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BPOF composite filter optimized with a genetic algorithm

RAFAŁ KOTYŃSKI, KATARZYNA CHAŁASIŃSKA-MACUKOW

Institute of Geophysics, Faculty of Physics, Warsaw University, ul. Pasteura 7, 02–093 Warszawa, Poland.

We introduce a new composite Fourier-plane binary phase-only correlation filter (BPOF). The filter is defined with the use of a nonquadratic energy function and is optimized numerically with a variant of the genetic algorithm. A possible application of the filter could include automatic postcode recognition. The paper also contains an introduction to composite filtering and a short overview of the present state of technology in the field of coherent optical correlators.

1. Introduction

In the last years the optical pattern recognition techniques are gaining new attention, *e.g.*, [1] – [7]. It is sometimes expected that optical pattern recognition may replace in the near future the traditional electronic techniques in applications that either involve large quantities of data or put critical requirements on the speed of operation. Examples of such applications include searching through large databases of, *e.g.*, fingerprints or DNA sequences, military applications including target tracking or conducting misssiles, examination of satellite images, robotic vision and many other. In fact, already with the present technology, the optical correlators could in principle operate at the frame-rates not reachable to electronic digital signal processing.

The two basic architectures of the coherent optical processors were introduced by VANDER LUGT [8] and WEAVER and GOODMAN [9] over thirty years ago. In the eighties, the popularization of spatial light modulators (SLMs) made it possible to operate the correlators from a computer and since then the correlators could be considered as specialized signal processors that might speed-up pattern recognition applications. Semiconductor lasers, spatial light modulators with XGA resolution and operating frequencies that exceed 1 kHz, and the high resolution CCD cameras became widely available in the last decade. In effect, it is finally possible to construct optical correlators that work in real-time and could be competitive to digital electronics. The recent paper of COLIN *et al.* [10] includes the comparison of operating speeds reached by the optical correlators constructed after 1992.

Composite filtering [1], [3] is a powerful pattern recognition technique closely related to techniques used in neural networks. Composite filtering extends simple matched filtering to complicated classification problems. It allows us to construct a single filter from multiple arbitrary reference objects. Recently, many approaches to the synthesis of composite filters have been proposed. They differ in the significance that is assigned to the noise robustness, discrimination capability, the shape of the detection signal, optical efficiency, or other criteria [11].

In this paper, we will introduce a new composite filter. We define the filter as the solution to an optimization problem. For this purpose we will introduce the energy function, select the domain of optimization and, finally, optimize the filter numerically. For the energy function we chose the ratio between the largest correlation plane energy for the reference objects that should be rejected and the lowest correlation peak for the reference objects that represent the true targets. As we will show, this criterion gives sufficient control over the correlation signal shape and does not involve the common but unnecessary degress of freedom, such as arbitrarily fixed phase or amplitude values corresponding to training target objects. For the domain of optimization we select the binary coding domain available to ferroelectric spatial light modulators. In the present paper, the acronym BPOF is used for any filter coded in this domain, and is not attributed to the classical BPOF filter of Horner and Leger [12]. We perform the optimization [13] numerically using a variant of the genetic algorithm [14].

The paper is organized as follows. In the next section, we review some basic information about optical pattern recognition, and overview the current state of technology, we compare the operating speeds of optical and electronic image processors, and cite some of the characteristics of the latest reported correlator devices. In Section 3, we introduce the basic composite filters designed for use in optical correlators, including the conventional SDF, MVSDF, MACE, MINACE, OT-SDF, EOF and MACH filters. Section 4 is a short introduction to genetic algorithms. In Section 5, we introduce the new filter and, in Section 6, we optimize the filter numerically and demonstrate its performance. Finally, Section 7 contains some concluding remarks for future research.

2. Optical correlation: the speed, applications, implementations

Research in the field of optical correlation is reasoned by its potentially great speed. The actual frequency of operation of the correlators is limited by the electronic I/O elements, *i.e.*, the SLM, CCD and the rate at which the reference images are extracted from a mass storage, such as a hard disk.

Most correlators reported recently use the ferroelectric or twisted-nematic SLMs or photorefractive-crystal SLMs. The spatial resolution of most currently produced SLMs is in the range between 128×128 and 1280×1024 pixels. It is easy to verify that the calculation of a two dimensional discrete Fourier transform with use of the FFT algorithm for a 1024×1024 image, at the frequency of 1 kHz, requires the processing speed of about $4.2\cdot10^{10}$ complex multiplications per second, or almost $1.7\cdot10^{11}$ real multiplications per second (170 GFlops). To calculate the correlation between two 1024×1024 images at 1 kHz we would need the power of 340 GFlops. A similar correlation at 5 kHz would correspond to the speed of 1.7 Tflops. In principle, this speed could be obtained with the use of on-table optics with currently existing elements. Moreover, in the future, with the miniaturization of the set-up, this speed might be further increased, because the current limitations are technological and not theoretical as in the case of electronic processing.

Speed is a strong advantage of optical correlation but probably the only one. Optical correlators are analog devices – the accuracy of one or even several percent does not seem to be attractive in comparison with the floating point digital arithmetic. Optical correlators are also not so compact as most electronic systems. However, a continuous trend for miniaturization can be observed. For example, a 0.8×1.7 cm^2 correlator integrating a ferroelectric SLM, two polarizers, a diffractive lens and a camera was presented in the OE Reports in July 1999.

The reported operating speeds of optical correlators that were actually built, or are commercially available are significantly lower than the 1.7 Tflops mentioned and in practice reach about 10^{10} operations per second (10 GOps). The speeds of the optical correlators made since 1992 in Thomson-LCR, Optical Institute of Sweden, Litton, Lockeed, QuantaImage, and Jet Propulsion Laboratory are compared in the recent paper of COLIN *et al.* [10], who report a 10GOps nonlinear JTC with a nonlinear BSO crystal and a single 256×256 ferroelectric SLM working at 1 kHz. A correlator of similar capabilities (256×256, binary, >1 kHz with use of the on-board memory) is advertised on the web by Boulder Nonlinear Systems (BNS) [15] as a plug-in card for PC computers (there is also a newer model with a lower resolution of 128×128 pixels but with an analog modulation – perhaps the first correlator with analog modulation operating at over 1 kHz). It is also interesting to note that the correlator from the Optical Institute of Sweden [16] (reconfigurable JTC/Vander Lugt architecture, 256×256 ferroelectric SLM operating at the frequency of 220 Hz – which is the limiting speed of the CCD, total size: 21×28 cm) is made available for collaborating laboratories for operation through Internet, at the web address [17] (a demonstration page is available for unregistered visitors).

The newest digital signal processors are capable of calculating up to about over five 1024×1024 correlations per second. Most civil applications do not require greater speeds. Optics is interesting for applications that involve huge computational powers. Some examples are seeking through large databases with fingerprint images [10] or DNA sequences [18], robot vision, or the reconstruction of synthetic aperture radar (SAR) images [19].

Optical correlation is also interesting for military applications. An example is the Standard Missile (SM) optical correlator co-processor program (OCCP) sponsored by US Navy [20]. The aim of the project is to develop an optical correlator for the processing of the signal from an infrared imaging seeker. The seeker is used in the new versions of SM-2 Surface to Air Missiles in the last seconds of flight. Because of the extreme closing velocities it is not possible to process the incoming data by means of electronic DSP.

It is an open question whether in the next years the optical correlators would become more attractive than digital signal processing. Currently digital processors are widely used in various applications, and optical correlators are still considered as experimental or prototype devices. However, it is widely expected that at some time the optical technology must at least partly replace the electronics. It is also clear now that it will be possible to implement high-speed correlation-based pattern recognition methods, by some means — either optically or digitally. This justifies the growing interest in the development of correlation algorithms that may be observed in optical literature, and to which we contribute in the present paper.

3. Composite filtering

The basic problems addressed in pattern recognition are related to image detection, localization and classification. The factors which are considered in the design of algorithms are:

1. The robustness to detector noise or background clutter compromised with tolerance to distortions of the target object [21].

2. Invariances against image transformations [22], *e.g.*, rotation, and scale invariance.

3. The hardware capabilities (computational cost at the design time and the possibility of efficient optical or digital implementation with existing hardware at the required speed).

4. The stability of the algorithm, and its robustness to assumptions (how the algorithm would work in other realistic situations than those assumed during the design time).

Correlation based pattern recognition methods may be roughly subdivided into simple matched filtering, and composite filtering (synthetic discriminant function filtering). Simple filters are matched to a particular reference object. The unknown location of a reference object $\mathbf{r}$ in the input signal $\mathbf{s}$ is determined by finding the location $\tilde{y}$ that maximizes the correlation between $\mathbf{s}$ and a filter $\mathbf{h}$ (we use the popular discrete vector notation to represent optical signals)

$$\tilde{y} = \arg\max_{y}(|c_y|^2), \tag{1}$$

$$\mathbf{c} = \mathbf{s} \star \mathbf{h}^*, \text{ where } \mathbf{h} = \mathbf{f}(\mathbf{r}) \tag{2}$$

where $\star$ denotes the complex correlation, and arg max(.) returns the value of y for which the function's argument is maximized. The filter $\mathbf{h}$ contains the information about the reference target $\mathbf{r}$ — we say that it is matched to the target object. In the simplest case of the classical matched filtering (CMF) [23], the impulse response of the filter is proportional to the reference object $\mathbf{r}$.

Assuming one filter for every possible reference object could be very inefficient. Alternatively, a single composite filter can be calculated for a class of reference objects (or perhaps the single object distorted or transformed in various possible ways, or viewed from various directions). The tutorial paper of KUMAR [1] reviews the classical composite filters and many related ideas. Here we will recall very briefly some of the most important solutions.

Suppose, we have a set of reference images $\{\mathbf{r}^{(\theta)}:\theta = 1 \ldots \theta_{\max}\}$. The reference images can be, *e.g.*, rotated, scaled or distorted versions of a single object, or can be divided into subclasses, *e.g.*, with two classes composed of the target and non-target objects. Filtering with a composite filter discriminates against different classes and should at the same time detect the location of the target object.

3.1. Synthetic discriminant function filters

Most correlation filters for pattern recognition are defined as the solution to a certain optimization problem. The best known composite filters are derived by optimization of a quadratic criterion with a linear constraint. The linear constraint consists of a set of underdetermined independent linear equations that define the correlation signal c_θ corresponding to every reference object $\mathbf{r}^{(\theta)}$ (objects $\mathbf{r}^{(\theta)}$ must be linearly independent and $\theta_{max} \ll d$, where d is the number of pixels in the images). There is no general method for the determination of the optimal values of c_θ. In particular, intensity detectors lose the phase-information, so the complex arguments of c_θ might be set independently for every reference $\mathbf{r}^{(\theta)}$ or class. The synthetic discriminant function (SDF) filter $\mathbf{h}$ is found as the solution to the linear equations, which additionally minimizes a criterion $\varepsilon(\mathbf{h})$ written as a quadratic form $\mathbf{h}^\dagger\mathbf{A}\mathbf{h}$, where $\mathbf{A}$ is a positive definite nonsingular matrix (in practice also a Toeplitz one)

$$\varepsilon(\mathbf{h}) = \mathbf{h}^\dagger \cdot \mathbf{A} \cdot \mathbf{h}, \text{ with } \mathbf{X}^\dagger \cdot \mathbf{h} = \mathbf{c} \tag{3}$$

where $\mathbf{X} = [\mathbf{r}^{(1)}, \mathbf{r}^{(2)}, \ldots \mathbf{r}^{(\theta_{max})}]$.

The analytic expression for the filter $\mathbf{h}$ can be found with use of the Lagrange's multipliers method

$$\mathbf{h} = \mathbf{A}^{-1} \cdot \mathbf{X} \cdot (\mathbf{X}^\dagger \cdot \mathbf{A}^{-1} \cdot \mathbf{X})^{-1} \cdot \mathbf{c}. \tag{4}$$

Depending on the choice of matrix $\mathbf{A}$, it is possible to obtain various criteria $\varepsilon(\mathbf{h})$ that lead to filters $\mathbf{h}$ that strongly differ in properties. The conventional-SDF filters were first introduced by HESTER and CASASENT [24]. Kumar introduced the minimum variance SDF (MVSDF) filters [25], by minimizing the mean-square-error (MSE) of the detection signal when the input noise is signal independent and stationary with the known power spectral density $\hat{\mathbf{p}}$. MAHALANOBIS [26] optimized the correlation-plane-energy (CPE) criterion averaged over the reference objects in order to get high and narrow correlation peaks for all the target objects. The filters obtained in this way are called the minimum average correlation energy filters (MACE). Averaging can involve equal weights ($\alpha_\theta = 1$), or weights that depend on the probabilities of the occurrence of the reference objects in the input signal ($\alpha_\theta = P_\theta$) – if available. The values of α_θ may also be found iteratively, to satisfy a MINI-MAX type criterion on the CPEs obtained with each of the reference objects [26].

Combined optimization of MSE and average CPE is more desired in practice, than the optimization of just one of these criterions. For this purpose REFREGIER [27] proposed an optimal-trade-off approach for these two criteria. Next, RAVICHANDRAN and CASASENT [28] introduced the MINACE filters.

All the filters enumerated in this subsection have the same form, given with Eq. (4), and only differ by the choice of matrix $\mathbf{A}$. In the spectral domain matrix $\mathbf{A}$ takes the form $\hat{\mathbf{A}} = \mathbf{F}\mathbf{A}\mathbf{F}^{-1}$ (where F is the matrix of the two-dimensional discrete Fourier transform, and the dash indicates the spectral representation). For the filters that we discussed $\hat{\mathbf{A}}$ is always diagonal and takes the following forms:

$$\begin{aligned}
&\hat{\mathbf{A}} = I && \rightarrow \text{conventional SDF}\\
&\hat{A}_{k,k} = \hat{p}_k && \rightarrow \text{MVSDF}\\
&\hat{A}_{k,k} = \Sigma_\theta \alpha_\theta \cdot |\hat{r}_k^{(\theta)}|^2 && \rightarrow \text{MACE}\\
&\hat{A}_{k,k} = \max(\hat{p}_k, |\hat{r}_k^{(1)}|^2, |\hat{r}_k^{(2)}|^2, |\hat{r}_k^{(3)}|^2, \ldots) && \rightarrow \text{MINACE}\\
&\hat{A}_{k,k} = \mu \cdot \hat{p}_k + \frac{(1-\mu)}{\theta_{\max}} \Sigma_\theta |\hat{r}_k^{(\theta)}|^2, \ \mu \in [0,1] && \rightarrow \text{OT-SDF}
\end{aligned} \tag{5}$$

Finally, it is interesting to note that if only a single target object is considered, *i.e.*, $\theta_{\max} = 1$, the conventional-SDF, MVCA, MASDF and OT-SDF reduce to the well known CMF – for white noise, CMF – for correlated noise, inverse filter (IF) and the optimal trade-off filters (OTF) [27], respectively.

3.1.1. Entropy optimized filters

In a different approach, FLAISHER, MAHLAB and SHAMIR [29] proposed to optimize composite filters in terms of the spatial entropy (considered as a heuristic criterion with no statistical background). The energy function $\varepsilon(\mathbf{h})$ which defines the entropy optimized filters (EOF) is equal to the difference of the normalized correlation entropies for objects to be rejected and objects to be detected, and is the following:

$$\varepsilon(\mathbf{h}) = \sum_{\theta \in \text{objects to reject}} S(\mathbf{c}^\theta) - \sum_{\theta \in \text{objects to detect}} S(\mathbf{c}^\theta) \tag{6}$$

where $S(\mathbf{c}^\theta) \equiv \sum_k c_k^\theta \ln(c_k^\theta)$ and $\mathbf{c}^\theta = \dfrac{|\mathbf{r}^\theta \star \mathbf{h}|^2}{\|\mathbf{r}^\theta \star \mathbf{h}\|^2}$.

Optimization must be performed numerically with use of, *e.g.*, simulated annealing or genetic algorithms. Even for the phase-only (POF) or binary -phase-only (BPOF) coding domains, the optimization was very time consuming, and the study of EOFs was not continued in later literature. However, the idea is worth citing, because the proposed criterion described the shape of the entire correlation signal, and at the same time did not impose any unnecessary constraints on the peak location and height, or amplitude or phase values at any point of the correlation signal.

3.2. Mean square error-SDF filters

A quadratic energy function that controls the correlation plane shape was proposed by KUMAR [30]. It is defined as the mean-square-error between the desired correlation shapes $\mathbf{c}^\theta$ and actual correllation shapes averaged over the set of reference objects $\mathbf{r}^\theta$

$$\varepsilon(\mathbf{h}) = \sum_\theta \|\mathbf{r}^\theta \star \mathbf{h} - \mathbf{c}^\theta\|^2. \tag{7}$$

The resulting filter is similar to other SDF filters and is called MSE-SDF. In particular with $\mathbf{c}^\theta = 0$, the above criterion is equivalent to the average CPE which is optimized with the MACE filter.

3.3. Maximum average correlation height filters

The design of the SDF filters, such as the MSE-SDF, includes unnecessary hard constraints on the correlation signal value at the origin (or other chosen points). MAHALANOBIS *et al.* [31] proposed to release these constraints and introduced the maximum average correlation height (MACH) filters, closely related to the MSE-SDF and MACE filters. The MACH filters depend only on the second order statistics calculated from the training images belonging to the two classes — one to be detected and the other to be rejected. The MACH filters maximize the ratio between the height of the average correlation peak for the target class $|\mathbf{h}^\dagger \mathbf{m}|^2$ (where $\mathbf{m}$ is the average of the target class training images), to the sum (possibly weighted, in the optimal trade-off approach) of the average CPE for the class to reject $\mathbf{h}^\dagger \mathbf{D}\mathbf{h}$ and the so-called average similarity measure (ASM) $\mathbf{h}^\dagger \mathbf{S}\mathbf{h}$ of the target class

$$\varepsilon(\mathbf{h}) = \frac{|\mathbf{h}^\dagger \mathbf{m}|^2}{\mathbf{h}^\dagger \mathbf{S}\mathbf{h} + \mathbf{h}^\dagger \mathbf{D}\mathbf{h}} \tag{8}$$

where: $\mathbf{m}$ — mean image from the target class, $\hat{S}_{k,k}$ — the average power spectrum of the target class measured relatively to $\mathbf{m}$, *i.e.*, the mean over θ of $|\hat{\mathbf{r}}^\theta - \hat{\mathbf{m}}|^2$, $\hat{D}_{k,k}$ — the average power spectrum of the class to be rejected.

Contrary to the other SDF filters, the MACH filter which optimizes (8) may be directly calculated in the frequency domain, without the need of matrix inversion

$$\hat{h}_k^{\mathrm{MACH}} \simeq \frac{\hat{m}_k^*}{\hat{S}_{k,k} + \hat{D}_{k,k}}. \tag{9}$$

Minimization of the ASM in the MACH filter optimizes the correlation plane shape in such a way that in the MSE sense this shape varies less for all the images from the target class.

4. Genetic algorithms

Genetic algorithms (GA) [32] form a powerful but computationally demanding family of optimization methods. In the next section, we will use a variant of a genetic algorithm to optimize a composite correlation filter defined with an energy function of the form specially chosen to speed-up the implementation of the GA.

The idea of the genetic algorithms is inspired by the natural evolution, which can be treated as a natural optimization process. Contrary to most other optimization methods, with the GA, a whole population of solutions takes part in the optimization in parallel. Although the GA are just a class of optimization algorithms, traditionally a specific terminology that underlines their biological origin is used for their description. The optimized energy function is called the fitness, the set of solutions is called the population, the binary represented solutions are called chromosomes, their elements (usually single bits) are called genes, *etc.* The genetic algorithms have many variants, usually oriented either to solve particular kinds of optimization problems [14] or to model particular aspects of the natural evolution [32].

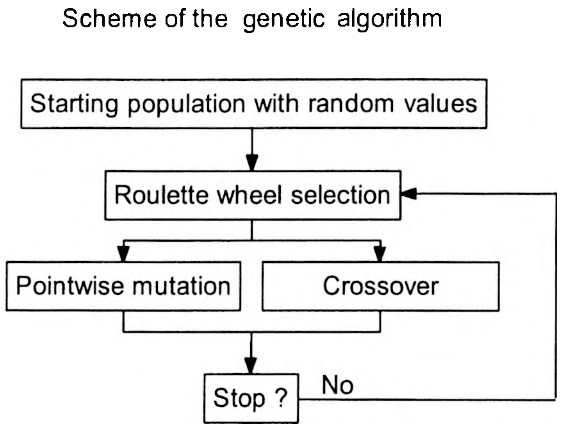

Fig. 1. Generic scheme of a genetic algorithm.

Figure 1 illustrates how a typical genetic algorithm works. The starting population of chromosomes is usually chosen randomly. Then, in every generation, with the "roulette wheel selection", a new population is created and replaces the old one. The probability of keeping a chromosome in the next generation is usually proportional to the (normalized) fitness value that corresponds to that chromosome. The purpose of the selection is to leave only the best solutions from the preceding population. The indeterministic selection helps to keep the diversity of the population. New solutions are introduced in every generation by the mutation, cross-over or certain other so-called genetic operators.

Optimization with the GA is simple, might be easily implemented on parallel machines if one has such, and can be used to solve difficult problems, with non-convex and non-differentiable fitness functions. The main problem with the GA is their low speed. However, here we will be interested in a class of criteria for which the effect of the point mutation on the change of the value of the criterion can be evaluated with only a few arithmetic operations. In effect, the convergence of the algorithm is obtained very quickly.

5. BPOF composite filter for a classification problem with 2-classes

We will consider a detection and classification problem with two classes [13]. We assume that the reference objects in the two classes are the digits $0-4$ and $5-9$, respectively. These digits are written as binary images with 64×64 pixels (see Fig. 2). Digits $5-9$ (class I) should produce a detection signal, while digits $0-4$ (class II)

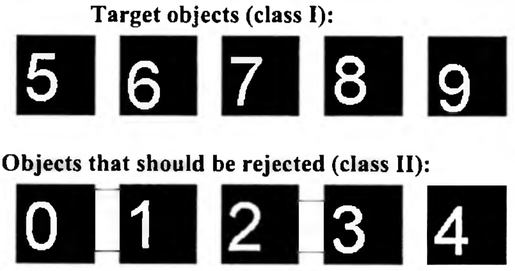

Fig. 2. Training objects used in the optimization process.

should be rejected. Our aim was to propose and optimize a composite filter (BPOF) that could discriminate objects from the class I against those from the class II. Automatic postcode recognition could be based on a cascade of filters optimized in this way. Although a simple matched filter might be used for each of the 10 digits, but an approach based on classification is more efficient as concerns the number of correlations to be calculated – 4 instead of 10. The number of reference images for each of the classes could be easily increased, *e.g.*, using a database of handwritten numbers. However, here we will only focus on the optimization of the BPOF composite filter for the already defined set of reference objects that are shown in Fig. 2.

In the current classification problem, it is not possible to apply the composite filters that we overviewed because we want to obtain a binary filter for use with a ferroelectric SLM.

We decided to select a more complex energy function than is used in the SDF filters, and optimize the filter numerically. We maximized the ratio between the squared modulus of the lowest correlation signal for objects from the class I to the highest correlation plane energy for objects from the class II. With use of the Parseval's theorem this ratio can be simply written in the spectral domain. The energy function to be minimized is the inverse of this expression and is equal to

$$\varepsilon(\hat{\mathbf{h}}) = \frac{\displaystyle\max_{\theta \in \text{objects to reject (class II)}} \sum_k |\hat{h}_k^* \hat{r}_k^\theta|^2}{\displaystyle\min_{\theta \in \text{objects to detect (class I)}} \Big|\sum_k \hat{h}_k^* \hat{r}_k^\theta\Big|^2}. \tag{10}$$

Our criterion has several practically important advantages. Because we use the min(.) and max(.) functions, the optimization always tends to improve the lowest correlation signal for class I, and to decrease the largest correlation plane energy for class II. In effect, after the filter is optimized, nearly equal correlation signals will be obtained for class I, and very similar correlation plane energies for class II. Our energy function does not impose unnecessary constraints on the exact value of intensity and phase of the correlation signals. Moreover, if we optimize the filter in the Fourier domain, the energy function is simple to evaluate – there is no need to calculate any Fourier transforms to find its value. If a single pixel of the filter is modified in an iterative optimization procedure (*e.g.*, by a mutation in a GA), the number of numerical operations needed to find the new value of the energy function is proportional to the number of reference images, and does not depend on their dimensions. After making this observation, the numerical optimization of the proposed energy function can be implemented in a very efficient way.

6. Optimization of the filter

We optimized the filter [13] with a genetic algorithm. The GA were already used for optimization of correlation filters and other diffractive elements with good results, *e.g.*, [33] – [35], however, the size of the optimized element was much below the 64×64 pixels that we have here. We have chosen a version of a GA with point mutation, proportional selection and a small probability of cross-over. The selection was forced not to eliminate the currently best solution from the population. The genetic operators that we used are schematically illustrated in Fig. 3. We varied the population size between 10 and 100 chromosomes. The size of about 20 chromosomes seemed to be the most reasonable to us – with a smaller population the optimization often got stacked at a local maximum. We also tried with a population of just one chromosome. Then the GA becomes similar to simulated annealing [36]. Then the optimization usually got stacked before at a local maximum, but if the procedure was repeated several times, the best result was similar to the result obtained with a larger population. The advantage of simulated annealing is that it is easier to implement and not so memory demanding as the GA. We also tried to implement a GA with no cross-over. Elimination of the cross-over did not affect

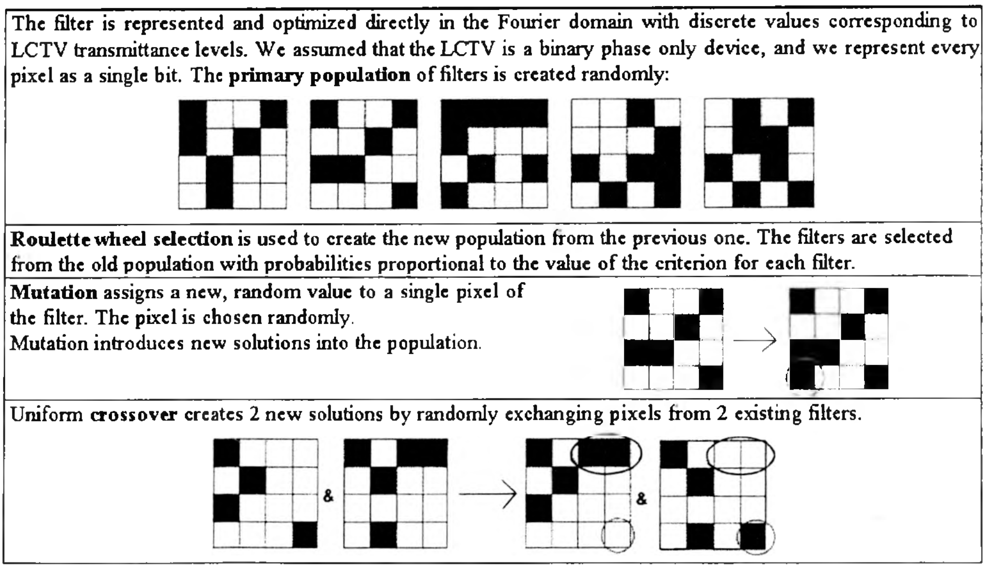

Fig. 3. Description of the binary representation of the filter and genetic operators used in our variant of the genetic algorithm.

the results and significantly increased the optimization speed. The algorithm converged in a few minutes on a Pentium 100 Mhz computer. A more systematic comparison of the optimization algorithms is out of the scope of this article, especially that there exists a large number of their variants and always there are some free parameters that can be chosen in many ways.

We will now discuss some of the results of the simulations. Figure 2 illustrates the two classes of reference objects that we assumed to find a binary filter $\hat{h}$ that maximizes the criterion from Eq. (10). After optimizing numerically the filter, we tested its performance on the training objects. In Figure 4, we compare the energies of the correlation peaks obtained with every reference object. Below we show the perspective view of the correlation planes obtained with a selected target-object and an object for rejection.

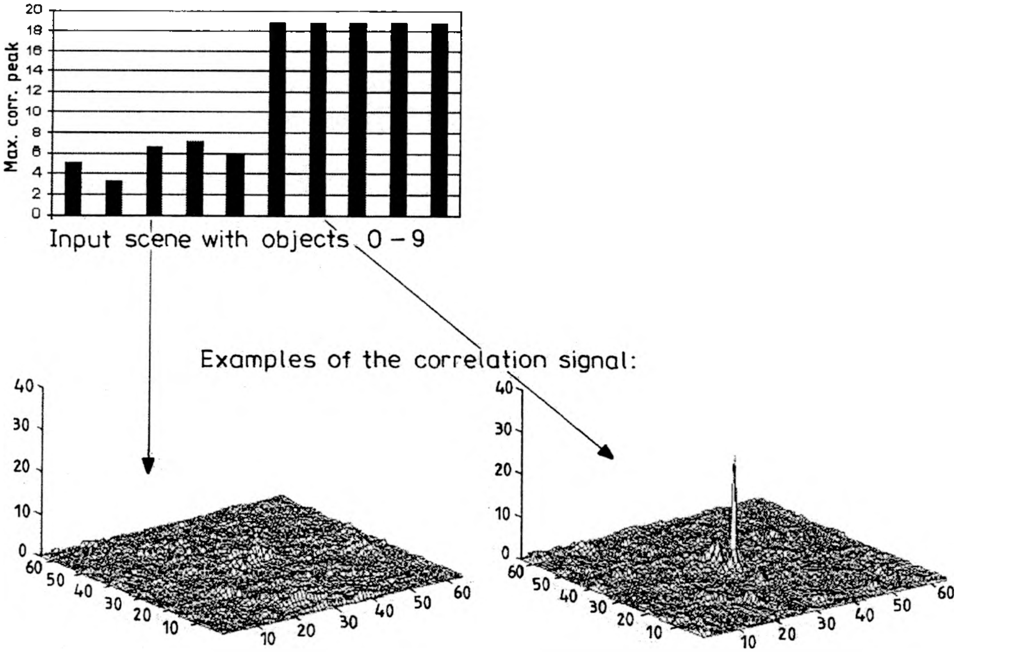

Fig. 4. Top: the maximal value in the correlation plane obtained with the optimized filter for each of the ten reference images. Bottom: examples of the correlation plane, when the input contains a false object (2) or a target object (6).

We notice that thanks to the appropriate choice of the fitness function that includes the nonlinear functions min (.) and max(.) we were able to achieve a filter that produces very uniform correlation signals for all the target objects from class I. We may contrast this result with a typical situation, when a SDF filter is calculated analytically from Eq. (4) and afterwards is projected onto the coding domain available to the SLM. For example, we verified the performance of the MACE filter projected onto the binary domain, and we obtained a significantly lower uniformity (see Fig. 5).

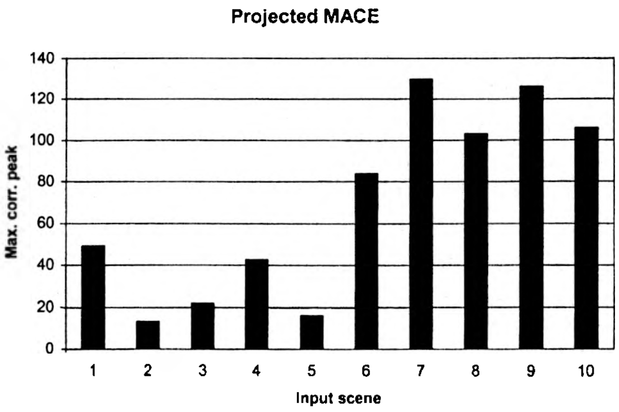

Fig. 5. Values of the correlation signals corresponding to the ten target objects, obtained with a MACE filter directly projected onto the binary coding domain.

The examples of the correlation planes plotted in Fig. 4 indicate that our filter has good discrimination properties and produces high and narrow detection signals for the target objects.

The criterion that we optimized could be easily modified to include a measure of noise robustness. Other (discrete) coding domains may be used, depending on the characteristics of the SLM. We would also impose a condition on the rotation symmetry of the filter to achieve rotation invariant recognition. Other mdified versions of the genetic algorithm or simulated annealing could be easily proposed. We intend to continue the research in some of these directions in the future.

7. Conclusions

Our main conclusion is that there exists a class of energy functions that give composite filters of interesting properties, and that can be very efficiently optimized with numerical methods such as the genetic algorithms or simulated annealing – which are usually considered to be compositionally expensive.

The speed of optimization is reasonable because the effect of point mutation on the change of the energy (fitness) function can be computed in just a few arithmetic operations. In fact, for any energy function that depends on the filter **h** through expressions such as: $\sum_k |\hat{h}_k^* \hat{r}_k^\theta|^2$ or $\sum_k \hat{h}_k^* \hat{r}_k^\theta$ or similar, the point mutation can be evaluated in a fast way, provided that the previous values of these sums are kept in the computer memory. The energy function may then depend on these sums in any way, and may include nonlinear or non-differentiable operations such as min or max or other.

An appropriate construction of the energy function lets us eliminate the hard constraints on the values or phases of the correlation peaks, at the same time giving more control of the correlation plane for each input object than, *e.g.*, we have in the MACE filters. Our numerical experiment based on the energy function (10) indicates that numerical optimization with quite arbitrary reference objects and realistic image dimensions leads to an interesting composite filter, and the optimization time is acceptable.

Further research is required to compare the performance of the new filter with other composite filters in terms of other criteria such as the SNR or light efficiency. We also intend to introduce other energy functions, and *e.g.*, include the information about the power spectral density of input noise into the optimized energy function.

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