*Gy. L. Bencze and P. Varga**

Delocalization of the Information Stored in a Holographic Memory**

By illuminating the data mask of a holographic memory by an array of coherent point sources, the information stored in a hologram can be delocalized, i.e. there exists a three dimensional region around the focal plane over which the information is distributed uniformly. In consequence of the method described storage redundancy can be achieved, and some properties of the Fourier spectrum can be, moreover, kept in the vicinity of the Fourier plane. The calculations have been verified by a model experiment.

1. Introduction

1 §. Information stored in holographic memory has the form of the Fourier spectrum of a primary data mask, containing transparent and opaque squares, corresponding to the bit content. When the data mask is illuminated by a plane or homocentric wave, then each square gives the same amplitude distribution in the Fourier plane, but the phase factor is proportional to the location of the bit-square. The Fourier spectrum of the whole data mask is produced by the interference of the individual bit spectra. Such a kind of spectrum will be called a *uniform* spectral distribution.

For reconstruction of a bit-square from its holographic record only the central part of the diffraction pattern is needed, i.e. the region defined by resolution requirements. From the uniformity it follows consequently that for the reconstruction of the whole data mask the same region is needed as for an individual bit-square. Thus, the system of a holographic memory can be the following [1]: A block of data is realized in the form of a data mask, the mask is then recorded in a hologram of finite extent, called subhologram. The whole storage area consists of side by side subholograms.

Difficulties arise, however in the realization of the storage redundancy. A mere increase in the size of the subholograms allows to record only spatial frequencies that are unimportant for the reconstruction; the reliability of the device does not increase while its capacity diminishes. A further problem is that the Fourier spectrum appears only in the image plane of the light source. Hence, the smallest misalignment of the recording plane in the direction of the light propagation leads to the recording of the Fresnel instead of the Fraunhoffer diffraction. In the Fresnel diffraction pattern the field scattered by the bit squares is not distributed uniformly any more, the patterns of the individual bits being locally shifted. A damage of the hologram plate may lead to the loss of some bits in the reconstructed image.

The enhancement of the region over which the information is distributed uniformly is called delocalization. Delocalization can be achieved in several ways. The simplest method is the illumination of the data mask through a ground glass plate. Then, however, the speckle noise of the reconstructed field will be very high, like in the case of rough surface objects [2]. A more advantageous procedure is the illumination of the mask through a diffraction grating [3], since due to various orders of the diffracted light Fraunhoffer patterns are shifted and consequently, the spectrum is multipled.

The method investigated by us [4] can be regarded as a generalization of the diffraction grating method. Each bit square is illuminated by an individual point source (Fig. 1). The Fourier spectrum of a bit, illuminated in this way, is the convolution of the spectrum of

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a single bit and that of the illuminating beam. This resultant spectrum consists of an infinite number of elementary holograms given by a plane wave source. Thus, the resultant spec-

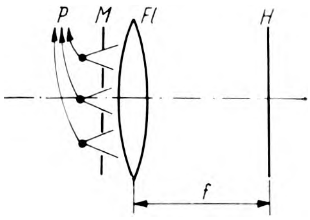


Fig. 1. Point matrix illumination
P – point sources, M – data mask; FL – Fourier transform lens, H – hologram plane, f – focal length of the Fourier transform lens

trum is getting blurred regularly. The coherent point sources can be produced by using a fly's eye optics, the distance between the lenslets being equal to the side of the bit squares. Such method was used for redundant recording of picture-like information [5]. The fly's eye illumination has also been described in [6] without detailed analysis.

2. Theory

2 §. At first we shall discuss the most popular Fourier (or quasi-Fourier) method of transparency transformation. A plane wave falls on the data mask (Fig. 2) and the hologram is

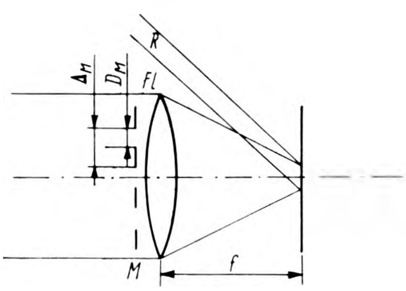


Fig. 2. Plane wave illumination
M – data mask, FL – Fourier transform lens, R – reference beam

registered in the focal plane of a lens. Let the transparency of the binary mask containing N^2 bits be

$$t(x_M, y_M) = \sum_{k=1}^{N}\sum_{l=1}^{N} C_{kl}\cdot \mathrm{rect}\left(\frac{x_M - \Delta_M k}{D_M}\right)\times \times\mathrm{rect}\left(\frac{y_M - \Delta_M l}{D_M}\right), \tag{1}$$

where

$$\mathrm{rect}(x) = \begin{cases} 1 & |x| \leqslant 1/2 \\ 0 & |x| > 1/2 \end{cases};$$

x_M, y_M are the coordinates on the binary mask; D_M is the size of a bit; Δ_M is the distance between the bit-centres; and $C_{kl} = 0$ or 1 depending on the content. Electric field in the hologram (x_H, y_H) is [7]:

$$E(x_H, y_H) = \frac{i}{\lambda f}\exp\left\{\frac{i\pi(x_H^2+y_H^2)}{\lambda f}\right\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0 t(x_M, y_M)\times \times\exp\left\{-\frac{2\pi i}{f}(x_M x_H + y_M y_H)\right\}dx_M dy_M, \tag{2}$$

where λ denotes the wavelength, E_0 the amplitude of the incident light wave and f is the focal length of the Fourier lens. Taking into account (1) we get for the field (in the following we neglect the proportionality factors):

$$E(x_H, y_H) = B(x_H y_H)\,C(x_H, y_H), \tag{3}$$

where

$$B(x_H, y_H) = \int_{-D_M/2}^{D_M/2}\int_{-D_M/2}^{D_M/2} \times \exp\left\{-\frac{2\pi i}{\lambda f}(x_M x_H + y_M y_H)\right\}dx_M dy_M \tag{4}$$

and

$$C(x_H, y_H) = \sum_{k=1}^{N}\sum_{l=1}^{N} C_{kl}\exp\left\{\frac{2\pi i}{\lambda f}\Delta_M(kx_H + ly_H)\right\}. \tag{5}$$

$B(x_H, y_H)$ function will be called the *bit spectrum function*, because it describes the spectrum of a single bit. The $C(x_H, y_H)$ function is called the *content spectrum function*, since it describes the spectrum of a mask on which the bits are represented by δ functions. As an example Fig. 3 shows the content spectrum

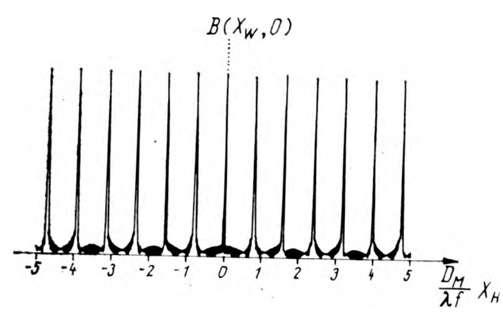


Fig. 3. A result of the one dimensional computer simulation of the content spectrum

function of 20 randomly selected bits of a one-dimensional mask. It can be seen that the content spectrum spreads over the whole hologram plane, so the spectrum to be recorded on the hologram for the reliable storage is determined by the bit spectrum function.

For plane wave illumination the bit spectrum function is

$$B(x_H, y_H) = D_M^2 \operatorname{sinc}\left(\pi \frac{D_M x_H}{\lambda f}\right) \operatorname{sinc}\left(\pi \frac{D_M y_H}{\lambda f}\right), \tag{6}$$

where $\operatorname{sinc}(x) = \sin x / x$.

The spectrum has appreciable values only in the vicinity of the optical axis ($x_H = 0$, $y_H = 0$).

The spectrum described in (3), (4), (5), (6), is recorded on the Fourier hologram of the data mask.

In the reconstruction process the intensity distribution at the detector x_D, y_D plane is

$$I(x_D, y_D) = \text{const} \left| \int\limits_{x_H^0-h}^{x_H^0+h} \int\limits_{y_H^0-h}^{y_H^0+h} B(x_H, y_H) \times C(x_H, y_H) e^{-\frac{2\pi i}{f^*}(x_H x_D + y_H y_D)} dx_H dy_H \right|^2, \tag{7}$$

where f^* is the focal length of the read out Fourier lens, x_H^0 and y_H^0 are the coordinates of the centre of the spectral band used for the reconstruction and h denotes the width of the reconstructed Fourier spectrum. The deformation of a bit of the reconstructed signal is shown in Fig. 4 as a function of the reconstructed spectral range (i.e. h) for the case $x_H^0 = 0$, $y_H^0 = 0$.

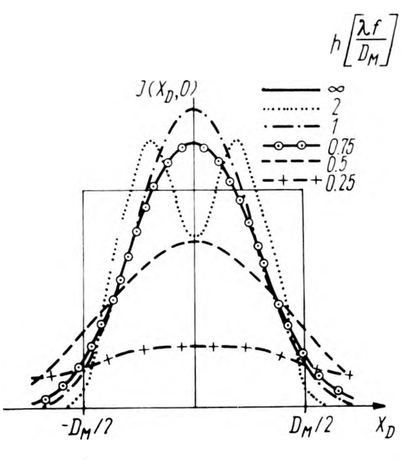


Fig. 4. Deformation of a bit of the reconstructed signal as a function of the reconstructed spectral range (calculation)

It can be seen that the main part of the information is contained within the spectral range determined by h $(0.8\text{–}1)\lambda f/D_M$; $x_H^0 = y_H^0 = 0$; there is no reason to take larger areas for reconstruction as the output signal increases only slightly.

Moreover, any lateral shift of the centre of the reconstructed area (from the origin of coordinates $x_H^0 = 0$, $y_H^0 = 0$) will lead to higher losses (Fig. 5). Therefore, from practical point of view the reconstruction region of the hologram can be defined by *

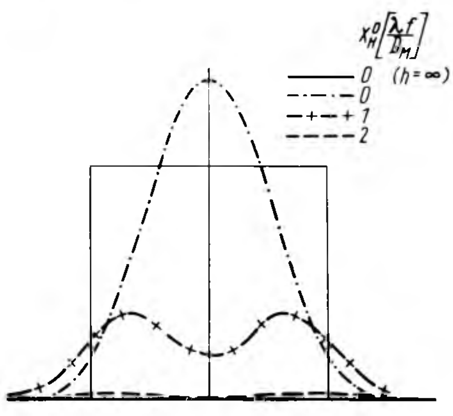


Fig. 5. Intensity distribution in the detector plane for several x_H^0 values
$h = \infty$ for the continuous curve, $h = 1\lambda f/D_M$ for the other curves (calculation)

$$|x_H| \leqslant \frac{\lambda f}{D_M}; \quad |y_H| \leqslant \frac{\lambda f}{D_M}. \tag{8}$$

Therefore there is also no reason to record larger subholograms, such subholograms are not redundant, and losses in the hologram yield information losses in the reconstructed image.

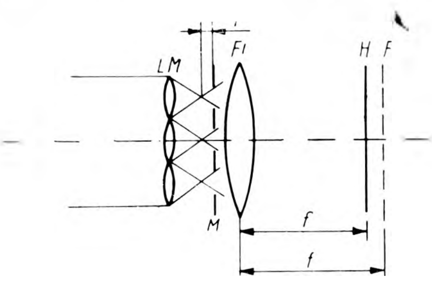


Fig. 6. Lens matrix illumination
LM – lens matrix, M – data mask, FL – Fourier transform lens, H – hologram plane, F – focal plane

3 §. Let now a matrix of lenses be placed in front of the data mask (see Fig. 6) so, that the axis of each lens intersect the data mask at the centre of the corresponding bit square. The bits

* The limit defined in (8) is twice that requested by the Rayleigh criterion.

are illuminated by spherical waves

$$E_0 \exp\left\{\frac{i\pi}{\lambda r}\left[(x_M - k\Delta_M)^2 + (y_M - l\Delta_M)^2\right]\right\},$$

where

r – denotes the distance between the focal plane of the lens matrix and the data mask. The field at a distance f' measured from the Fourier lens is then

$$E(x_H) = E_0 \sum_k \int_{k\Delta_M - \frac{D_M}{2}}^{k\Delta_M + \frac{D_M}{2}} \exp\left[\frac{i\pi}{\lambda r}(x_M - k\Delta_M)^2\right] \times C_k \exp\left\{-\frac{i\pi}{\lambda f} x_M^2\right\} \exp\left\{\frac{i\pi}{\lambda f'}(x_M - x_H)^2\right\} dx_M. \tag{9}$$

(For simplicity we discuss only a one-dimensional case, the relations for two dimensions are obvious.) The first factor in the integrand in (9) describes the spherical wave, the second is the transparency of the mask, the third describes the focusing action of the Fourier transform lens, and the fourth is the propagation term. After some boring calculations for the field diffracted by the k-th square we get

$$E_k(x_H) = \exp\left\{-\frac{i\pi}{\lambda f}\left[\left(k\Delta_M(a+\varphi) - x_H(1+\varphi)\right)^2 + x_H^2(1+\varphi)\right]\right\} \times \left\{\Phi\left[\frac{\sqrt{2}}{\sqrt{\lambda f}}\left(\frac{D_M\sqrt{a}}{2} + \frac{k\Delta_M\varphi + x_H(1+\varphi)}{\sqrt{a}}\right)\right] + \Phi\left[\frac{\sqrt{2}}{\sqrt{\lambda f}}\left(\frac{D_M\sqrt{a}}{2} - \frac{k\Delta_M\varphi + x_H(1+\varphi)}{\sqrt{a}}\right)\right]\right\} \tag{10}$$

where

$$\Phi(v) = \int_0^v e^{i\frac{\pi}{2}t^2} dt$$

is the Fresnel integral, φ is the defocusing factor characterizing the misalignment of the hologram plate, defined as

$$\varphi = \frac{f - f'}{f'}$$

and a stands for

$$a = \frac{f}{r} + \varphi. \tag{11}$$

The formula (10) is hardly disputable. We shall concentrate the discussion on the role of the parameter a containing both the defocusing factor and the spherical wave illumination of the bit. If $a \to 0$ an assimptotic approximation is to be used [8] and [10] turns into the Fourier transform of the square(6). The case, $a = 0$ but $\varphi \neq 0$ and $r \neq \infty$, corresponds to the realization of the Fourier transformation, when the Fourier plane is the image plane of the source [7]. The case $r \to \infty$ and $\varphi \to 0$ has been discussed in § 2.

A more important case is when a is a large number, i.e. the distance r is much less than the focal length f, but the misalignment is not too high ($\varphi \ll 1$). The distribution remains nearly uniform, if for the two (independent of the coordinates x_H) terms (11) of the Fresnel integral argument the following condition holds:

$$\left|\frac{k\Delta_M\varphi}{\sqrt{a}}\right| \ll \frac{D_M\sqrt{a}}{2}$$

i.e. when the defocusing factor

$$|\varphi| \ll \frac{a}{2k}\frac{D_M}{\Delta_M} \approx \frac{f}{r}\frac{D_M}{2k\Delta_M}.$$

For $\Delta_M/D_M = 1.5$, $k = 50$ and $f/r = 30$ the limit of the relative misalignment is

$$|\varphi| \ll 1/15 \sim 6\%,$$

which can be easily achieved.

In the absence of the lens matrix ($r = \infty$) the term dependent on the location of the square will be

$$k\Delta_M\sqrt{\varphi}$$

(see (10) and (11)), instead of

$$\frac{k\Delta_M\varphi}{\sqrt{a}} \sim \sqrt{\frac{r}{f}}\,k\Delta_M\varphi;$$

thus the accuracy of the longitudinal alignment is facilitated by the factor r/f.

The field distribution must be calculated from Fresnel integrals. In Fig. 7 we show some calculated distributions for some values of f/r. As it can be expected, the witdh of the intensity distribution curve increases with the f/r ratio. (It is clear, that the case, when the approximation of the geometrical optics is valid, the illumination of the Fourier plane by a bit between the shadow margines is constant.)

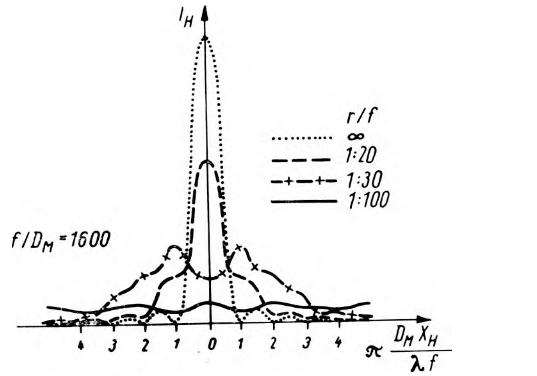


Fig. 7. Dependence of the intensity distribution of the bit spectrum on the focal length of the fly's eye lenses ($D_M = 0.19$ mm, $f = 300$ mm, $\lambda = 6328$ Å) (calculation)

When the whole data mask is illuminated by a lens matrix, the field distribution in the hologram plane is

$$E(x_H) = \sum_k E_k(x_H),$$

where $E_k(x_H)$ is given by (10).

The reconstructed intensity distribution in the detector plane is

$$I(x_D) = \text{const}\left|\int\limits_{-h+x_H^0}^{h+x_H^0} E(x_H) e^{-\frac{2\pi i}{\lambda f} x_H x_D} dx_H\right|^2. \tag{12}$$

The reconstructed intensity distribution (12) for the case $f/r = 100$, $q = 0$ was calculated

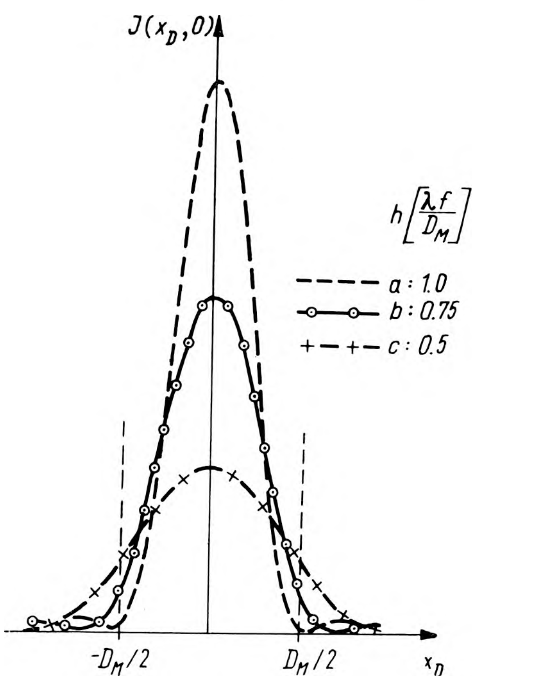


Fig. 8. Calculated intensity distribution at the detector plane in the case of fly's eye lens illumination for several h values; $x_H^0 = 0$

numerically. Fig. 8 presents the calculated distribution in the case of $x_H^0 = 0$ for several values of h. Comparing Figs 4 and 8 we can see, that a satisfactory reconstruction with plane wave and fly's eye lens illumination methods requires the same spectral band ($h = 1$). The enlargement of the reconstructed area ($h > 1$) gives an enlarged signal in the detector proportional to the enlargement of the reconstructed area, i.e. the storage redundancy is proportional to the enlargement of the hologram size. In Fig. 9 the reconstructed intensity distribution is plotted for bandwidth $h = f\lambda/D_M$ but the centre shifted up to $x_H^0 = 3.75\ \lambda f/D_M$. The graph shows no significant distortion compared with the centred band (curve *a* in Fig. 9), in contradiction to the case of the plane wave illumination (Fig. 5) which proves the equivalence of different parts of the spectrum.

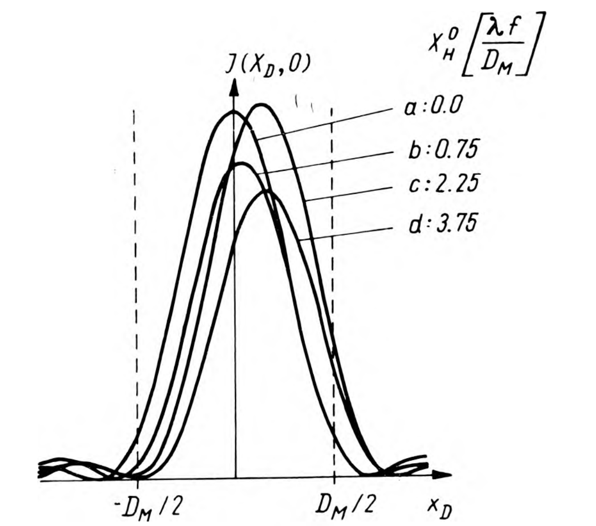


Fig. 9. Calculated intensity distribution at the detector plane for several x_H^0 values ($h = 0.75\lambda f/D_M$; $D_M = 0.19$ mm; $f = 300$ mm; $\lambda = 6328$ Å; $f/r = 100$)

3. Experimental

§ 4. For the sake of simplicity in the experiments we have used a pinhole instead of the fly's eye lens array. The arrangement of the pinholes on an opaque mask was identical to that of the bits in the binary mask. Placing the pinhole array instead of the lens array the bits were illuminated by the beams diffracted from the pinholes. The distance between the pinhole array and the binary mask was chosen so that each bit be illuminated by light coming only from the pinhole. It is obvious that pinhole array cannot replace the fly's eye lens array in every respect because of the loss in light power.

The data mask used for test is shown in

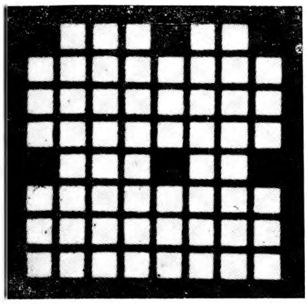
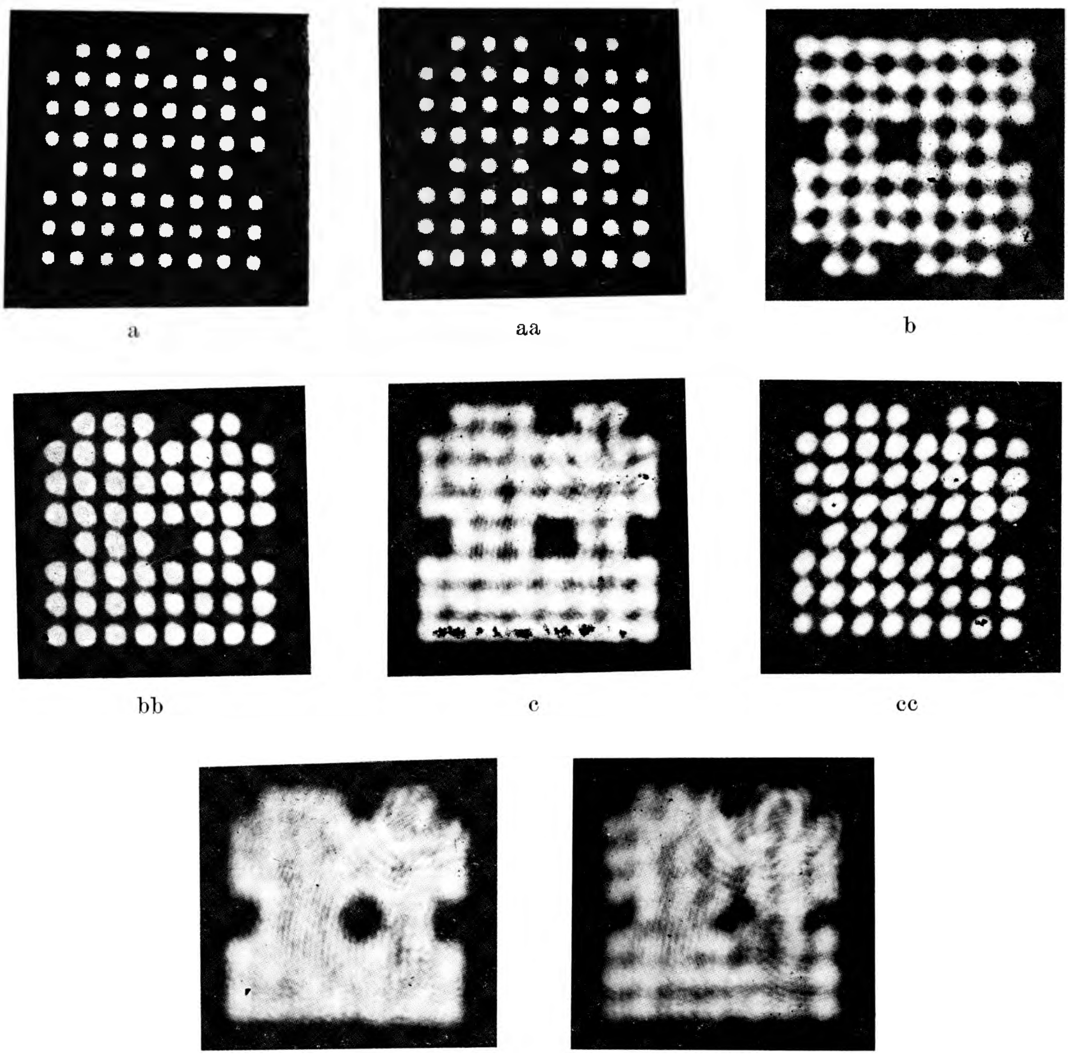


Fig. 10. The test mask

Fig. 10, while Fig. 11 demonstrates its image when centred stops of various diameter were put in the focal plane of the Fourier transform lens. For pictures a-d parallel wave and for pictures aa-dd matrix illumination were used. According to § 3, no significant deviation for symmetrical spectra is observed at both kinds of illumination. The size $h = \lambda f/D_M$ is satisfactory for making fair records.

Fig. 12 is a side-band picture using stop of diameter $2\lambda f/D_M$ shifted by $3/4\lambda f/D_M$. Fig. 12a was taken with parallel beam, Fig. 12b with a matrix. The difference between the two records is obvious. The further advantage of the matrix illumination is demonstrated in Fig. 13, showing the result of the off focal plane misalignment. The image of a plane wave illuminated mask is cut off at the corners and strongly distorted in the sides, indicating the absence of the uniformity, while the picture taken with the matrix shows no local losses in the content, the uniformity being observed.

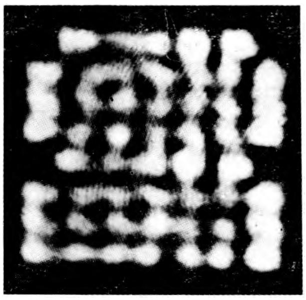


a aa b

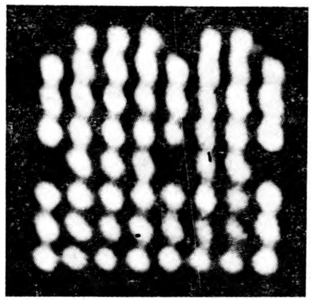
bb c cc

d dd

Fig. 11. Intensity distribution at the detector plane: a–d: parallel wave illumination, aa–dd: matrix illumination, a, aa : $h = 1(\lambda f/D_M)$, b, bb : $h = 0.85(\lambda f/D_M)$, c, cc : $h = 0.7(\lambda f/D_M)$, d, dd : $h = 0.6(\lambda f/D_M)$, $x_H^0 = 0$

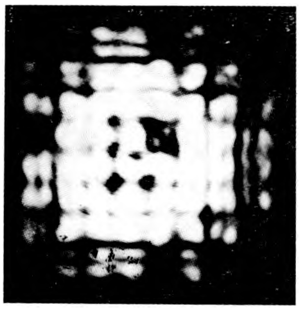


a

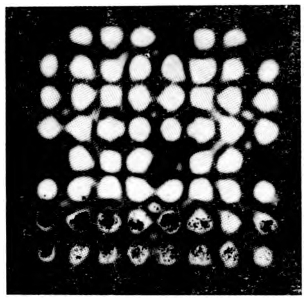


b

Fig. 12. Experimental comparison of the plane wave (a) and matrix (b) illumination method ($h = 0.75\ (\lambda f/D_M)$, $x_H^0 = 0.75\,(\lambda f/D_M)$).



a



b

Fig. 13. Experimental comparison of the plane wave (a) and matrix (b) illumination method ($h = 1\,(\lambda f/D_M)$, $x_H^0 = 0$, $f/r = 100$, $\varphi = 0.15$)

4. Conclusions

Calculation and experiments show that a satisfactory storage redundancy can be achieved when each bit of the binary mask is illuminated separately by an identical divergent beam. The use of divergent illumination does not increase the minimal size of the hologram, so this method does not decrease the maximally attainable storage density. The required storage redundancy can be achieved by a proportional enlargement of the hologram size. Using our method the spectrum remains uniform for the holograms of out-of Fourier planes just as well. Consequently, Fourier transform lenses with large angular aperture can be used, and the recording material need not be matched to the curved focal plane, because the properties of the Fourier transform are conserved in depth.

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Le déplacement des informations magasinées dans une mémoire holographique

Au moyen de l'éclairage de la masque contenant des données de la mémoire holographique par un système des sources de lumière ponctuelles cohérentes il est possible de déplacer les informations stockées dans un hologramme; cela veut dire qu'autour du plan focal il existe une zone de trois dimensions, sur laquelle les informations sont disposèes uniformément. En se servant de cette méthode on peut obtenir un surplus de mémoire; en plus, certaines propriétés du spectre de Fourier peuvent être préservées dans l'entourage du plan de Fourier. Les calculs ont été vérifiés pratiquement sur un modèle.

Перемещение информации, хранимой в голографической памяти

Путем освещения маски с данными голографической памяти системой когерентных точечных источников можно перемещать информацию, хранимую в голограмме. Иначе говоря: существует трехмерная зона вокруг фокальной плоскости, на которой информация одинаково размещена. Применением описанного метода можно получить избыток памяти, а некоторые свойства спектров Фурье могут, сверх того, сохраниться вблизи плоскости Фурье. Расчеты проверены эмпирически на модели.

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