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Aberration effect of a holographic lens for Fourier transform

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The optical Fourier transform operation must ensure a linear mapping between spatial frequencies produced in the Fourier plane. The paper considers an aberrated holographic collimating lens in the Fourier transform setup. In the case of aperture error in the collimating beam that illuminates the object transparency, the patterns of the spatial frequencies formed in the back focal plane of the Fourier transform lens, are not of good quality. In particular, the influence of the third order aberration coefficient on the intensity of the diffraction pattern is determined.

1. Introduction

The optical transform of any amplitude transmittance is always burdened with aberrations, analogously to the image of an object formed by a lens. In an ideal lens system there exists the Fourier transform relationship between the coherent light distribution in the object and image focal planes of this system. Otherwise, it is possible to produce spatial frequencies of an arbitrary complex amplitude transmittance introduced in the input focal plane of a lens, but they are formed with departures in the quality of the focused spots. The description of the diffraction at the input aperture by the Fourier transform integral and propagation of light from the front focal plane to the back focal plane of the lens is valid only for small diffraction angles. For large diffraction angles in the well corrected lens, the diffraction pattern forming the Fourier spectrum in the output focal plane must be aplanatic as well as anstigmatic, and the principal rays must fulfil the sine condition; but an amount of distortion is allowed to exist.

The magnitude of spherical aberration of the lens depends on the diameter of the bundle in the entrance pupil and can be defined as a variation of focus with lens aperture [1]. This paper discusses aberration in terms of the wave nature of light in the form of the optical path difference (OPD). If the longitudinal spherical aberration is a function of the spatial coordinate x in the entrance pupil of the lens and can be represented by the series

$$\mathrm{LA} = a_1 x^2 + a_2 x^4 + a_3 x^6 + \dots, \tag{1}$$

then the OPD with respect to its paraxial focus is given as

$$\mathrm{OPD} = \left(\frac{a_1}{4}x^2+\frac{a_2}{6}x^4+\frac{a_3}{8}x^6+\ldots\right)n\sin^2\theta$$

where n is the index of refraction of the image medium, and θ is the final slope of the marginal ray in the image space. A lens that performs the Fourier transform operation between the front and the back focal planes must the plane waves diffracted at the input aperture to different points in the image focal plane of the lens. But the quality of the focused spots at all locations in the Fourier plane is not the same and differs from each other. Therefore departures in the quality of the focused spots can be described by the OPD errors. The allowance for the tolerable depth of focus is usually established by Rayleigh's criterion that permits not more than $\lambda/4$ of OPD over the wave front with respect to a reference sphere about a selected image point. The permissible focus shift with respect to the paraxial focus in air is then determined by the equation

$$(a_1' x^2+a_2' x^4+a_3' x^6+\ldots)\sin^2\theta = \lambda.$$

The optical path difference introduced by a diffractive lens is given by

$$\mathrm{OPD} = m\frac{\lambda}{\lambda_0}(Ax^2+Bx^4+Cx^6+\ldots), \tag{2}$$

where the diffraction order $m = 1$ in our case, and λ_0, λ are the wavelengths of recording and reading waves, respectively.

2. Beam expander

Expansion of a laser beam for transparency illumination should be accomplished without loss of power and without significant alteration of the intensity distribution in the cross-section of the beam. The beam expansion can be performed with the use of a microscope objective or a holographic lens (see Fig. 1). If the holographic optical

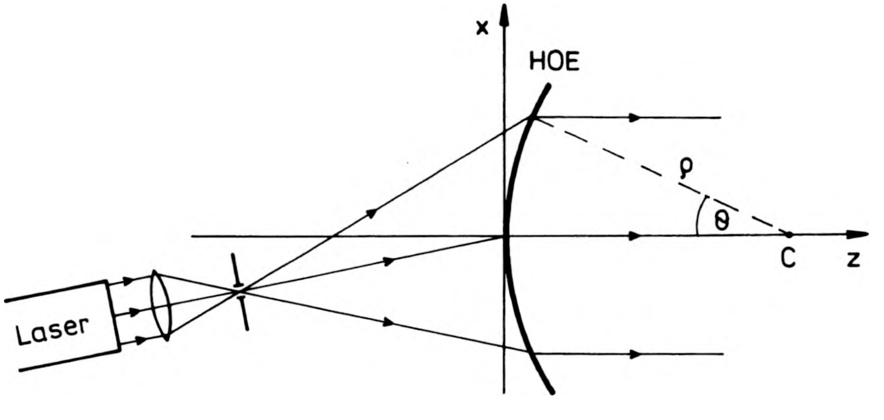

Fig. 1. Ray tracing in the collimating holographic lens. C is the centre of the spherical substrate, and ρ is the curvature radius.

element is recorded on the spherical substrate with well matched radius curvature, as shown in the figure, then spherical aberration can be corrected [2]. Such a holographic lens is recorded using two waves: an object plane wave (on-axis) with the complex amplitude $u_O(x,y)$ whose phase alters from point to point at the spherical recording surface, and the off-axis spherical reference wave $u_R(x,y)$ emerging from the off-axis point source at coordinates (x_R, y_R, z_R). The intensity of the interference pattern of the holographic collimating lens is then given by the equation

$$I(x,y) = |u_O+u_R|^2 = A_O^2+A_R^2+2A_OA_R\cos\left\{\frac{\pi[(\rho-z_R)\sin\varphi+2x_R]x}{\lambda z_R}\right.$$
$$\left.+\frac{\pi[(\rho-z_R)\sin\varphi]y}{\lambda z_R}\right\} \tag{3}$$

where the spatial frequencies of the fringe pattern are described as

$$\nu_x = \frac{2x_R+(\rho-z_R)\sin\varphi}{2\lambda z_R},$$
$$\nu_y = \frac{(\rho-z_R)\sin\varphi}{2\lambda z_R}. \tag{4}$$

The holographic lens formed in the way described above can be applied to collimation of light beams of another wavelength, different from that which has been used for recording the lens at spherical substrate. Let us assume that the HOE was recorded by the two waves of wavelength λ_0, and that the illumination was accomplished by the light of wavelength λ. The phases of the two spherical waves of which the first is emerging from the reference point source during recording, and the second one from the illuminating point source during illumination, are given by the equations:

$$\Phi_0(x,y) = -\frac{\pi}{\lambda_0 z_R}(x^2+y^2+2x_R x),$$
$$\Phi(x,y) = -\frac{\pi}{\lambda z_C}(x^2+y^2+2x_C x) \tag{5}$$

where z_C is the distance of the illuminating point source from the vertex of the spherical surface (holographic lens). The last two equations describe the phases of spherical waves over the holographic surface. In the case of $z_C = z_R$ the phase difference between the two waves produces aberrations (because of $\lambda_0 \neq \lambda$). When the wave diffracted by HOE is collimated without aberrations, then the illuminating wave is identical with the reference one (*i.e.*, $\lambda = \lambda_0$, $z_C = z_R$). If the light wavelength used for illumination differs from that applied to record the HOE, then in order to obtain the same spatial frequencies for the two (reference and illuminating) waves, the location of the second point source should be shifted in z-direction ($z_C \neq z_R$). Therefore the relationship between the two wavelengths must fulfil the condition:

$$\frac{\lambda_0}{\lambda} = \frac{z_C}{z_R} \rightarrow z_C = \frac{\lambda_0}{\lambda} z_R.$$

We see that for the magnification different from unity ($\lambda \neq \lambda_0$), the distance z_C alters with the wavelength λ and is inversely proportional to the wavelength. Simple, not expensive lenses (such as telescope doublets or flat holographic optical elements) are not perfect collimating and transform lenses. The effect of departures in the input wave fronts from a plane wave determines the aberration that influences the quality of the Fourier transform spectrum.

3. OPD as a phase error

The problem of optimal configuration for Fourier transform has been described by JOYEUX and LOWENTHAL [3], and two extreme cases with respect to aberrations have been analysed: the parallel-beam and the converging-beam Fourier transform lenses. Now, we consider the parallel-beam Fourier transform setup based on the diffraction of a plane wave by the object frequency components. After diffraction, the plane waves emerging from the object frequency components at different angles are focused by the Fourier lens to the corresponding points in the Fourier plane, as shown in Fig. 2. Usually, a conventional photographic objective that is always corrected for all five monochromatic aberrations can be applied to the Fourier operation. In particular, the distortion is minimized so that the plane wave diffracted at the angle α by the input aperture is focused in the back focal plane, and at the same time the distance of the focal point is defined by equation

$$x_f = f \tan\alpha.$$

But for the Fourier transform lens, the sine condition must be satisfied

$$x'_f = f \sin\alpha. \tag{6}$$

We see that the difference between the Fourier transform lens and other types of

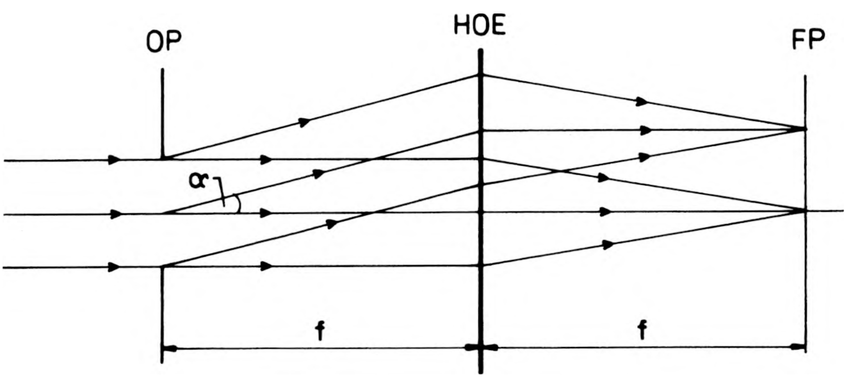

Fig. 2. Input and output planes in the Fourier transform setup. OP — object plane, FP — Fourier plane, α — angle of diffraction.

lenses lies in the fact that the image height is proportional to the sine of the incoming field angle, as shown in Eq. (6). This condition ensures a linear mapping between the spatial frequencies of an object in the Fourier plane. Therefore, the third order transverse aberration for Fourier transform lens must be

$$\Delta x_f = f\tan\alpha - f\sin\alpha = \frac{1}{2}f\alpha^3.$$

In this case, the resolvable spot size of the image point (representing the corresponding spatial frequency) in the Fourier plane is defined by

$$\Delta x'_f = \lambda\frac{f}{D}$$

where D is the diameter of the entrance pupil of the setup. Thus, it is evident that any plane wave diffracted at an angle α and focused by the Fourier lens represents the circular spatial frequency of the amplitude transmittance expressed by

$$\omega = \frac{2\pi}{\lambda}\left(\frac{\lambda}{D} + \sin\alpha\right). \tag{7}$$

Now, let us discuss the OPD phase error introduced in the collimated beam by the expander. The OPD is calculated with respect to the spherical aberration and is defined as the difference between the optical path lengths from points of the wave front emerging from the lens to the reference sphere centred on the paraxial focus point. For the error of the collimating lens we consider the meridional rays parallel to the optical axis with the OPD described by Eq. (2). When the error consists of only the third order aberration, the corresponding expression is approximately limited to the first term and becomes

$$\mathrm{ODP} = Ax^2.$$

Therefore, the phase of the amplitude transmittance placed in the front focal plane of the Fourier lens is disturbed by an additional error, and the distribution of the field amplitude across the Fourier plane (see Fig. 2) is

$$U(x_f, y_f) = C\iint\limits_{-\infty}^{\infty} P(x,y)\exp[iA(x^2+y^2)]\exp\left[-\frac{2\pi}{\lambda f}(xx_f + yy_f)\right]\mathrm{d}x\mathrm{d}y \tag{8}$$

where the pupil function $P(x,y) = 1$ inside the lens aperture and $P(x,y) = 0$, otherwise. For the rectangular aperture of sides $2b_1$ and $2b_2$ parallel to the coordinate axes with origin at the centre of the aperture, and by omitting the constant term in the integral, we have

$$U(\omega_x, \omega_y) = \int\limits_{-b_1}^{b_1}\int\limits_{-b_2}^{b_2} \exp[i\mathrm{A}(x^2+y^2)]\exp[-i(\omega_x\mathrm{x} + \omega_y y)]\mathrm{d}x\mathrm{d}y, \tag{9}$$

or in one dimension

$$U(\omega) = \int_{-b}^{b} \exp(iAx^2)\exp(-i\omega x)\mathrm{d}x. \tag{10}$$

Thus, using the Fraunhofer approximation in the lens without aberrations, the diffraction pattern of the aperture is seen to be

$$U(\omega)_{A=0} = U^{(0)}(\omega) = \int_{-b}^{b} \exp(-i\omega x)\mathrm{d}x = \frac{\sin(\omega b)}{\omega b}. \tag{11}$$

In the general form of the collim ating lens with the third order aberration, we expand the exponential phase error in a power series and Eq. (10) becomes

$$U(\omega) = U^{(0)}(\omega) - \frac{A^2}{2!}U^{(4)}(\omega) + i\left[AU^{(2)}(\omega) - \frac{A^3}{3!}U^{(6)}(\omega)\right] \tag{12}$$

where $U^{(n)}(\omega)$ means the (n)-th derivative of $U^{(0)}(\omega)$ with respect to the circular spatial frequency ω. The intensity of the Fraunhofer diffraction pattern in the Fourier plane is then given by

$$I(\omega) = U(\omega)U^*(\omega) = \left[U^{(0)}(\omega) - \frac{A^2}{2!}U^{(4)}(\omega)\right]^2 + \left[AU^{(2)}(\omega) - \frac{A^3}{3!}U^{(6)}(\omega)\right]^2.$$

But it is more convenient here to use the completely sufficient notation

$$I(\omega) = [U^{(0)}]^2 - A^2U^{(4)}\left[U^{(0)} - \frac{A^2}{4}U^{(4)}\right] + A^2U^{(2)}\left[U^{(2)} - \frac{A^2}{3}U^{(6)}\right] + \frac{A^6}{36}[U^{(6)}]^2, \tag{13}$$

and for small values of the coefficient A we have

$$I(\omega) = [U^{(0)}(\omega)]^2 - A^2[U^{(0)}(\omega)U^{(4)}(\omega)] + A^2[U^{(2)}(\omega)]^2. \tag{14}$$

It follows that the intensity of the diffraction pattern in the Fourier plane as a function of the spatial frequency is of the form

$$I(\omega) = \left[\frac{\sin(\omega b)}{\omega b}\right]^2 + \frac{4A^2}{\omega^4}\left\{\left[\frac{\sin(\omega b)}{\omega b}\right]^2(2\omega^2b^2 - 5) + 4\left[\frac{\sin(\omega b)}{\omega b}\right]\cos(\omega b) + \cos^2(\omega b)\right\} \ldots . \tag{15}$$

Equation (15) shows how the intensity of the Fourier spectrum depends on the spherical aberration introduced to the Fourier transforming lens. The first term of the right-hand side of the equation determines the intensity of the diffraction pattern given by a single slit without errors (or by a rectangular aperture in one dimension). The minima of the curve correspond to $\sin(\omega b) = 0$, for $\omega b = m\pi$ (where $m = \pm 1, \pm 2, \ldots$) but not for $\omega b = 0$. We see that they (the minima of the intensity) do not accept the null values as in the case of an ideal curve [4], but they become

$$I_{\min}(m) = \frac{4A^2b^4}{m^4\pi^4}, \tag{16}$$

and decrease very fast for higher order of the minima.

The principal maximum occurs at $\omega b = 0$, and the secondary maxima are located at points satisfying the equation

$$\omega b = \tan(\omega b), \tag{17}$$

i.e., for $\omega b = 1.43\pi, 2.46\pi, 3.47\pi, \ldots$. Inserting expression (17) in Eq. (15), we obtain the intensity of the maxima diffraction pattern in the Fourier plane

$$I_{\max}(\omega) = \left(1+\frac{8A^2b^2}{\omega^2}\right)\cos^2(\omega b)$$

which shows how it depends on the aberration coefficient. The contribution to the intensity from the aberrated bundle (second term in the above equation) decreases with an increase of the spatial frequency (higher diffraction order).

4. Conclusions

The holographic Fourier transform lens that must ensure a linear mapping between patterns representing spatial frequencies in the Fourier plane is described. But the use of an off-axis spherical holographic lens for beam collimation that is burdened with phase error degrades its performance. Apart from locating the object transparency in the front focal plane of the Fourier transform lens, the image height in its back focal plane must be proportional to the sine of the incoming diffraction angle. The paper considers the effect of the third order aberration in the collimating beam and its influence on the quality of Fourier transformation results. In conclusion, we have presented that the Fourier transformation with aperture errors cannot take the null values of minima of the intensity distribution even for a higher order of diffraction pattern.

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