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Analytical formulation of bound solitons in a lossy birefringent optical fiber

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A variational approach is followed to study soliton propagation in a lossy birefringent fiber for an illustrative model. In particular, the criterion for the coupling of two polarized soliton pulses to form a bound state has been obtained. The implications of this model havealso been discussed.

1. Introduction

Soliton propagation in birefringent optical fibers has attracted much attention in recent years towards the development of fiber-optic communication lines, generation of short pulses and soliton lasers [1]–[9]. It is well known that single-mode fibers in reality are bimodal because of the presence of linear birefringence [10]. This can lead to the splitting of fundamental mode into two orthogonal polarization modes with weak intermodal dispersion, and either of the two modes captures the other one through the Kerr effect such that the two pulses can propagate together in spite of the group-velocity mismatch [11]. The coupling between the two polarization modes in a birefringent single-mode optical fiber is an important effect since the coupling between the two modes is possible over long propagation distances. As is well established, propagation of solitons in birefringent optical fibers is governed by a set of coupled nonlinear Schroedinger (CNLS) equations. This set of CNLS equations is nonintegrable in the sense that methods such as inverse scattering transform (IST) cannot be applied to it. Furthermore, particle like nature of solitons has been exploited in order to derive, from conservation laws, ordinary differential equations (ODEs) for the adiabatic variation of soliton parameters during the propagation. The propagation of solitons in a highly birefringent fiber has been studied theoretically [12] as well as experimentally [13] in the past within the framework of CNLS equations without taking into account fiber losses in their formulations.

In this paper, an analytical model of the dynamics of soliton coupling in a lossy highly birefringent optical fiber, based on variational formalism, is developed for the first time in the framework of CNLS equations to study the threshold effect of soliton trapping of two polarized pulses having the same amplitude. This study should be of considerable interest to the experimental efforf on soliton ring networks.

2. Analytical formulation

The copropagation of two optical pulses in a highly birefringent lossy fiber is described by the following basic system of coupled NLS equations [1]:

$$i(u_z+\delta u_t)+\frac{1}{2}u_{tt}+(|u|^2+\varepsilon|v|^2)u = -i\gamma u, \tag{1a}$$

$$i(v_z-\delta v_t)-\frac{1}{2}v_{tt}+(|v|^2+\varepsilon|u|^2)v = -i\gamma v, \tag{1b}$$

where $\varepsilon = \frac{2}{3}$, u and v are the slowly varying amplitudes of the wave envelopes in the two polarizations, δ corresponds to the group velocity detuning due to the birefringence, and γ is the dissipative coefficient characterizing losses in the fiber.

Using the transformation: $u \to pe^{-\gamma z}$ and $v \to qe^{-\gamma z}$, as is well known that losses in the fiber can lead to exponential decreasing of the soliton amplitude, the above equations can be rewritten into the following form

$$i(p_z+\delta p_t)+\frac{1}{2}p_{tt}+(|p|^2+\varepsilon|q|^2)pe^{-2\gamma z} = 0, \tag{2a}$$

$$i(q_z-\delta q_t)+\frac{1}{2}q_{tt}+(|q|^2+\varepsilon|p|^2)qe^{-2\gamma z} = 0. \tag{2b}$$

The variational approach [2], [12], [14] is based on the possibility to present Eqs. (2) as Lagrange equations corresponding to the Lagrangian

$$L = \int_{-\infty}^{\infty}\Big[\frac{i}{2}(p_zp^*-p_z^*p)+\frac{i}{2}(q_zq^*-q_z^*q)+\frac{i\delta}{2}(p_tp^*-p_t^*p)-\frac{i\delta}{2}(q_tq^*-q_t^*q)-\frac{1}{2}|p_t|^2 -\frac{1}{2}|q_t|^2+\frac{1}{2}|p|^4e^{-2\gamma z}+\frac{1}{2}|q|^4e^{-2\gamma z}+\varepsilon|p|^2|q|^2e^{-2\gamma z}\Big]dt \tag{3}$$

where: $p_z = \frac{\partial p}{\partial z}$, $p_t = \frac{\partial p}{\partial t}$, and so on.

The appropriate trial function [2], [12] describing the temporal form of the solitons was chosen as

$$p = \frac{2\eta_1}{\sqrt{1+\varepsilon}}\operatorname{sech}[2\eta_1(t-\zeta_1)]\exp[2iV_1(t-\zeta_1)+iD_1], \tag{4a}$$

$$q = \frac{2\eta_2}{\sqrt{1+\varepsilon}}\operatorname{sech}[2\eta_2(t-\zeta_2)]\exp[2iV_2(t-\zeta_2)+iD_2] \tag{4b}$$

where the evolution parameters η_r, ζ_r, D_r ($r = 1$, 2 correspond to p and q solitons, respectively) represent amplitude, central position, velocity of soliton's central position as it propagates along the fiber, and phase, respectively. Substitution of Eqs. (4) into Eq. (3) and using Euler–Lagrange equations, yields the following system of coulped ODEs for the evolution of p and q soliton parameters:

$$-8\frac{\mathrm{d}}{\mathrm{d}z}(\eta_r V_r)+\frac{\partial}{\partial \zeta_r}L_{pq}=0, \tag{5}$$

$$\frac{\mathrm{d}\zeta_r}{\mathrm{d}z}-\delta-2V_r=0, \tag{6}$$

$$\frac{\mathrm{d}D_r}{\mathrm{d}z}=2V_r^2-2\eta_r^2+\frac{4\eta_r^2}{(1+\varepsilon)}e^{-2\gamma z}+\frac{1}{4}\frac{\partial}{\partial \eta_r}L_{pq}=0, \tag{7}$$

where $L_{pq}=\frac{16\varepsilon\eta_1^2\eta_2^2}{(1+\varepsilon)}e^{-2\gamma z}\int_{-\infty}^{\infty}\mathrm{sech}^2[2\eta_1(t-\zeta_1)\mathrm{sech}^2[2\eta_2(t-\zeta_2)]\mathrm{d}t.$

The resulting system (5)–(7) of ODEs for the evolution of soliton parameters models Eqs. (2).

Considering $\eta_1=\eta_2=\eta=\mathrm{const}$, $V_1=-V_2=V$, and $\zeta_{1,2}=\pm\frac{\rho}{4\eta}$, where ρ is the relative distance between the p and q polarization maxima. Equation (6) can now be expressed in trems of the symmetric parameters

$$\frac{\mathrm{d}\rho}{\mathrm{d}z}=4\eta\delta+8\eta V. \tag{8}$$

Differentiating the above equation with respect to z and on substituting the value of $\mathrm{d}V/\mathrm{d}z$ from Eq. (5), one can obtain

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}z^2}=4\eta\frac{\partial}{\partial\rho}L_{pq}. \tag{9}$$

Equation (9) can also be written as

$$\frac{\mathrm{d}^2\rho}{\mathrm{d}z^2}+\frac{\partial}{\partial\rho}U_{\mathrm{int}}(\rho)=0 \tag{10}$$

where $U_{\mathrm{int}}=4\eta[L_{pq}(0)-L_{pq}(\rho)]$ is defined as the effective interaction energy of the soliton with p and q polarization components.

Evaluating $L_{pq}(0)$ and $L_{pq}(\rho)$ for the case $\eta_1=\eta_2=\eta$, and on substituting their values into the expression for effective interaction energy, one can get

$$U_{\mathrm{int}}=\frac{128\varepsilon\eta^2}{(1+\varepsilon)}e^{-2\gamma z}\left[\frac{1}{3}-\frac{\cosh\rho}{\sinh^2\rho}(\rho-\tanh\rho)\right]. \tag{11}$$

For small ρ, $\frac{\cosh\rho}{\sinh^3\rho}(\rho-\tanh\rho)\simeq\frac{1}{3}-\frac{2}{15}\rho^2$. Hence

$$U_{\text{int}}(\rho) = \frac{256\varepsilon\eta^4}{15(1+\varepsilon)}\rho^2 e^{-2\gamma z}. \tag{12}$$

Also, as $\rho \to \infty$, the interaction energy given by (11) tends to a finite value

$$U = \frac{128\varepsilon\eta^4}{3(1+\varepsilon)} e^{-2\gamma z}. \tag{13}$$

Equation (10) when multiplied throughout by $\mathrm{d}\rho/\mathrm{d}z$ and integrated with respect to z yields the energy conservation law

$$\frac{1}{2}\left(\frac{\mathrm{d}\rho}{\mathrm{d}z}\right)^2 + U_{\text{int}}(\rho) = \text{const}, \tag{14}$$

the constant (total energy) is determined from the initial conditions at $\rho = 0$, $U_{\text{int}}(0) = 0$, and $\left(\frac{\mathrm{d}\rho}{\mathrm{d}z}\right)_{z=0} = 4\eta\delta$.

It follows from Equation (14) that the two partial pulses trap each other to form a bound state provided $E_{\text{Kin}} \leqslant U$ and obtained an analytical expression for threshold amplitude A_{thr} of the input pulse in a lossy birefringent fiber

$$A_{\text{thr}} = \frac{1}{\sqrt{(1+\varepsilon)}} + \frac{1}{2}\sqrt{\frac{3}{2\varepsilon}}\delta e^{\gamma z}. \tag{15}$$

It is important to note at this point that for amplitude $A < A_{\text{thr}}$, the two polarizations interact weakly whereas for $A \geqslant A_{\text{thr}}$ the solitons form a bound state due to intermode coupling described by the effective potential energy. Also when $\gamma = 0$, Eq. (15) reduces to

$$A_{\text{thr}} = \frac{1}{\sqrt{(1+\varepsilon)}} + \frac{1}{2}\sqrt{\frac{3}{2\varepsilon}}\delta, \tag{16}$$

which was obtained earlier [12] by Kivshar without taking into account fiber losses in his formulations.

3. Conclusions

The proposed model describing propagation dynamics of soliton coupling in a lossy birefringent fiber is simplified making it possible to obtain analytical results at the expense of some quantitative accuracy. Although approximate, the analogy with classical mechanics is satisfying. If more precision is desired, then one has to look for a new trial function for soliton's wave form through numerical simulations and follow the technique given here. This may be necessary for solitary wave propagation in a lossy birefringent fiber over long propagation distances with different initial conditions. This analytical model though describes optical pulse propagation in the anomalous dispersion regime of a birefringent fiber, but its results can also be applied to a wide array of different physical problems modelled by Eqs. (1) and (2), opens the door for further examination through numerical simulations.

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