distribution is of a particular significance in holography and in all the cases, when information contained in an electromagnetic field scattered by diffusing object is registered in media possessing limited dynamic range of registration.

The paper presents experimental results concerning radiant power density distribution of coherent monochromatic light scattered by reflecting or translucent stochastic diffusers of different shapes. There is a total agreement with Rayleigh's anticipation of this phenomenon [5].

1. Introduction

Speckle patterns, observed in imaging systems using coherent light occur when illuminating light is scattered by translucent or reflecting objects. According to many authors this phenomenon is a coherent noise, disturbing the quality of the images. If an optical system is equipped with additional diffusers, for instance, for a diffused illumination of the objects during holographic recording, then of course, the speckle pattern acquires an over-informational character; one of its component grows because of the presence of the diffuser, and may therefore be treated as non-informative noise. The other component, however, contains the information on the object in speckle form. In the cases when holograms of scattering objects illuminated by non-informative (plane or spherical) waves are made, the speckle pattern contains total information on the object and it is difficult to consider this pattern as a non-informative coherent noise.

In all the cases when the speckle pattern may be considered as an information carrier, the problem, how this form of information is to be registered, may be a matter of importance. Let us consider a simple experiment. Fig. 1 shows three photographs of the same speckle pattern achieved by a coherent illumination of an arbitrary diffusing object. The differences between the photos concern solely their exposure times; the exposure time of photo (b) was chosen to get an average optical density of the film equal to about 0.5; the exposure times of photos (a) and (c) compared with that of photo (b) were many times shorter and longer, respectively. As it can be seen from photo (a), there are separate areas strongly exposed, whereas the photo (c) proves the existence of non exposed areas. This experiment leads to conclusion, that the dynamic range of the radiant power density in speckle pattern is very high, and that the measurement of the average value of radiant power density of the coherent radiation scattered by diffusing objects do not supply sufficient information, necessary for registration. The knowledge of the statistical distribution of the radiant power density in speckle patterns may be of a great significance in many applications, especially in all holographic processes. This distribution ought to be taken into consideration when laser beams are dissipated by scattering surfaces and safety factors are to be

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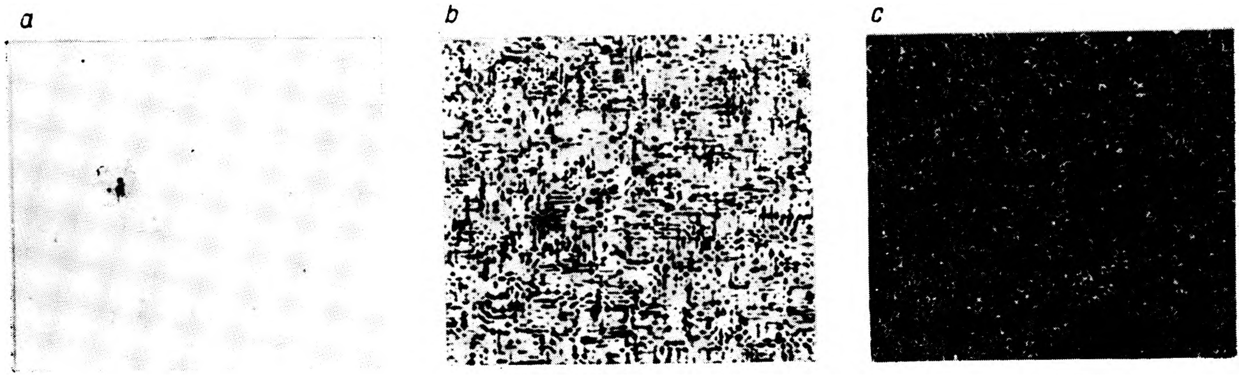

Fig. 1. Negative images of speckle pattern resulting from coherent light random scattering at a cross-shaped object
a) underexposed, $t_a = t_b/32$, b) normally exposed, c) overexposed, $t_c = 32 \cdot t_b$

defined; in these cases Lambertian rule is in power only when average values of radiant power densities are determined.

2. On mathematical formalization of the properties of scattering objects

The model of scattering surface considered in terms of geometrical optics is rather simple, being defined as an irregular surface by which the incident light is scattered in all possible direction. According to this definition the only property of the surface is its irregularity. This assumption, however, proves to be inadequate if the problem is considered in terms of wave optics, because of finite dimensions of scattered waves. Thus, it should be assumed that the irregularities of the scattered surface are comparable with the scattered waves, and moreover, that both amplitude and phase functions describing those surfaces are continuous.

It is generally assumed that the amplitude function of a diffuser is constant and close to unity, and that its phase function changes only along the coordinates. This assumption being in agreement with real properties of a common diffuser does not involve any restrictions.

The continuity of the phase function allows to divide the total area of a translucent diffuser into a multitude of separate areas of arbitrary shapes. Each elementary area might be described approximately by any regular phase function for spherical and cylindrical lenses, wedges, plane-parallel plates, and so on. In case of reflective diffuser those areas could be described by phase functions for spherical and cylindrical mirrors and adequately inclined plane mirrors.

It is obvious that to achieve the exact solution of the problem established in the way presented above, basic parameters and the statistics of elementary optical elements should be known. This, in turn, requires a number of new assumptions, which might render the subject unreal. What is more, the problem is so complicated from mathematical point of view that its usefulness becomes disputable.

From the continuity of the diffuser phase function it follows also that a certain area of the diffuser may be reduced to one point and thus treated as a coherent point source, both amplitude and phase functions being defined by the integral over this area. The diffuse surface model assumed by L.I. GOLDFISCHER [1] is based on such an assumption. The surface of interest is overlapped with a regular orthogonal grid with scatterers centred at its vertices. Every scatterer replaces an area $\Delta u \cdot \Delta v$, where Δu and Δv are the separations of grid lines. It has been assumed that the phase angle $\varphi_{u,v}$, associated with the scatterer at point (u, v), is random for $0 \leqslant \varphi_{u,v} \leqslant 2\pi$ and independent of the adjacent scatterers. In the following analysis the distances between grid lines, i.e. Δu and Δv have been assumed to be infinitesimally small, hence the density of scatterers in indefinitely large. By virtue of the above, and assuming that the scattering surface is illuminated by a monochromatic plane wave of a wavelength λ, the author considerers the resultant pattern at a distance h from the scattering surface, and uses the second-order approximation in expansion series. From the vector sum of all the contributions from every scatterer at the observation point (x, y) it may be inferred that the field at this point consists of two parts: the first represents the mean radiant power density which can be defined by applying Lambertian rule of hemispherical scattering, while the second one results from the coherent superposition of a large number of interference fringes oriented in all directions, and generated in the plane of observation by multiplicity pairs of scatterers present on the diffuse surface.

It is worth to mention the assumption on the infinitesimally small distance between the adjacent scatterers. Considering random phase of each scatterer, this assumption is contradictory to the physical sense of a real diffuser, since there must exist a relation between the phases of adjacent scatterers. Nevertheless, the infinitesimally small separation between the adjacent scatterers and the illumination of the diffuser by plane wave, as well as second-order approximation in expansion series, assumed by the author, permitted to define exactly the autocorrelation function and the spectral power density in speckle patterns, considered in a certain plane of observation.

Another model of a diffusing object was accepted by L. H. ENLOE[2]. According to his assumption the monochromatic coherent light is scattered by a random set of point scatterers. Each scatterer is many wavelengths distant from its neighbour; the coordinates x_i, y_i of i-scatterer and its θ_i relative phase are random variables uniformly distributed within the intervals $(-X, +X)$, $(-Y, +Y)$ and (0.2π), respectively; moreover, the relative phase of the wave scattered by each scatterer is a random variable, statistically independent of the phase of the waves scattered by other scatterers. It should be noticed that under the above assumptions the statistical properties of the scattered field are independent of any deterministic variations in the illuminating field. In view of the above model and assuming the Poisson distribution of the scatterers the author defines the autocorrelation function and the spatial power spectral density in the Fraunhofer region. In paper by Enloe, like in work by GOLDFISCHER [1] the radiant power density distribution is not calculated either. B. ELIASSON and F. M. MOTTIER [3] who based their calculations on the Goldfischer's model and still assume infinitely small separations between the adjacent scatterers have determined the autocorrelation function of the field irradiance for each point and its immediate surroundings in space. They have found that the "grains" in speckle pattern are cigar-shaped and directed normally to the diffuser. The calculations being not limited to the Fraunhofer region are valid also for near field region, except for the space spread immediately in front of the diffuser.

An interesting theoretical analysis concerning statistic properties of coherent light reflected by a system of similar but randomly distributed scatterers has been published by J. W. GOODMAN [5]. From the above analysis it follows that under the assumption of a full randomness of radiation phase reflected by separate scatterers (objects) a negative exponential distribution of the probability of radiant power density should be expected in the reflected wave field. The above work is purely theoretical, so the author refers to the paper by G. GOULD et al. [6] on the detection of coherent light reflected by scattering surfaces.

H. FUJII and T. ASAKURA [7] have published their experimental results on the influence of surface roughness on statistical distribution of radiant power density in the speckle pattern arising at the transmission. Ground-glass plates of different degree of roughness performed the role of scattering elements. Although for high degree of roughness (at which the statistical distribution of the values of phase operator on the diffuser surface may be considered to be homogeneous) the radiant power density distribution in measuring plane has an exponential form, nevertheless the optical measuring systems applied by the above authors may raise some objections.

3. Experimental methods used in definition of radiant power density distribution in speckle pattern

In view of all the difficulties involved in formulation of the exact definition of the mathematical model of scattering surfaces, and regarding the fact that any mathematical result based on a certain accepted model may be in contradiction with the existing physical values, the author performed some experimental measurements in order to define radiant power density distribution in the field scattered by randomly scattering surfaces. Two methods were used simultaneously. The first one was based on measurements of integral transmittances $\tilde{T}_e$ of specially processed photofilms to get a high contrast coefficient γ. By applying multiple copying the following conditions may be achieved:

$$T_e = 0 \quad \text{for} \quad H < H_0, \tag{1a}$$

$$T_e = 1 \quad \text{for} \quad H > H_0, \tag{1b}$$

where H_0 is a certain level of the exposition. Conditions (1a and b) will be obviously achieved if an even number of photoprocesses is performed to get a positive structure of the measuring transparencies. Because

$$H = p \cdot t, \tag{2}$$

where

p — local radiant power density,

t — exposure time,

thus

$$p_0 = \frac{H_0}{t} \tag{3}$$

is a boundary level of radiant power density depending on exposure time. If the exposure time is changed so that its reciprocal changes linearly, then the changes in the boundary level will be linear. The average transmittance $\tilde{T}_e$ of a transparent slide will be:

$$\tilde{T}(p_0) = \frac{1}{S} \iint_S T_e(x, y)\, dx \cdot dy = \frac{S(p > p_0)}{S} = s(p > p_0), \tag{4}$$

where

$S(p > p_0)$ — the surface corresponding to the areas where radiant power density is higher than p_0,

S — total surface of the measuring slide,

$s(p > p_0)$ — the relative surface of the areas where power radiant dendensity is higher than p_0.

By the method described above several structures of speckle patterns arising from the coherent illumination of random diffusers of different apertures have been analysed. Eight "cross-sections" of radiant power density distribution of speckle pattern resulting from the scattering at a circular diffuser are shown in Fig. 2. The ratio of exposure times of the marginal photos is 1000 : 1. Experimental measurements of three series of transmittances, corresponding to three different apertures limiting a randomly scattering surface are presented in Fig. 3. All te diagrams show linear dependence between natural logarithms of transmittances and reciprocals of the corresponding exposure times:

$$\ln \tilde{T}_e = -\frac{t_0}{t}, \tag{5}$$

where t_0 — the exposure time, at which transmittance $\tilde{T}_e$ is equal to $1/e$.

Taking into account the relation (2) and introducing a relative value of power radiant density

$$\varrho = \frac{p}{p_a} = \frac{t_0}{t} \tag{6}$$

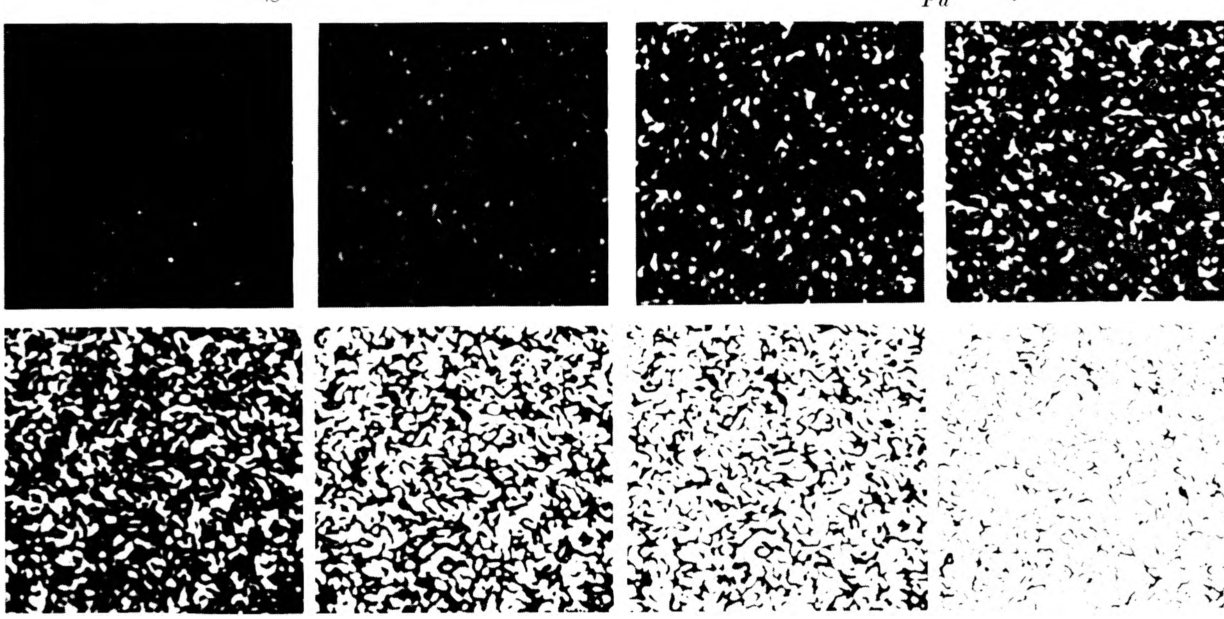

Fig. 2. Distributive exposures of the same speckle pattern but at different boundary levels of radiant power density

we obtain

$$\tilde{T}_e(\varrho_0) = s(\varrho > \varrho_0) = \exp(-\varrho_0). \qquad (7)$$

The second method used by the author to define the radiant power density distribution in speckle patterns was based on scanning a rectangular sector of a speckle pattern by a pho-

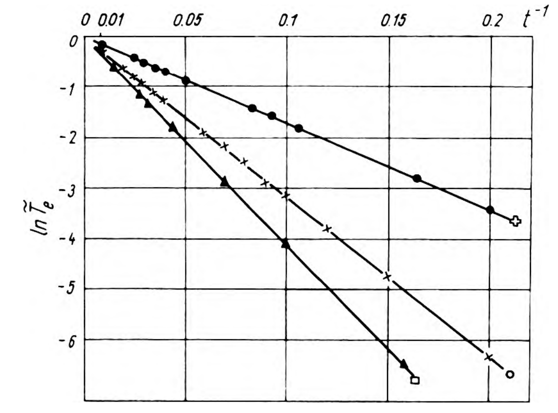

Fig. 3. Dependence of the natural logarithm of mean transmittance versus the reciprocal exposure time

tomultiplier. A general scheme of a measuring arrangement is shown in Fig. 4. As in previously described integrating method a randomly scattering diffuser limited by circular, rectangular or x-shaped apertures was used as a so-

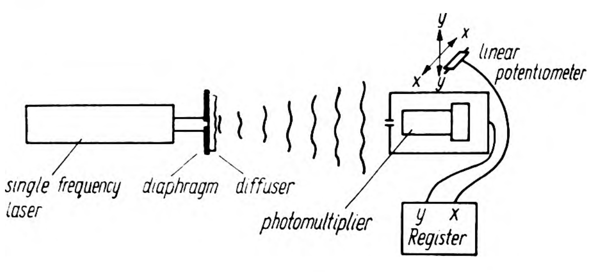

Fig. 4. Measuring arrangement scheme intended for the analysis of the radiant power density of the speckle pattern

urce of different speckle patterns. Dimensions of the aperture, placed in front of a photomultiplier cathode, were much smaller than linear dimensions of separate speckles arising in the measuring sector. Fig. 5 shows a cut from a number of successive records located one over another. All the records were copied on granulated paper of well defined specific surface gravity and cut out. A sector taken from the masks prepared in this way is shown in Fig. 6. The masks were used for two purposes:

1. to define from their weight the average power radiant density in the measuring speckle pattern, and

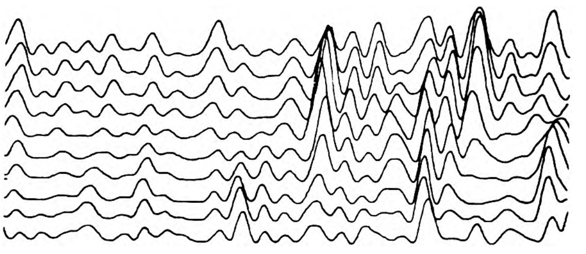

Fig. 5. Successing records of radiant power density traced along X–X coordinate, located one over another one for different values of Y–Y coordinates

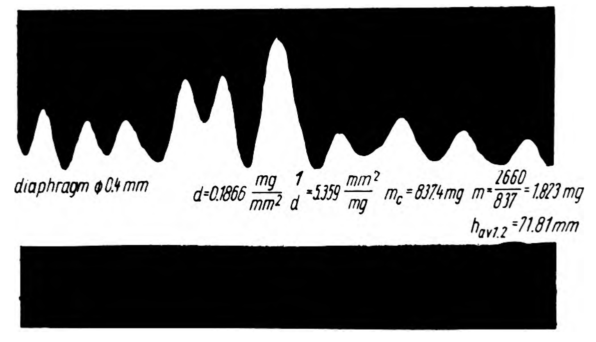

Fig. 6. The measuring mask made from the record speckle pattern resulting from scattering at a circular aperture of a 0.4 mm diameter

2. to measure the Fourier spectrum of the signals by applying the masks to an oscilloscope function generator connected with an acoustic spectrum analyzer. Thereupon, relative values p/p_{av} of radiant power densities in all the records were defined and the sums of sections s_i, corresponding to the areas where radiant power densities were higher than cutoff level p/p_{av} calculated. The measuring procedure and final results are presented in Figs. 7 and 8.

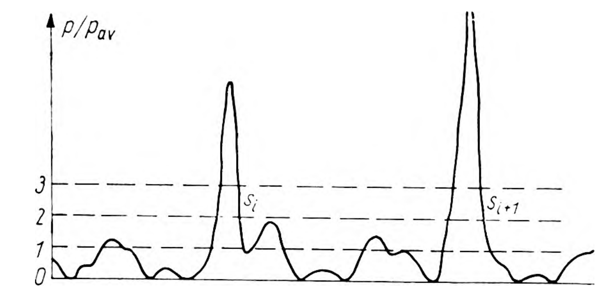

Fig. 7. A scheme illustrating defining of procedure relative surfaces occupied by relative radiant power density higher than a certain level ϱ_0

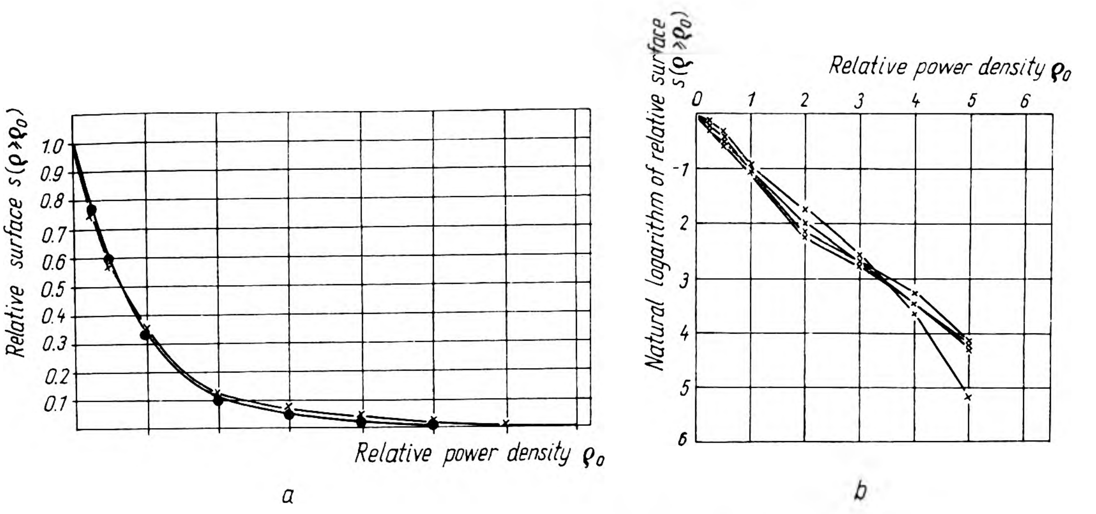

Fig. 8. Final results presenting the relative surface $s(\varrho \geqslant \varrho_0)$ and a) its natural logarithm, b) versus the relative level of radiant power density

It should be noticed that because of the limited recording dynamic of this method the linear dependence between the natural logarithm of a relative surface $s(\varrho \geqslant \varrho_0)$ and relative radiant power density ϱ_0 is kept for ϱ_0 ranging from 0 to about 2. At higher values of ϱ_0 the number of sections S_i decreases and for $\varrho_0 > 4$ the results can hardly be treated as statistical data, because of only several single peaks present in the analysed field. This method, however, led to the definition of value p_a in the equation (6). Since the relative radiant power density in scanning method was normalized towards the average radiant power density p_{av} then from the beginning of the diagrams shown in Fig. 8b it may be clearly seen that

$$p_a = p_{av}. \tag{8}$$

Equation (7) is given in a distributive form, and shows how much of relative surface of the speckle pattern image is occupied by the relative radiant power density higher than a certain relative level ϱ_0. If we take the interval closed by $s(\varrho > \varrho_0)$ and $S(\varrho > \varrho_0 + \Delta\varrho_0)$ values, then we get

$$\begin{aligned} &\Delta s(\varrho_0 < \varrho < \varrho_0 + \Delta\varrho_0) \\ &= s(\varrho > \varrho_0) - s(\varrho > \varrho_0 + \Delta\varrho_0) \\ &= [1 - \exp(-\Delta\varrho_0)] \cdot \exp(-\varrho_0) \end{aligned} \tag{9}$$

and if $\Delta\varrho_0 \to 0$

$$s(\varrho) = \frac{ds(\varrho_0 < \varrho < \varrho_0 + \Delta\varrho_0)}{d\varrho_0} = \exp(-\varrho). \tag{10}$$

The equation (10) may be treated as a distribution of probability with which a point with relative illumination equal to ϱ can be found in speckle pattern. It is clear, that a speckle pattern considered with field amplitude values would imply the Gaussian form of the radiant power density distribution.

It is worth knowing the amount of the relative total radiant power included in the sector $s(\varrho, \varrho + \Delta\varrho)$ of speckle pattern. Because

$$dp = \varrho(s) \cdot ds \tag{11}$$

then

$$p(\Delta s) = \int_{s_1}^{s_2} \varrho(s) \cdot ds. \tag{12}$$

But

$$s(\varrho) = e^{-\varrho}, \; ds = -e^{-\varrho} \cdot d\varrho$$

and integrating (12) by parts we obtain finally

$$p(\Delta\varrho) = [(1 + \varrho) \cdot \exp(-\varrho)]_{\varrho_2}^{\varrho_1}. \tag{13}$$

4. Conclusions

The radiant power density distribution in speckle pattern, formed by scattering of coherent radiation at a diffuser of an arbitrary shape, was defined by applying two different experimental methods. The results may be of a particular importance in many coherent light applications: they may facilitate the definition of optimal conditions during the production of holograms of diffusing objects; they may also be useful in all cases in which laser radiation is applied, and the definition of safety factor is required.

Statistique de la distribution de puissance d'irradiation du champ électromagnétique diffusé par des objets diffusants

Quand on éclaire un objet diffusant avec la lumière monochromatique cohérente, il y apparaît une tache de projection. Dans de nombreux cas il est nécessaire de connaître des valeurs telles que la moyenne carrée de densité de la puissance d'irradiation, la fonction d'autocorrelation et la distribution de densité de la puissance d'irradiation. En holographie et dans tous les cas, où les informations contenues dans le champ électromagnétique diffusé par les objets diffusants sont enregistrées dans les centres dont l'étendue dynamique de mesure est limitée, la connaissance de densité de la puissance d'irradiation est d'une importance particulière.

On a présenté des résultats d'expériences concernant la distribution de densité de la puissance d'irradiation monochromatique cohérente de la lumière, diffusée par des diffuseurs reflétant ou semitransparents de formes différentes. On a constaté la conformité absolue avec l'approche de Rayleigh.

Статистика распределения плотности мощности излучения электромагнитного поля, рассеянного выборочно рассеивающими объектами

Если какой-либо рассеивающий объект осветить когерентным монохроматическим светом, то появится отображающее пятно. Во многих нужно знание таких значений, как средний квадрат плотности мощности излучения, функция автокорреляции и распределение плотности мощности у излучения. В голографии и во всех случаях, в которых информации, содержащиеся в электромагнитном поле, рассеянном рассеивающими объектами, регистрируются в средах с ограниченным динамическим диапазоном регестрирования, знание распределения плотности мощности излучения имеет особое значение.

В работе приведены опытные результаты, касающиеся распределения плотности мощности когерентного излучения, монохроматического света, рассеиваемого отражающими или полупрозрачными диффузорами различной формы. Выявлено полное соответствие с подходом Рейли.

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