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Modelling of laser pumped intracavity frequency doubled laser*

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A model of intracavity frequency doubled laser pumped by laser beam was elaborated. The effects connected with mode matching, thermal lensing, pump induced diffraction losses, as well as parameters of frequency converter and geometry of cavity were considered. An analytical model of cavity with internal lens and thermal lensing was derived to optimize output characteristics. Two types of cavity (with and without internal lens) were analyzed and the best configurations with respect to II harmonic output power were found for both cases. The pump induced diffraction losses were found to limit output power of frequency doubled laser for high pump levels.

1. Introduction

The enormous interest of scientists has been focused on diode pumped frequency doubled laser (FDL) for the last 10 years (see, *e.g.*, [1]–[6]). The intracavity conversion scheme is one of typical methods for increasing conversion efficiency in such lasers. Although theory of intracavity FDL was formulated in 1970 [7], it has taken more than 20 years to build the practical, stable, highly efficient FDL's (see, *e.g.*, [4]–[6]). Despite a great number of theoretical as well as experimental works devoted to this topic that have been published to date some important problems still require additional elucidation. We intend to focus attention in this paper on one aspect, namely an analysis of the influence of cavity scheme parameters on conversion efficiency in intracavity FDL in the case of cw pumping and high thermal lensing. For that purpose, a model of cw intracavity FDL was developed, as presented in Sect. 2. In Section 3, a model of cavity with thermal lensing and pump dependent diffraction losses has been formulated. Section 4 presents numerical investigations of the performance of intracavity FDL's for two typical cavities. In the last chapter some conclusions are derived.

2. Model of intracavity frequency doubled laser

We intend to elaborate a simple theoretical model of intracavity frequency doubled laser enabling an analysis of the following effects:

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– pump dependent mode matching efficiency,
– frequency converter parameters including walk-off efficiency,
– several types of losses including pump dependent diffraction losses,
– geometry of cavity including thermal lensing.

We restrict our discussion to the simplest case of a single spatial and longitudinal mode, thus the problems caused by the sum frequency generation are out of our interest. Moreover, we assume a four-level scheme of laser action and near threshold approximation. In this case, the total cavity round trip losses γ can be divided into three components: linear round trip losses L_{lin}, pump dependent diffraction L_{dif} and nonlinear losses proportional to internal power on I harmonic $P_{1\omega}$ as follows:

$$\gamma = L_{\mathrm{lin}} + L_{\mathrm{dif}} + K_{2\omega} P_{1\omega} \tag{1}$$

where $K_{2\omega}$ denotes II harmonic conversion function (see, *e.g.*, [4], [8]) given by

$$K_{2\omega} = \frac{2\beta\eta_{\mathrm{woff}}}{A_{\mathrm{c}}} l_{\mathrm{c}}^2 K' \tag{2}$$

where: A_{c} denotes the mode area in nonlinear crystal, l_{c} – the length of nonlinear crystal, β – enhancement factor resulting from the interference of backward and forward flows on I harmonic wavelengths inside nonlinear crystal (see, *e.g.*, [2], [4], [7]), η_{woff} – the walk-off efficiency and K' – the II harmonic conversion parameter dependent solely on crystal data (see, *e.g.*, [4], [8]) as follows:

$$K' = \left(\frac{2\pi}{\lambda_{1\omega}}\right)^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{d_{\mathrm{eff}}^2}{n^3} \tag{3}$$

where: n is the refractive index of nonlinear crystal, d_{eff} – nonlinear coefficient of crystal and ε_0, μ_0 – dielectric and magnetic permeabilities of vacuum, respectively. To estimate the influence of walk-off efficiency we will use the simplified formula (see, *e.g.*, [4], [8]):

$$\eta_{\mathrm{woff}} = \frac{1}{1 + \frac{\rho l_{\mathrm{c}}}{\sqrt{2A_{\mathrm{c}}}}} \tag{4}$$

where ρ denotes the walk-off angle of nonlinear crystal.

To fulfil the energy conservation law in near threshold approximation under steady state condition, the total power dissipated inside cavity as a result of linear and nonlinear losses should be equal to pump power above threshold multiplied by excitation efficiency as follows:

$$\gamma P_{1\omega} = \eta (P - P_{\mathrm{th}}) \tag{5}$$

where: P – incident pump power, P_{th} – threshold pump power, η – excitation efficiency given by

$$\eta = \eta_{\mathrm{p}} \eta_{\mathrm{m}} \tag{6}$$

where η_m denotes the mode matching efficiency and η_p – the pumping efficiency given by

$$\eta_p = \eta_{abs}\eta_{del}\eta_{St} \tag{7}$$

where: η_{del} – delivering efficiency of the pump radiation from pump source to active medium, η_{abs} – absorption efficiency of pump radiation inside active medium, $\eta_{St} = \lambda_p/\lambda_{1\omega}$ denotes Stokes' efficiency equal to the ratio of pump wavelength λ_p to I harmonic wavelength $\lambda_{1\omega}$.

The mode matching efficiency η_m depends on spatial distribution of absorbed pump density and laser mode intensity inside active medium (see, *e.g.*, [9]–[11]). We will use the simplified formula for η_m derived by LAPORTA and BRUSSARD [10] for the case of end pumping geometry and circular Gaussian shapes of pump as well as laser mode distributions, as follows:

$$\eta_m = \frac{1+2m_p}{(1+m_p)^2} \tag{8}$$

where: $m_p = A_p/A_m$ and A_m is the laser mode area and A_p the pump area averaged over active medium. The threshold pump power P_{th} is defined in the following way (see, *e.g.*, [9], [10]):

$$P_{th} = (A_m + A_p)\gamma\frac{I_{sat}}{\eta_p} \tag{9}$$

where I_{sat} denotes the saturation power density for I harmonic wavelength given by

$$I_{sat} = \frac{h\nu_{1\omega}}{\sigma\tau} \tag{10}$$

where: h – Planck's constant, $\nu_{1\omega}$ – frequency of I harmonic, σ – emission cross-section and τ – lifetime of the upper laser level. Let us note that the threshold power depends on pump power via diffraction losses L_{dif} and on I harmonic power $P_{1\omega}$ via II harmonic nonlinear losses.

After substituting the losses γ given by formula (1) to energy conservation law (5) we obtain a simple quadratic equation for the I harmonic power $P_{1\omega}$ as follows:

$$K_{2\omega}P_{1\omega}^2 + (L_{1\omega} - \eta K_{2\omega}P_{sat})P_{1\omega} - \eta(P - L_{1\omega}P_{sat}) = 0 \tag{11}$$

where: $P_{sat} = (A_p + A_m)I_{sat}/\eta_p$ denotes the saturation pump power and $L_{1\omega} = L_{lin} + L_{dif}$ denotes the total losses for I harmonic wavelength. Knowing the I harmonic power $P_{1\omega}$ we can determine the II harmonic power $P_{2\omega}$ according to well known formula (see, *e.g.*, [8])

$$P_{2\omega} = \eta_{ext}K_{2\omega}P_{1\omega}^2 \tag{12}$$

where η_{ext} denotes extraction efficiency of the II harmonic radiation out of cavity.

Solving Eq. (11) with respect to $P_{1\omega}$ and substituting the result into (12) we obtained the final formula for the II harmonic power $P_{2\omega}$ as follows:

$$P_{2\omega} = \frac{\eta_{ext}}{4K_{2\omega}} [\sqrt{(L_{1\omega} - \eta K_{2\omega} P_{sat})^2 + 4\eta K_{2\omega} P} - (L_{1\omega} + \eta K_{2\omega} P_{sat})]^2. \tag{13}$$

The II harmonic conversion efficiency with respect to pump power is given by

$$\eta_{2\omega} = \frac{P_{2\omega}}{P} = \frac{\eta_{ext}}{4PK_{2\omega}} [\sqrt{(L_{1\omega} - \eta K_{2\omega} P_{sat})^2 + 4\eta K_{2\omega} P} - (L_{1\omega} + \eta K_{2\omega} P_{sat})]^2. \tag{14}$$

Let us note that both formulae (13), (14) are complicated functions of pump power, material data of active medium and nonlinear crystal, sizes of mode area inside active medium and nonlinear crystal. Thus, to analyze the FDL we propose to use two additional, much simpler merit functions, namely: K_p – pump-conversion factor, $L_{1-2\omega}$ – relative losses defined as follows:

$$K_p = \eta K_{2\omega}, \tag{15}$$

$$L_{1-2\omega} = \frac{L_{1\omega}}{K_{2\omega} P_{1\omega}}. \tag{16}$$

We should maximize the pump-conversion factor K_p whereas the relative losses $L_{1-2\omega}$ should be minimized. To achieve efficient intracavity frequency doubled output we should precisely examine cavity parameters, especially the thermal lensing magnitude and non-parabolic thermally induced aberration contents. Knowing these data, we have to find cavity parameters such that would simultaneously maximize K_p and minimize $L_{1-2\omega}$ for required pump level. Let us note that the cavity optimized with respect to I harmonic (*i.e.*, work with fundamental wavelength) may be quite different from the cavity optimized for II harmonic output.

3. Model of cavity with internal lens and thermal lensing

As was mentioned in Section 2, before starting an analysis of frequency doubling we have to know the fundamental mode sizes in active medium and in frequency converter, dependent also on pump power via thermal lensing effect. Let us restrict our discussion to a simple cavity with only one internal lens of optical power m placed at distance l_0 from the active medium and l_1 from the flat output coupler OC (see Fig. 1), respectively. An additional thermal lensing power denoted by M_t is located in the middle of active medium. Such a scheme is typical of the FDL with folded cavity of V- or L-shape of cavity. In particular case, when we put $m = 0$, we have a simple cavity with thermal lensing only. The round trip *ABCD* matrix parameters (starting from the output coupler plane OC) for such a cavity are given by

$$\begin{aligned} A &= (1-x-l_{eqv}M_t)(1-y) - m\, l_{eqv}, \\ B &= (2(1-x) - M_t l_{eqv}) l_{eqv}, \\ C &= -(1-y)(2m - (1-y)M_t), \\ D &= (1-x)(1-y) - (m + (1-y)M_t) l_{eqv} \end{aligned} \tag{17}$$

where: $x = ml_0, \quad y = ml_1, \quad l_{eqv} = (x+y-xy)/m.$ (18)

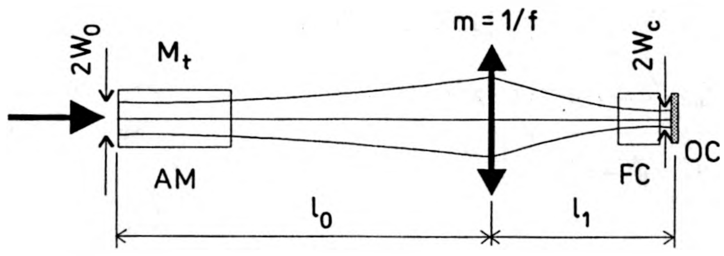

Fig. 1. Scheme of cavity with internal lens.

Then, following the well known *ABCD* method (see, *e.g.*, [12]), we can determine fundamental mode diameter at OC plane $2W_c$ and at active medium plane $2W_o$. Note that both radii W_o, W_c depend on pump power via thermal lensing optical power M.

3.1. Pump induced diffraction losses

As a result of non-homogeneous temperature profile inside active medium, induced by heat generation of absorbed pump power, the paraxial thermal lensing M_t, as well as additional non-paraxial phase aberration occur. In the case of homogeneous pump density, typical of circularly symmetric side pumping geometry, phase aberration is dominated by parabolic term, whereas in the case of end pumping scheme non-homogeneous pump distribution results in a more complicated shape of phase aberration. The exact phase aberration function can be found integrating optical path difference over active medium, with thermally induced refractive index change (see, *e.g.*, [11]) or solving the ray equation problem for such a type of thermally induced refractive index profile [13]. To describe this type of phase distortion a thermal axicon model of aberration was proposed [14] as follows:

$$W_{\mathrm{TA}}(x, T_a) = T_a \begin{cases} x^2 & \text{for } x < 1, \\ 2x - 1 & \text{for } x \geqslant 1 \end{cases} \tag{19}$$

where: $x = r/W_p$, r denotes the radius and T_a – the magnitude of aberration, given by

$$T_a = \frac{\beta_T W_p^2 M_t}{\lambda} \tag{20}$$

where β_T is the correction coefficient (in our calculations we take $\beta_T = 1/2$). It is worth noting that the parabolic component of thermal axicon aberration describes paraxial thermal lensing effect, whereas the linear term describes net phase aberration oustide pump area. Thus, to estimate an effect of non-parabolic phase aberration, the parabolic component was subtracted as follows:

$$W_{\mathrm{npTA}}(x, T_a) = T_a \begin{cases} 0 & \text{for } x < 1, \\ -(x-1)^2 & \text{for } x \geqslant 1. \end{cases} \tag{21}$$

For such a phase aberration function W_{npTA} we can determine Strehl ratio for incident

Gaussian beam with radius W_0 (see, *e.g.*, [15]) as follows:

$$\mathrm{SR}(a,b,T_a) = \frac{\left|\int_0^b \exp(i W_{\mathrm{apTA}}(ax,T_a))\exp(-x^2)x\mathrm{d}x\right|^2}{\left|\int_0^b \exp(-x^2)x\mathrm{d}x\right|^2} \tag{22}$$

where:

$$a = \frac{W_o}{W_p} = m_p^{-1/2}, \quad b = \frac{R_{\mathrm{rod}}}{W_o}, \tag{23}$$

R_{rod} denotes the radius of active medium. We have found in numerical experiments that no further approximations of formula (21) are allowed, because the magnitude T_a can be quite large. Knowing Strehl ratio we can calculate the diffraction losses L_{dif} (see, *e.g.*, [11]) as follows:

$$L_{\mathrm{dif}}(a,b,T_a) = 1-\mathrm{SR}(a,b,T_a). \tag{24}$$

The diffraction losses depend on pump power via dependence of W_p, W_o, T_a on thermal lensing M_t. However, it is interesting to examine the dependence of L_{dif} on the ratio of mode to pump radii a (see Fig. 2). Note that logarithmic diffraction

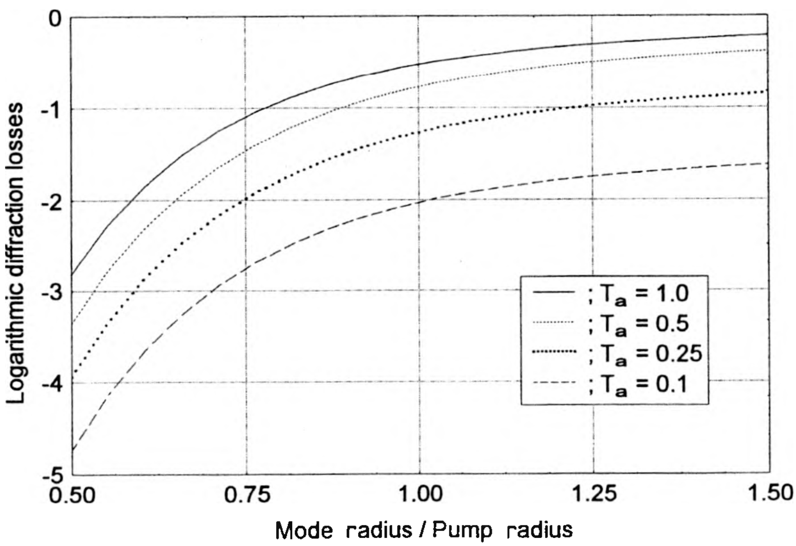

Fig. 2. Logarithmic diffraction losses versus the ratio of mode radius to pump radius W_0/W_p for several thermal axicon aberration magnitudes.

losses diminish nonlineary with a decreasing radius a. It means that, in contradiction to the mode matching efficiency optimization requirement, the mode radius should be less than the pump radius in the case of minimization of diffraction losses. Such a conclusion is consistent with the result of CLARKSON [16] obtained for a laser with high thermal load. However, in our case of intracavity FDL the situation is much more complicated because of nonlinear character of II harmonic losses.

4. Optimization of cavity parameters for frequency doubled laser

Let us limit the scope of our analysis to intracavity frequency doubling (1.064 μm → 0.532 μm) in a cavity consisting of Nd:YAG crystal as active medium, optional internal lens and frequency converter placed in the vicinity of output coupler plane. Knowing material data of nonlinear crystals we can find the best candidate for our particular application. Let us take a frequency converter made of KTP crystal for which parameter $K' = 2.1 \cdot 10^{-8}$ W^{-1} and walk-off angle $\rho = 4.5$ mrad (see, *e.g.*, [4]). Our aim is to find an optimized cavity configuration for a given pump unit emitting up to 15 W of pump power. Let us assume that the pump beam averaged over active medium has radius $W_p = 0.5$ mm. To start the analysis we should, moreover, know dependence between the exact thermal lensing function M_t and pump power P. The magnitude of thermal lensing depends on several technical parameters such as: diameter of pump caustics, sizes, dopant level and quality of active medium, sizes and quality of heat sink contacts, *etc.* As a rule, the function M_t is approximately linear with respect to P as follows: $M_t = a_t P + b_t$ for low and medium pump densities. To be close to our experimental situation, we assume that parameters of M_t are the following: $a_t = 0.7$ 1/m/W and $b_t = -1.8$ 1/m. Such values were obtained in separate measurements of thermal lensing carried out for Nd:YAG rod of 10 mm in length and 4 mm in diameter pumped by 10 W fiber coupled diode SDL 3450.

4.1. Optimization of cavity with thermal lensing only

In such a case we want to find the best cavity length with respect to maximum of the II harmonic output power for a given pump level. The dependence of the II

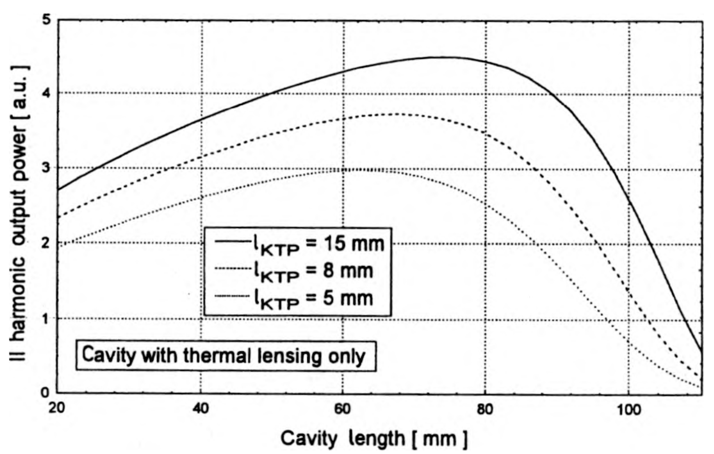

Fig. 3. Dependence of II harmonic output power on cavity length for pump power of 15 W and several lengths of KTP crystal.

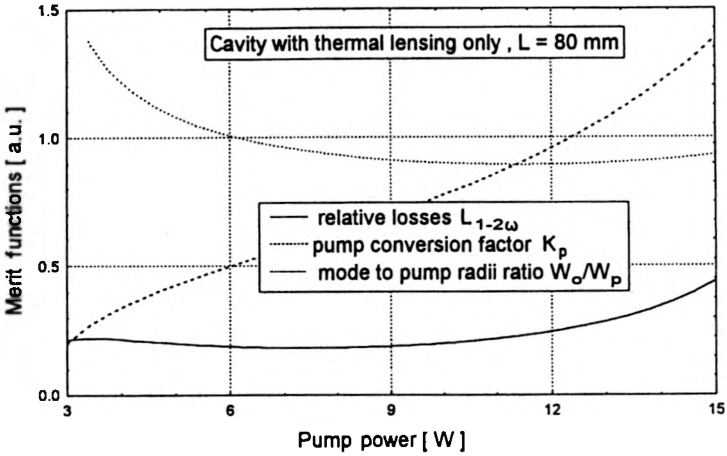

Fig. 4. Dependence of pump conversion factor K_p, relative losses $L_{1-2\omega}$ and the mode to pump ratio W_o/W_p on pump power for KTP crystal of 8 mm in length, and level of linear losses equal 0.01; cavity with thermal lensing only, cavity length 80 mm.

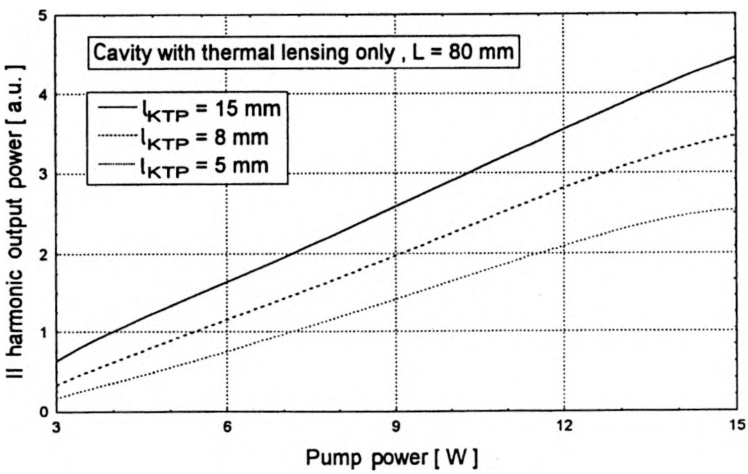

Fig. 5. The II harmonic output power vs. pump power for several KTP crystal lengths; level of linear losses equal 0.01, cavity with thermal lensing only, cavity length 80 mm.

harmonic power on cavity length is shown in Fig. 3. We can see that the best cavity length is approximately 80 mm for the given pump level of 15 W and it does not depend on crystal length. Changing the cavity length by 5 mm (about 10% of length) causes a 5% drop of the maximum, thus such a cavity is slightly sensitive to lenght changes. The dependences of merit functions K_p, $L_{1-2\omega}$ and W_o/W_p on pump power are presented in Fig. 4. The maximum value of relative losses $L_{1-2\omega} \sim 0.5$ for the highest pump level occurs for the ratio of beam radii $W_o/W_p = 0.95$. Such a level of diffraction losses does not cause any observable decrease of II harmonic output. As is shown in Fig. 5, the II harmonic output depends approximately on the pump power. Despite the decrease of walk-off efficiency, the II harmonic output significantly increases with crystal length. A direct conclusion from this analysis appear to be such that it is advisable to use nonlinear crystals of the greatest possible length.

4.2. Optimization of cavity with thermal lensing and internal lens

A short length of cavity required for maximum output for the II harmonic can be a serious drawback in practical applications. Thus, we tried to find a cavity configuration with folding spherical mirror (equivalent to internal lens) such that could be optimized with respect to II harmonic output. Let us assume the radius of curvature of the folding mirror to be $R = 200$ mm, *i.e.*, equivalent optical power of internal lens $m = 10\ \mathrm{m}^{-1}$. As a result of step by step optimization it was found that the optimized cavity length should be equal to 380 mm, with the mirror located in the middle of cavity. The dependence of II harmonic output on pump power is shown in Fig. 6. We can see that in the range of ± 5 mm changes of cavity length

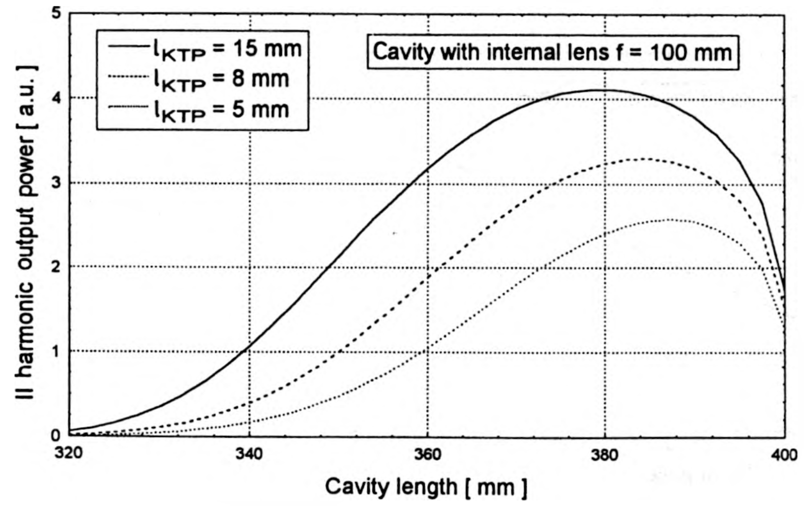

Fig. 6. Dependence of II harmonic output power on cavity length for pump power of 15 W and several lengths of KTP crystal, level of linear losses equal 0.02, cavity with internal lens of optical power $m = 10\ \mathrm{m}^{-1}$.

from the optimal value the output drops not more than 5%. Thus, such a solution is also inconsiderably sensitive to cavity length variation. The ratio of the optimal mode to pump radii, shown in Fig. 7, is also less than 1, and relative losses are less than 0.25 for maximum pump level. However, the pump conversion factor is nearly constant with respect to pump power and it is smaller compared to short cavity case. As is shown in Fig. 8, the II harmonic output power vs. pump power characteristics are more linear compared to the previous ones, with the slightly higher output power for maximum pump power.

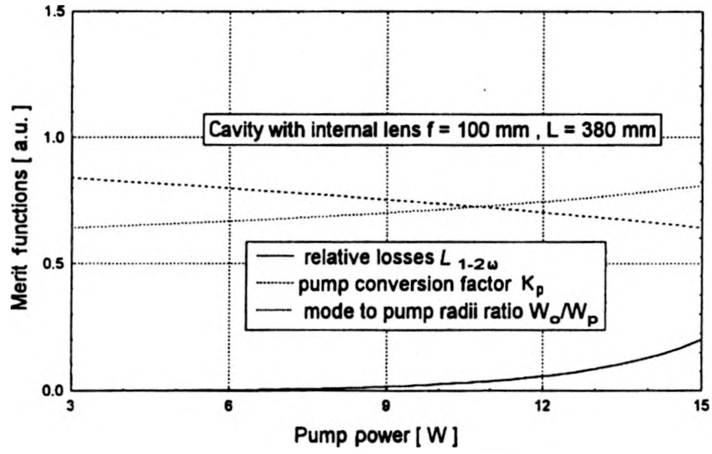

Fig. 7. Dependence of pump conversion factor K_p, relative losses $L_{1-2\omega}$ and the mode to pump radii ratio W_o/W_p on pump power for KTP crystal of 8 mm in length, level of linear losses equal 0.01; cavity with internal lens of optical power $m = 10\ \mathrm{m}^{-1}$ and length of 380 mm.

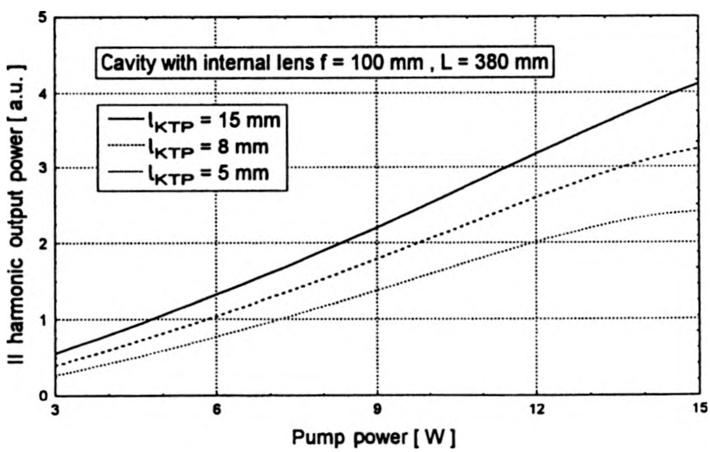

Fig. 8. The II harmonic power vs pump power for several KTP crystal lengths, level of linear losses equal 0.01, cavity with internal lens of optical power $m = 10\ \mathrm{m}^{-1}$ and length of 380 mm.

5. Conclusions

The model of intracavity frequency doubled laser, with cw laser pumping, was elaborated. The effects connected with mode matching, thermal lensing, pump induced diffraction losses, as well as parameters of frequency converter and geometry of cavity were taken into account. The *ABCD* model of cavity with internal lens and thermal lensing was derived to optimize output characteristics. The analytical model of pump induced diffraction losses assuming thermal axicon phase aberration was worked out. It was shown that for FDL the appropriate ratio of fundamental mode radius to pump radius in respect of minimization of pump induced diffraction losses should be less than 1. Two types of cavity (with and without internal lens) were analyzed assuming the Nd:YAG rod as active medium, KTP crystal as frequency converter and given, determined in separate experiments, dependence of thermal lensing on pump power. The best configurations with respect to II harmonic output power were found for both cases. The optimized length of cavity of 80 mm, for a simple resonator with thermal lensing only, may be too short for practical applications, however, Chen *et al.* [5] showed that such a short cavity laser could be very efficient for great heat load. The cavity with internal folding mirror of 200 mm radius and optimized length was shown to have the same performance as the previous one. The pump induced diffraction losses were found to be the main factor limiting performance of FDL for high pump levels. To optimize FDL it is necessary to determine in experiments the thermally induced pump aberrations for given pump parameters. The elaborated model can be a useful tool in analysis and optimization of several types of frequency doubled, coherently pumped lasers.

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