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Evaluation of the Elements of Direct Recovery Matrices by a Spot-Diagram Method for Selected Types of Optical Systems

In the paper a method of reconstruction matrix calculation is given for the case of incoherent imaging. The method proposed is based on spot-diagram technique.

The influence of such factors as number of rays traced through the system, the size of elementary cell of division and the diameter of the integrating element, on the results obtained has been analyzed.

1. Introduction

The problem of direct recovery of an arbitrary region in airial images for the case of incoherent imaging in absence of any a priori information about the object has been analyzed in [1–3]. For the convenience we will summarise the main idea of the direct recovery procedure.

An unknown incoherent object of intensity distribution $I(\alpha, \beta)$ in the plane P is imaged into plane P_I by the optical imaging system C (α, β). Thus the image intensity distribution $I_{\text{im}}(p, q)$ appears at II (see Fig. 1). Next the image is subject to a sampling by an observing system (II), which results in a set of N measurements $x(a_i b_i)$ corresponding to particular discrete locations $(a_i b_i)$ of the observing system II. The set $x(a_i b_i)$, for $i = 1, \ldots, N$ constitutes a so-called measurement representation of the image not identical, by definition, with the image intensity distribution.

The observing system consists of an imaging part and an integrating element E. The later is supposed to absorb he whole incident light flux and to convert it into a signal of a different nature.

According to the principle of direct recovery we are aiming in finding extreme intensity distributions in image which is consistent with the given measurement representation. The following procedure has been applied:

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a) Upper bound reconstruction procedure.

Let the object consist of a set of point sources distributed at the points (α_i, β_i) related to the points (a_i, b_i) in the image plane by eqs. $\alpha_i = a_i/M$ and $\beta_i = b_i/M$, where M is the linear magnification of the imaging system (I). Then

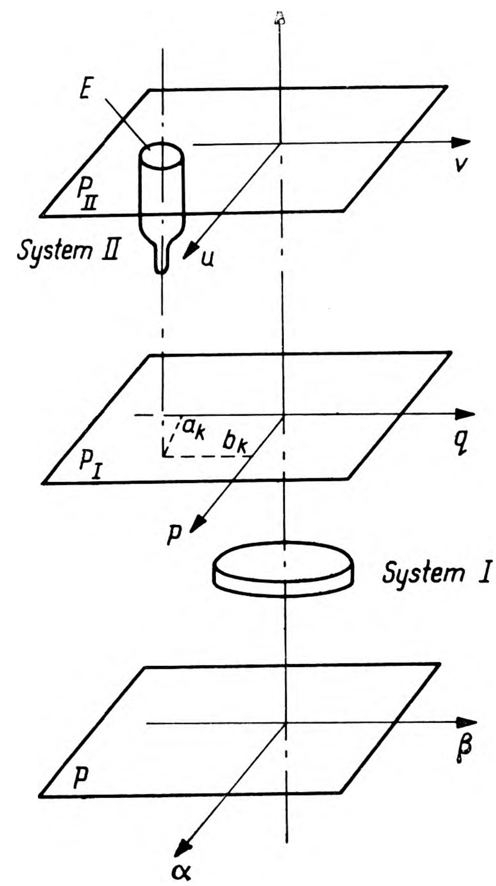

Fig. 1

the object intensity distribution is given by (see [1])

$$I(\alpha, \beta) = \sum_{i=1}^{N} c_i \delta(\alpha-\alpha_i)\delta(\beta-\beta_i) \tag{1}$$

and, consequently, the image intensity distribution takes the form (see [1])

$$y(p, q) = \sum_{i=1}^{N} c_i \varphi_{(\alpha_i \beta_i)}(p, q), \tag{2}$$

where $\varphi_{(\alpha i \beta i)}(p, q)$ is the intensity spread function of the imaging system and c_i are unknown weighting factors which are to be determined.

The measurement representation produced by the observing system (which is called also the observed image (see [2])) is given by

$$x(a_k b_k) = \sum_{i=1}^{N} c_i \int_{p} \varphi_{\alpha_i \beta_i}(p, q) \times$$

$$\times \Phi(p-a_k, q-b_k)\, dp\, dq = \sum_{i=1}^{N} c_i R_{k,i} \tag{3}$$

$$k = 1, \ldots, N$$

$$i = 1, \ldots, N$$

where $\Phi(p-a_k, q-b_k)$ is the instrumental function of the system II defined as: (see [2])

$$\Phi(p-a_k, q-b_k) = \int_{E} \varphi_{(p-a_k, q-b_k)}(u, v)\, du\, dv \tag{4}$$

and

$$R_{ik} = \int_{pI} \varphi_{(\alpha_i \beta_i)}(p, q) \Phi(p-a_k, q-b_k)\, dp\, dq$$

are the elements of the upper bound incoherent reconstruction matrix defined by properties of both the imaging and observing systems. Given the measurement representation $x(a_k, b_k)$, the Equations (4) is a linear system with respect to weighting factors which are to be determined.

Thereafter the image intensity at the sampling points is found from the formula

$$I^{\max}(a_k b_k) = \sum_{i=1}^{N} c_i^{\max} \varphi_{(\alpha_i \beta_i)}(a_k b_k), \tag{5}$$

(see [1]) which exhibits the property of representing the maximum values of intensity at the points a_k, b_k permissible by the condition of consistency with the measurement representation $x(a_k, b_k)$.

b)Lower bound reconstruction procedure.

On the other hand, by assuming that the object consists of a set of point-sources located at (α_k', β_k') $k = 1, \ldots, N$, which are distributed exactly in-between the previous set of object points (α_i, β_i) and repeating the above considerations we end up with an image representation

$$I^{\min}(a_k, b_k) = \sum_{i=1}^{N} c_i^{\min} \varphi_{(\alpha'_i \beta'_i)}(a_k b_k), \tag{6}$$

which has the property of representing the minimum possible values of image intensity at the sampling points (a_k, b_k) still consistent with the same measurement representation.

The average value of the recovered intensity distribution may be defined as (see [2])

$$I_{\text{im}} = \tfrac{1}{2}[I_{\text{im}}^{\max}(a_k b_k) + I_{\text{im}}^{\min}(a_k b_k)], \tag{7}$$

while its absolute error is given by

$$\Delta I_{\text{im}}(a_k b_k) = \pm\tfrac{1}{2}[I^{\max}(a_k b_k) - I^{\min}(a_k b_k)]. \tag{8}$$

2. Evaluation of the intensity spread function $\varphi_{(\alpha,\beta)}(p, q)$ of the imaging system I

As it is easily seen from formulae (5)–(8) in the recovery procedure an essential part is performed by the intensity spread function. Its evaluation is then of special importance.

As it is well known for the diffraction limited imaging the intensity spread function on the axis has the form

$$\varphi(p, q) = \left\{2\, \frac{J_1(K_1\sqrt{p^2+q^2})}{K_1\sqrt{p^2+q^2}}\right\}^2$$

$$= \varphi(\sqrt{p^2+q^2}, K_1), \tag{9}$$

where

$$K_1 = \frac{\pi}{\lambda M_1},$$

λ – is wavelength of the light used, J_1 – is the Bessel function of the first kind and first order, M_1 – is the f-number of the system if the object is at infinity.

Unfortunately, for the real aberrated optical systems, an exact analytical form of the intensity spread function cannot be given. Instead, a ray-tracing technique (e.g. according to the well-known Feder formulae [5]) may be easily used to determine the passage of a number N rays through the optical system for an arbitrary point in the object plane.

Let the intersection points of the rays (emerging from an object point) with the image plane form a set contained within the region π defined by the coordinates of the extreme points. For the sake of simplicity, it is assumed that the region π is a square as (Fig. 2) whose side

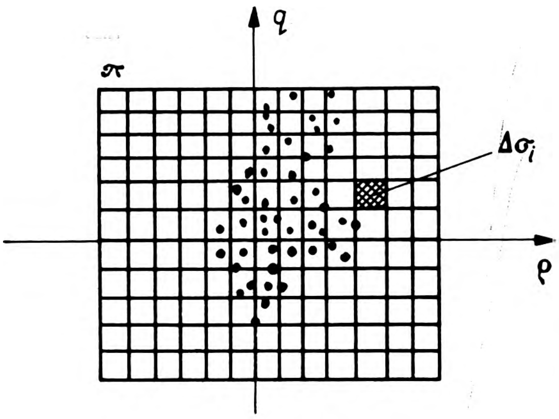

Fig. 2

is two times greater than the longer coordinate of the ray most distant from the optical axis. Let the region π be divided in a regular way shown in Fig. 2. Let $\Delta\sigma_i$ denote the area of the i-th division cell and Δn_i denote the number of rays contained within the i-th cell.

By assuming that each ray carries the same portion of the light energy it is evident that the number of rays Δn_i is proportional to the mean intensity within the corresponding area. Thus, the normalized intensity spread function φ of the imaging system may be defined as

$$\varphi_i^I = \frac{1}{N_c^I}\frac{\Delta n_i}{\Delta\sigma_i} \tag{10}$$

or

$$\varphi_i^I = \frac{1}{N_c^I}\frac{dn_i}{d\sigma_i} \tag{11}$$

where N_c^I denotes total number of the rays which passed the imaging system. In this way the discrete values of the function φ may be found for points whose coordinates are equal to coordinates of middle points in the respective cells.

3. Evaluation of the instrumental functions of the observing system II

According to the definition given in [1] an instrumental function has the form

$$\Phi(p-a_k, q-b_k) = \int\limits_E \varphi^{II}{}_{(p-a_k, q-b_k)}(u, v)\,du\,dv, \tag{12}$$

where $\varphi^{II}_{(p-a_k, q-b_k)}(u, v)$ is an intensity spread function of the imaging part of the observing system, while E denotes the integrating element area.

The spread function φ^{II} may be evaluated in the way shown in the previous section. Thus, denoting the elementary cell area of the function by $\Delta\sigma_i$, as shown in Fig. 2, we may put the normalized spread function of the system II in the form

$$\varphi^{II} = \frac{1}{N_c^{II}}\frac{dn_i'}{d\sigma_i'}, \tag{13}$$

where

dn_i' – denotes the number of rays striking the area,

N_c^{II} – denotes the total number of rays which passed through the observing system.

With the known discrete values of the intensity spread function of the imaging system its instrumental function can be determined, according to def. (12)

$$\Phi(a_k b_k) = \int\limits_E \frac{dn_i'}{d\sigma_i}\frac{1}{N_c^{II}}\,d\sigma_i, \tag{14}$$

which is equivalent to

$$\Phi(a_k b_k) = \frac{1}{N_c^{II}} N_E, \tag{15}$$

where N_E denotes the number the rays falling immediately on the integrating element E.

The method suggested allows to find the discrete values of the instrumental function at the point (a_k, b_k) defining the position of the integrating element.

4. Evaluation of the reconstruction matrix elements R_{ki}

By definition [3] the upper bound reconstruction matrix may be described as

$$R_{ki} = \int \varphi_{(a_i\beta_i)}(p, q)\,\Phi(p-a_k, q-b_k)\,dp\,dq.$$

From the previous discussion it is clear how to estimate the definiteness region of the functions φ and Φ.

One of the possible variants of their mutual position is illustrated in Fig. 3.

The matrix elements R_{ik} are different from zero only within the common region $\sum = \pi \cap T$

and thus, the numerical evaluation should be restricted to this region. By dividing the region $\sum$ in the way analogical to that used for calculation of the functions φ^I and Φ i.e., by assuming the elementary cell area equal to $\Delta\sigma_i$ the respective values of functions φ^I and

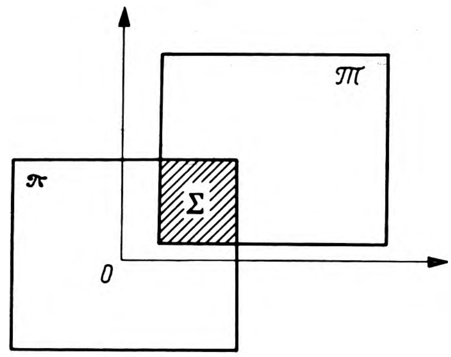

Fig. 3

Φ may be estimated at the middle points of each elementary cell. It should be noted, that while the definiteness region of φ^I was identical with the area at the image plane covered by the rays passing through the optical system, the region of definiteness associated with the instrumental function exceeds the region covered by the rays passing through the observing system by the area equal to a zone of width defined by the size of the integrating element. This is shown in Fig. 4.

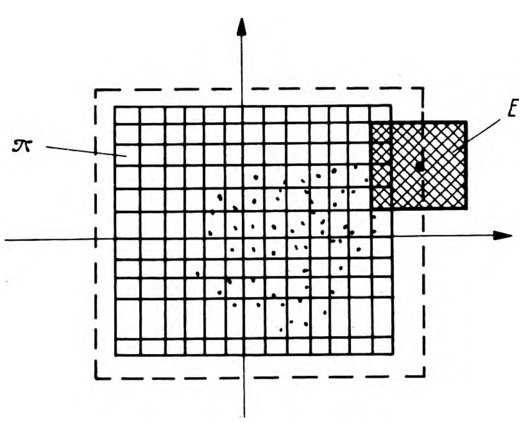

Fig. 4

5. Conclusions

In case of real (aberrated) optical system contrary to diffraction limited system, neither intensity spread function nor instrumental function can be given in analytical forms. Consequently, the integrand in expression for matrix elements R_{ik} is usually unknown.

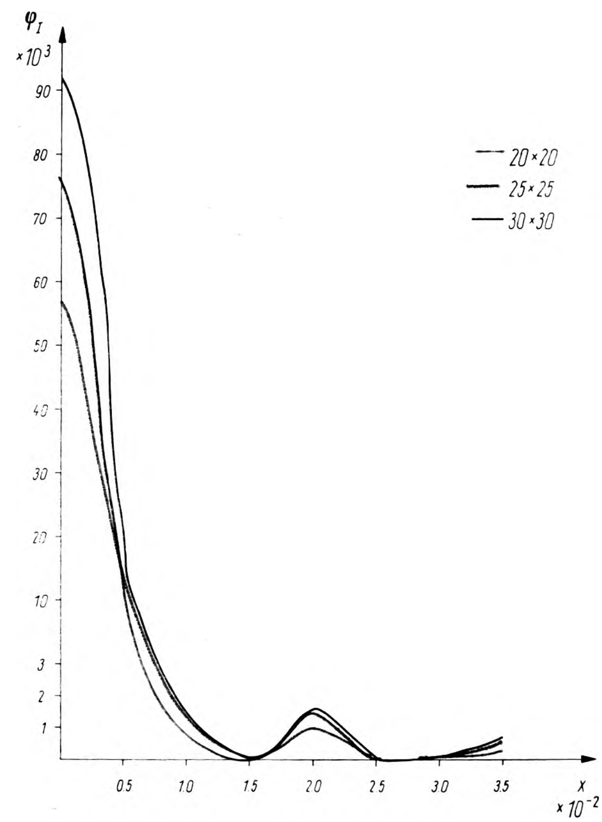

Fig. 5. Intensity spread function of the imaging system vs. the number and dimensions of the elementary cells in the domain of the axis aberration spot. Calculations made for $n = 1009$ rays

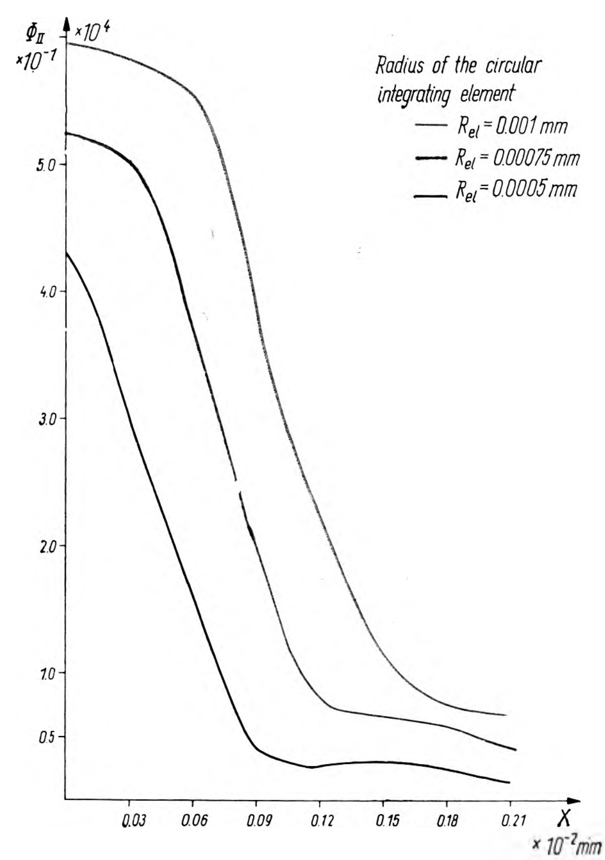

Fig. 6. Changes in the instrumental function of the observing system vs. the dimensions of the circular integrating element (on the axis)

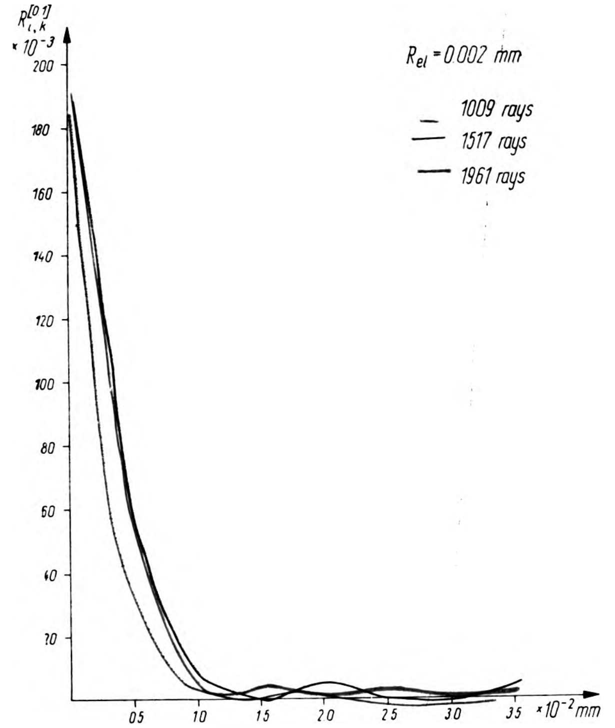

Fig. 7. Changes in the values of the reconstruction matrix elements $R^{0;1}_{ik}$ depending on the mutual separation of the centres of the domains of the functions φ and Φ for different numbers of traced rays (the symbol $R^{0;1}_{ik}$ denoting an observing system reduced to the integrating element alone)

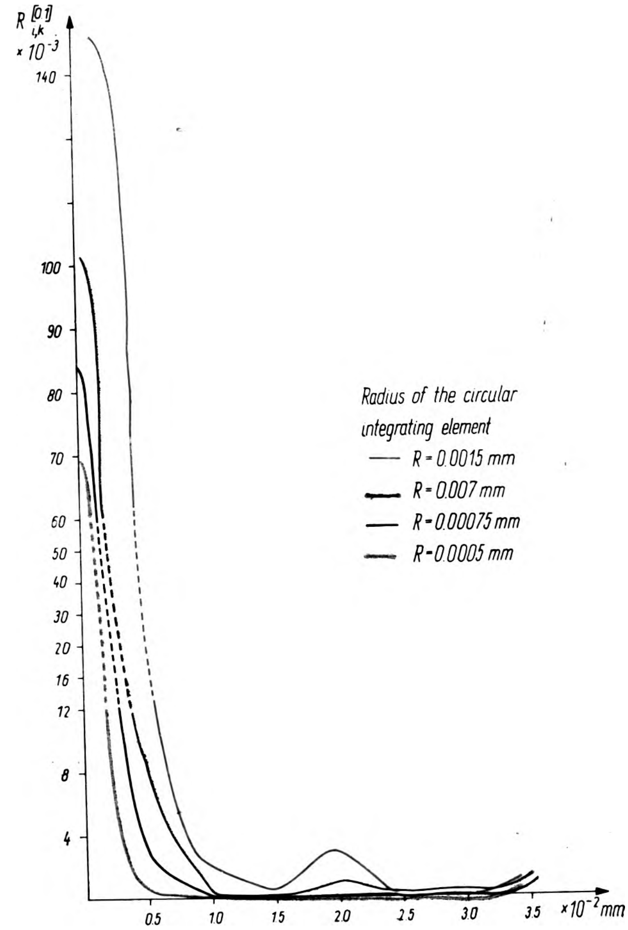

Fig. 8. Changes in the value of the reconstruction matrix elements vs. the mutual separation of the φ and Φ function domains for different sizes of the integrating element (point-object on the axis)

Nevertheless, for the aberrated systems the approximated method of reconstruction matrix estimation appeared to be much simpler (from numerical view point) than the exact analytic method used in diffraction limited case. However, the application of the above method to evaluating the intensity spread function, the instrumental function and the matrix elements $\boldsymbol{R}_{ki}$, requires some additional preliminary studies.

It appears that final results may be completely useless in the case of inproper choise of elementary cell sizes i.e. too small or too large in the division structure in the image planes. As a rule, the number of rays and, consequently, the division should be determined for each system separately.

It should be noticed that the greater the number of rays the higher accuracy of evaluation, but the longer the computation time.

As an illustrating example consider an imaging system consisting of a telescope triplet lens ($f = 100$, f-number 3,5), the optical part of observing system being represented by a 40 × microscope objective. Fig. 7 illustrates the dependence of $R^{[0,1]}_{ki}$ on the mutual separation of the centres of functions φ^I and Φ and the number of traced rays for the case when the observing system is reduced to the integrating element. It may be noticed that the estimation does not depend on the number of rays (above 1000 in this case) up to the first minimum, so the further increment of the number of rays gives only a slight improvement in evaluation accuracy. Fig. 5 illustrates the changes in the intensity spread function introduced by changing the division into elementary cells.

It should be emphasized that making a finer division for fixed number of rays renders an additional fluctuation in the results, due to

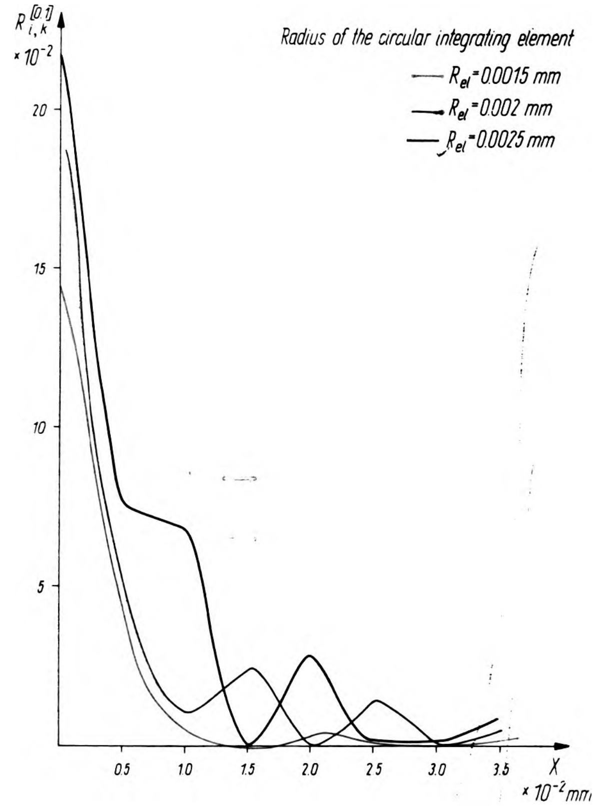

Fig. 9. Changes in the values of the reconstruction matrix elements as a function of the mutual separation of the φ and Φ function domains for different sizes of the integrating element (point-object on the axis)

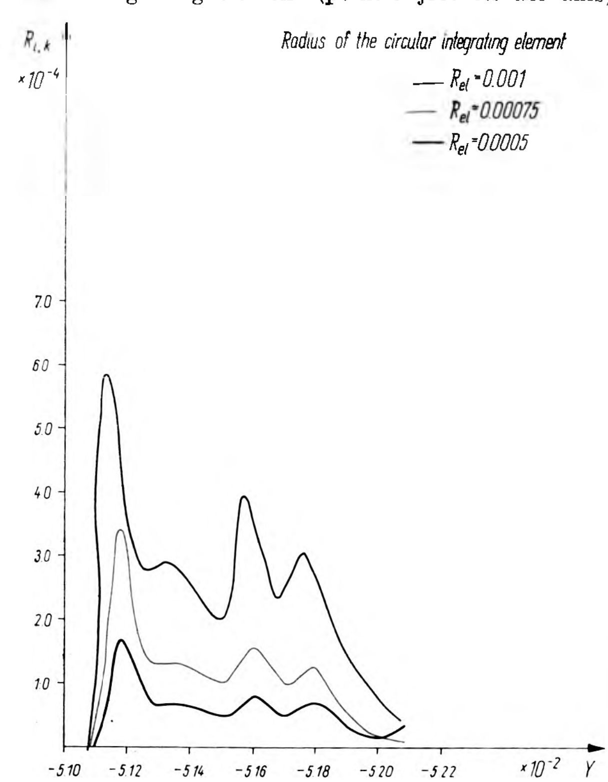

Fig. 10. Changes in the value of the reconstruction matrix elements vs. the mutual separation of the φ and Φ function domains for different sizes of the integrating element (on the axes)

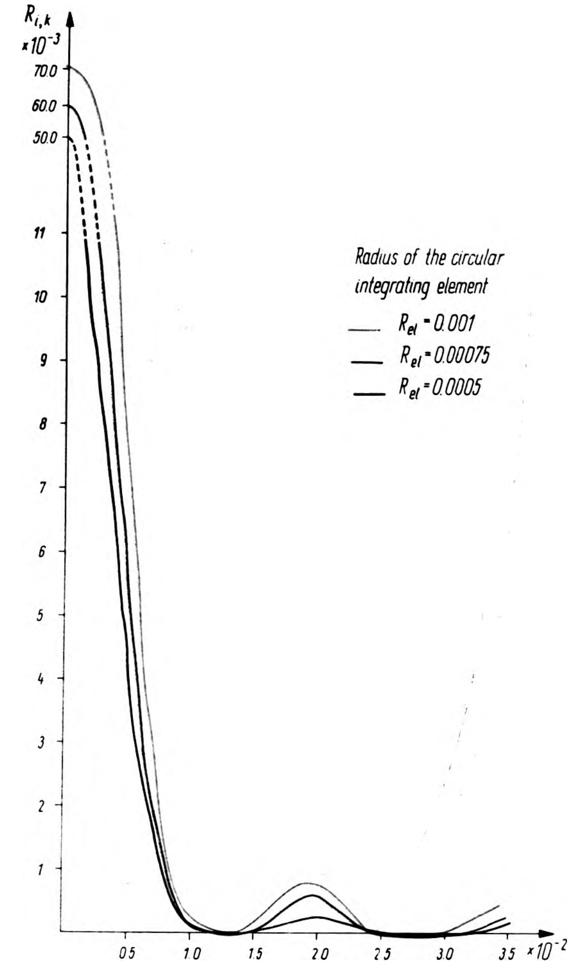

Fig. 11. Changes in the value of the reconstruction matrix elements as a function of the mutual separation of the φ and Φ function domains for different sizes of the integrating element (on the axes)

the fact that the part of the rays, which hit the division lines, increases in a random way. Then the problem arises to which cell those rays should be assigned, the number of common rays being accidental and different for each division. Another equally important point is the right choice of the integrating element area. This is illustrated in Figs. 8 and 9. It is easy to note that the area of the integrating element should not exceed the domain of the instrumental function. It is also seen that for too small integrating element (for instance $R_{el} = 0.00075$; $R_{el} = 0.0005$) a part of information gets lost ($R_{ki} = 0$) while for too great size of the element (for instance $R_{el} = 0.002$; $R_{el} = 0.0025$) accidental maxima occur.

The right choice of integrating elements allows to obtain the proper results which are illustrated in Fig. 10 and 11.

Calcul des éléments de la matrice de reconstruction directe par la méthode des spot-diagrammes pour les systèmes optiques choisis

L'ouvrage présente des méthodes de calcul des éléments de la matrice de reconstruction directe pour un cas du la correspondance non-coherente.

La technique suggérée, basée sur une méthode des spot-diagrammes permet le calcul numérique des éléments de la matrice de reconstruction pour des systèmes réels des types photographique et d'agrandissement.

Les auteurs ont analysé l'influence des facteurs particuliers (comme le nombre des rayons calculés, taille de la cellule élémentaire de division, etc) sur des résultats obtenus.

Расчет элементов матрицы прямой реконструкции методом спот-диаграмм для избранных оптических систем

В работе представлены методы расчета элементов матрицы реконструкции для случаев с некогерентным отображением. Предложенный способ, основанный на методе спот-диаграмм, делает возможным применение цифровых методов для расчета элементов матрицы для действительных систем фотографического и увеличительного типов. В статье проанализировано влияние отдельных факторов (число пересчитанных лучей, величина элементарной ячейки разделения и т. п.) на полученные результаты.

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