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Theoretical model of thermodetection system

Rafał Dulski, Henryk Madura, Tadeusz Niedziela, Zbigniew Sikorski

Military University of Technology, ul. Kaliskiego 2, 00-908 Warszawa, Poland.

A theoretical model of thermodetection system based on the linear filter theory is proposed. The modulation transfer function of a thermodetection system is expressed as a product of the modulation transfer functions of its subsystems. A numerical technique for computing the optical transfer function of an objective based on the wave optics is proposed, which takes into account spectral characteristics of the imaging chain. Infrared images obtained with the model simulate well the experimentally obtained images.

1. Introduction

The modelling and analysis of thermal imaging have fundamental significance for construction and investigation of thermodetection systems of automatic target recognition systems. This goal is fulfilled by, *e.g.*, the model of thermodetection system (MTS) proposed in this paper.

Most of the models of thermodetection systems known [1]–[3] assume that the thermodetection system is a sequence of subsystems converting the signal in a linear way. It is also assumed that the imaging is isoplanatic and deterministic.

Depending on the foreseen application of the model, different factors influencing distortion of the signal are taken into account. Each of these factors can be described using its modulation transfer function (MTF_n). The resulting modulation transfer function of the whole system (MTF_{syst}) is a product of the modulation transfer functions (MTF_n) of its individual subsystems ($\mathrm{MTF}_{syst} = \mathrm{MTF}_1 \cdot \mathrm{MTF}_2 \cdot \ldots \cdot \mathrm{MTF}_n \cdot \ldots \cdot \mathrm{MTF}_N$) [4].

The process of thermal imaging leads inevitably to the loss of a portion of information being included in the thermal scene (the thermal target together with the background). This can be due to the following reasons:

– The thermal imaging is influenced by spectral characteristics of transmission of the atmosphere, objective of the thermodetection system, and spectral responsiveness of the detection module.

– The objective is not a perfect optical system (the image of a target's point is not a point but a spot; the image can also be distorted geometrically).

– The detector has finite dimensions, which causes integration of the incident radiation over its surface. As a result, the information at one point of the thermal image originates from many points of the thermal scene.

— The electronic subsystem distorts the signal from the detector. The distortions are described by the amplitude-phase frequency characteristic of the electronic subsystem.

The diagram of the process of creating the model thermodetection system and its capabilities is presented in Fig. 1. In the MTS, the thermal image is determined on the basis of the ideal image of the target, parameters of the thermodetection system, and conditions of the measurement being modelled.

It is assumed that the ideal image of the target is created in a perfect optical system and differs from the target only in scale (equal to the lateral magnification of the system) and location in the optical system [5]. The ideal image can also be obtained experimentally (by means of a high-quality thermovision measuring camera) or theoretically (by means of the faceted thermal target model, FTTM [6]).

The thermal image of a target is most often rather strongly distorted in relation to the ideal image. These distortions are described by the point spread function (PSF) of the system.

As a result of applying the MTS, one obtains theoretical distribution of luminance in the thermal image. The authors of this paper performed verification of the MTS by comparison of theoretical distributions with distributions of luminance obtained experimentally.

2. Assumptions and structure of the model

The process of thermal imaging is a train of transformations of the image (the angular spatial distribution of radiometric quantities, *e.g.*, the traget's luminance) or electric signal by subsystems of the thermodetection system.

The analysis of the process of thermal imaging can be easily carried out in the domain of angular spatial frequencies. This consists in decomposition of the target image ($J(x, y)$) and ideal image of the target ($I(x, y)$) into sine functions of angular locations x and y of different period and phase. The reciprocals of these periods are called the angular spatial frequencies (f_x, f_y) in x and y directions, respectively. To simplify calculations, real functions are replaced with their complex representations — the analytic signal [7], [8]. Decomposition of the ideal image of the target ($I(x, y)$) into analytic signals representing sinusoidal functions takes the form of the inverse Fourier transformation (FT^{-1})

$$I(x, y) = \mathrm{FT}^{-1}[F(f_x, f_y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(f_x, f_y) e^{i2\pi(f_x x + f_y y)} \mathrm{d}f_x \mathrm{d}f_y \tag{1a}$$

where $F(f_x, f_y)$ is the spectrum of the ideal image.

The inverse relation also takes place

$$F(f_x, f_y) = \mathrm{FT}[I(x, y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x, y)\, e^{-i2\pi(f_x x + f_y y)} \mathrm{d}x\, \mathrm{d}y \tag{1b}$$

where FT denotes the direct Fourier transformation.

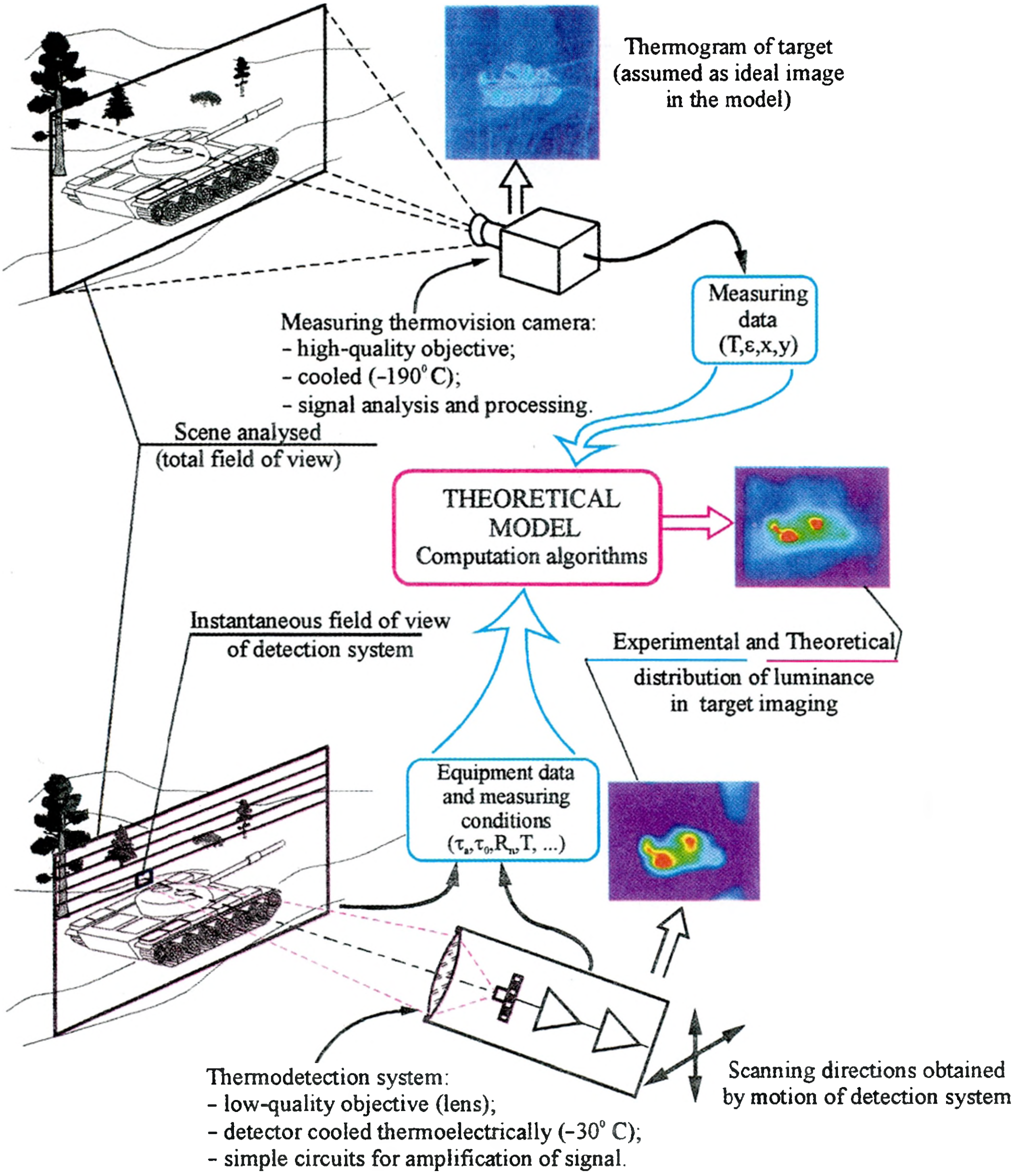

Fig. 1. Diagram of construction of the theoretical model of thermodetection system.

Analogously, the image ($J(x,y)$) and its spectrum ($G(f_x, f_y)$) are connected with a pair of coupled Fourier transformations:

$$J(x,y) = \mathrm{FT}^{-1}[G(f_x, f_y)] = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G(f_x, f_y)\, e^{i2\pi(f_x x + f_y y)}\, \mathrm{d}f_x \mathrm{d}f_y, \tag{1c}$$

$$G(f_x, f_y) = \mathrm{FT}[J(x,y)] = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} J(x,y)\, e^{-i2\pi(f_x x + f_y y)}\, \mathrm{d}x\, \mathrm{d}y. \tag{1d}$$

The relation between the spectrum of the image ($G(f_x, f_y)$) and the spectrum of the ideal image ($F(f_x, f_y)$) has the form

$$G(f_x, f_y) = \mathrm{OTF}(f_x, f_y)\, F(f_x, f_y) \tag{1e}$$

where $\mathrm{OTF}(f_x, f_y)$ is the optical transfer function of the system which performs linear, stationary, spatially invariant transformation and for which the rules of geometric optics of ideal-image creation determine the mutual unambiguous correspondence between the points of target and its ideal image.

The optical transfer function of the thermodetection system ($\mathrm{OTF}_{\mathrm{syst}}(f_x, f_y)$) is a product of the optical transfer functions of individual subsystems

$$\mathrm{OTF}_{\mathrm{syst}}(f_x, f_y) = \mathrm{OTF}_{\mathrm{ob}}(f_x, f_y)\, \mathrm{OTF}_{\mathrm{det}}(f_x, f_y)\, \mathrm{OTF}_{\mathrm{el}}(f_x). \tag{1f}$$

When the linear system does not introduce a phase shift between the components of image spectrum, or when they can be omitted, it is convenient to replace the optical transfer function with its module – the modulation transfer function, $\mathrm{MTF}(f_x, f_y)$. Then, relation (1f) takes the form

$$\mathrm{MTF}_{\mathrm{syst}}(f_x, f_y) = \mathrm{MTF}_{\mathrm{ob}}(f_x, f_y)\, \mathrm{MFT}_{\mathrm{det}}(f_x, f_y)\, \mathrm{MTF}_{\mathrm{el}}(f_x). \tag{1g}$$

The above formulae enable us to determine the thermal image if the optical transfer functions of the thermodetection subsystems are known. The way of computing the $\mathrm{OTF}(f_x, f_y)$ functions of the subsystems is specific for each subsystem since it depends on the physics of phenomena introducing distortions in the process of thermal imaging.

Knowing the $\mathrm{OTF}_{\mathrm{syst}}(f_x, f_y)$ function, one can also calculate the spatial resolution of the thermodetection system, being the basic factor influencing its capability to perform the task of automatic target recognition.

The optical transfer function ($\mathrm{OTF}(f_x, f_y)$) is a Fourier transform of the point spread function ($\mathrm{PSF}(x,y)$). For incoherent illumination

$$\mathrm{OTF}(f_x, f_y) = \mathrm{FT}[\mathrm{PSF}(x,y)] = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \mathrm{PSF}(x,y)\, e^{-i2\pi(f_x x + f_y y)}\, \mathrm{d}x\, \mathrm{d}y. \tag{1h}$$

This relation enables the $\mathrm{OTF}(f_x, f_y)$ function to be derived from the known $\mathrm{PSF}(x,y)$ function. This is an alternative way of determining the optical transfer functions of individual subsystems occurring in formula (1f).

3. Optical transfer functions of the thermodetection system

The first subsystem modelled is the objective of thermodetection system. Optical assemblies occurring in thermodetection systems are the subsystems in which optical aberrations, defocusing and effects of diffraction on the objective aperture have essential influence on distortion of the image. The analysis of such subsystems by means of the geometric optics is not sufficient since the geometric optics is not suitable for description of diffraction effects. The known analytic solutions of the wave optics (*e.g.*, for a diffraction-limited system) cannot be applied as well since, in the case being considered, the influences of optical aberrations, defocusing and diffraction on the aperture compensate each other partially and they should not be considered separately (as is done even in advanced models described in the literature [9]).

In this paper, we propose a numerical method for determining the optical transfer function of the objective ($\mathrm{OTF_{ob}}(f_x, f_y)$) for incoherent illumination.

Due to the fact that diffraction on the objective aperture depends on the radiation wavelength λ, the optical transfer function ($\mathrm{OTF_{ob}}(f_x, f_y)$) is defined as a normalized weighted integral of the objective optical transfer functions for individual wavelengths ($\mathrm{OTF_{ob}}(f_x, f_y, \lambda)$)

$$\mathrm{OTF_{ob}}(f_x, f_y) = \frac{\int_0^\infty \mathrm{OTF_{ob}}(f_x, f_y, \lambda)\, L_\lambda \tau_a(\lambda)\tau_o(\lambda)\, R_n(\lambda)\,\mathrm{d}\lambda}{\int_0^\infty L_\lambda \tau_a(\lambda)\tau_o(\lambda) R_n(\lambda)\,\mathrm{d}\lambda} \tag{2a}$$

where: L_λ – the spectral density of target radiance, $\tau_a(\lambda)$ – the spectral coefficient of transmission of the atmosphere, $\tau_o(\lambda)$ – the spectral coefficient of transmission of the optics, $R_n(\lambda)$ – the spectral responsivity of the detector.

The way of determining the $\tau_a(\lambda)$, $\tau_o(\lambda)$ and $R_n(\lambda)$ functions is presented in [6], which also includes their plots for verifying conditions of experimental investigations. Figure 2 presents the normalized product of the L_λ, $\tau_a(\lambda)$, $\tau_o(\lambda)$, and $R_n(\lambda)$ functions denoted with the symbol S_λ.

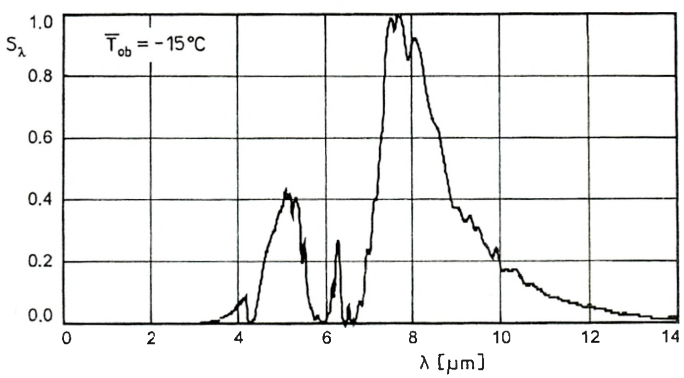

Fig. 2. The spectral characteristic function S_λ computed for verification of the conditions of experimental investigations.

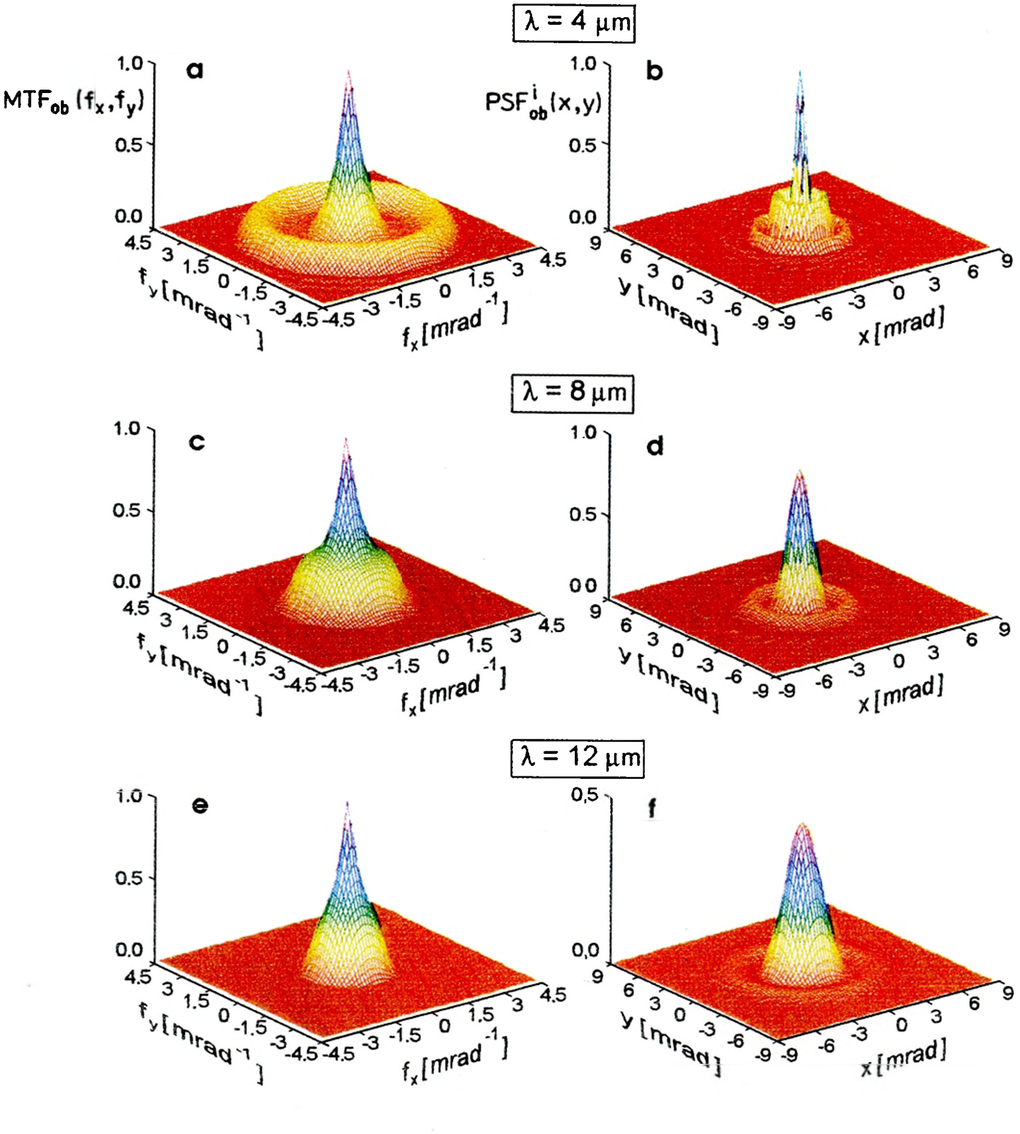

Fig. 3. Modulation transfer functions ($MTF_{ob}(f_x, f_y)$) – **a,c,e**, and intensity point spread functions (PSF^{i}_{ob} (x,y)) – **b,d,f**, for the objective of thermodetection system modelled.

This characteristic illustrates the relative contribution of radiation of different wavelengths to creation of the detector's output signal. In paper [6], the authors show that the S_λ-function dependence on wavelength is approximately independent of average temperature of the target. The above conclusion allows us to avoid multiple determination of the S_λ function for the points of the target surface of different temperatures in the range from 250 K to 320 K.

Formula (2a) can be converted into the form

$$\mathrm{OTF}_{\mathrm{ob}}(f_x, f_y) = \frac{\int\limits_{\lambda_1}^{\lambda_2} \mathrm{OTF}_{\mathrm{ob}}(f_x, f_y, \lambda) S_\lambda \mathrm{d}\lambda}{\int\limits_{\lambda_1}^{\lambda_2} S_\lambda \mathrm{d}\lambda}. \tag{2b}$$

The determination of the $\mathrm{OTF}_{\mathrm{ob}}(f_x, f_y)$ function is usually a complex problem of the wave optics.

The optical transfer function of objective for incoherent illumination ($\mathrm{OTF}_{\mathrm{ob}}(f_x, f_y)$) is the autocorrelation of the optical transfer function of objective for coherent illumination ($\mathrm{OTF}^{\mathrm{c}}_{\mathrm{ob}}(f_x, f_y)$)

$$\mathrm{OTF}_{\mathrm{ob}}(f_x, f_y) = \mathrm{OTF}^{\mathrm{c}}_{\mathrm{ob}}(f_x, f_y) \otimes \mathrm{OTF}^{\mathrm{c*}}_{\mathrm{ob}}(-f_x, -f_y)$$

$$= \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \mathrm{OTF}^{\mathrm{c}}_{\mathrm{ob}}(f'_x + f_x, f'_y + f_y)\, \mathrm{OTF}^{\mathrm{c*}}_{\mathrm{ob}}(f'_x, f'_y) \mathrm{d}f'_x \mathrm{d}f'_y\ . \tag{2c}$$

The optical transfer function of objective for coherent illumination ($\mathrm{OTF}^{\mathrm{c}}_{\mathrm{ob}}(f_x, f_y)$) is a Fourier transform of the point spread amplitude function ($\mathrm{PSF}_{\mathrm{ob}}(x, y)$) of this objective, and the $\mathrm{PSF}_{\mathrm{ob}}(x, y)$ function is proportional to the Fourier transform of the generalized objective transmittance ($T_u(\rho_x, \rho_y)$). Hence we obtain

$$\mathrm{OTF}^{\mathrm{c}}_{\mathrm{ob}}(f_x, f_y) = C \iint\limits_{-\infty}^{+\infty} \iint\limits_{-\infty}^{+\infty} T_u(\rho_x, \rho_y) e^{-2\pi i(f'_x x + f'_y y)} e^{2\pi i(f_x x + f_y y)} \mathrm{d}f'_x \mathrm{d}f'_y \mathrm{d}x \mathrm{d}y. \tag{2d}$$

It results from the properties of the Fourier transformation that

$$\mathrm{OTF}^{\mathrm{c}}_{\mathrm{ob}}(f_x, f_y) = C T_u(-\rho_x, -\rho_y) \tag{2e}$$

where the coefficient of the proportionality C depends on defocusing.

The above formulae enable determination of the optical transfer function of objective for incoherent illumination ($\mathrm{OTF}(f_x, f_y)$).

The intensity point spread function of objective ($\mathrm{PSF}^{\mathrm{i}}_{\mathrm{ob}}(x, y)$) is derived from the following dependence:

$$\mathrm{PSF}^{\mathrm{i}}_{\mathrm{ob}}(x, y) = \mathrm{FT}^{-1}[\mathrm{OTF}_{\mathrm{ob}}(f_x, f_y)] = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \mathrm{OTF}_{\mathrm{ob}}(f_x, f_y) e^{i2\pi(f_x x + f_y y)} \mathrm{d}f_x \mathrm{d}f_y. \tag{2f}$$

Figures 3**a**,**c**,**e** present the optical modulation transfer functions of objective, $\mathrm{MTF}_{\mathrm{ob}}(f_x, f_y) = |\mathrm{OTF}_{\mathrm{ob}}(f_x, f_y)|$, for three wavelengths: $\lambda = 4$, 8 and 12 µm.

With an increase in the wavelength, the angular spatial frequencies, f_x and f_y, transferred by the system decrease. This is manifested by the narrowing of the $\mathrm{MTF}_{ob}(f_x, f_y)$ function (Fig. 3e). Simultaneously, the influence of optical aberrations becomes less significant (the shape of $\mathrm{MTF}_{ob}(f_x, f_y)$ function in Fig. 3**e** gets closer to the shape of $\mathrm{MTF}_{ob}(f_x, f_y)$ function of the diffraction-limited systems). Figures 3**b**,**d** and **f** present the intensity point spread functions of objective $\mathrm{PSF}^{i}_{ob}(x, y)$ corresponding with the $\mathrm{MTF}_{ob}(f_x, f_y)$ functions in Figs. 3**a**,**c** and **e**.

The transmission seen (Fig. 3**f**) from the system burdened with aberrations for $\lambda = 4$ μm to the diffraction-limited system for $\lambda = 12$ μm shows that approximations of the geometric optics and diffraction-limited system cannot be applied in the case considered and proves the correctness of the modelling method assumed.

The computation of the objective optical transfer functions ($\mathrm{OTF}_{ob}(f_x, f_y, \lambda)$) performed for the spectral range from $\lambda = 3$ μm to $\lambda = 13$ μm were employed in determining the effective optical transfer function ($\mathrm{OTF}_{ob}(f_x, f_y)$) of the objective of the thermodetection system being modelled (Fig. 4).

The effective intensity point spread function of objective ($\mathrm{PSF}^{i}_{ob}(x, y)$) was computed using formula (2f). Its normalized module is presented in Fig. 5. The $\mathrm{OTF}_{ob}(f_x, f_y)$ and $\mathrm{PSF}^{i}_{ob}(x, y)$ functions are the basic characteristics of the objective quality.

Finite dimensions of the detector of infrared radiation cause integration of radiation incident on the detector's surface. This is described by the detector point spread function ($\mathrm{PSF}_{det}((x, y)$).

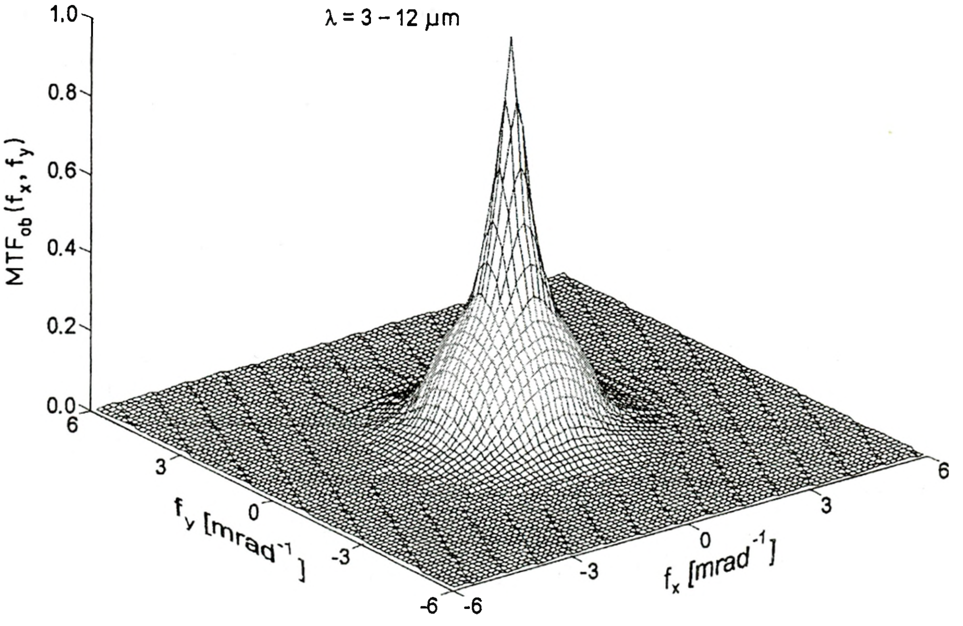

Fig. 4. Normalized effective modulation transfer function of the objective of thermodetection system ($\mathrm{MTF}_{ob}(f_x, f_y)$).

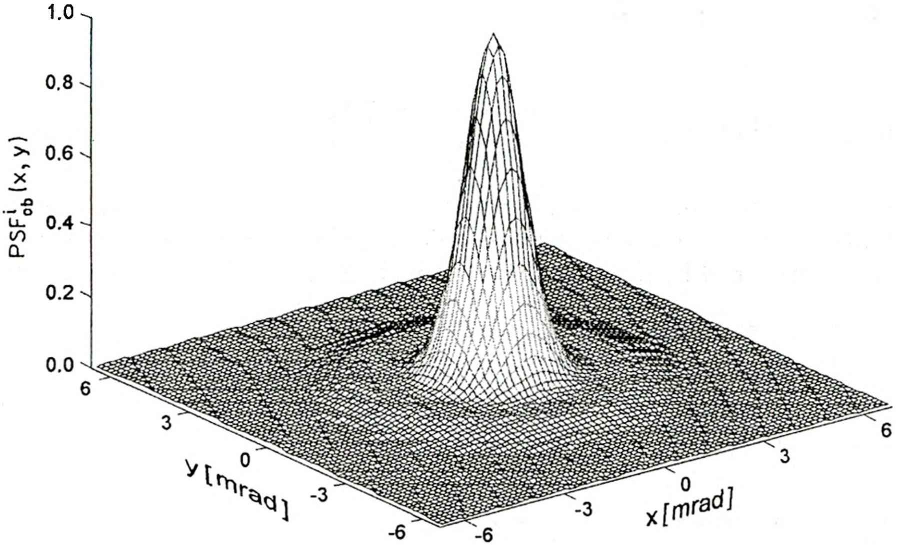

Fig. 5. Normalized effective intensity point spread function of the objective of thermodetection system ($PSF^i_{ob}(x, y)$).

For the detector of a rectangular shape of the active surface, this function has the form

$$PSF_{det}(x, y) = \frac{1}{\alpha_x \alpha_y} \mathrm{rect}\left(\frac{x}{\alpha_x}\right) \mathrm{rect}\left(\frac{y}{\alpha_y}\right) \tag{2g}$$

where α_x and α_y denote angular dimensions of the detector.

In the thermodetection system being modelled, we applied a detector of a large area (0.5 mm × 0.5 mm). The plot of the normalized point spread function for the detector ($PSF_{det}(x, y)$) of such dimensions is presented in Fig. 6.

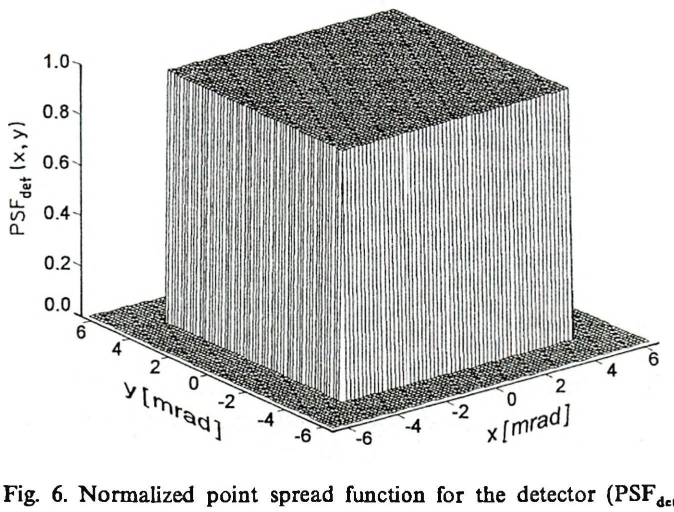

Fig. 6. Normalized point spread function for the detector ($PSF_{det}(x, y)$).

The optical transfer function of detector (OTF_{det}) can be expressed by the formula

$$OTF_{det}(f_x, f_y) = FT[PSF_{det}(x, y)] = \frac{\sin(\pi\alpha_x f_x)}{\pi\alpha_x f_x} \frac{\sin(\pi\alpha_y f_y)}{\pi\alpha_y f_y}. \tag{2h}$$

Its dependence on the angular spatial frequency f_x at $f_y = 0$ is presented in Fig. 7.

The amplitude-phase frequency characteristic of preamplifier of the electronic subsystem was determined by means of MicroCap III program. In order to be able

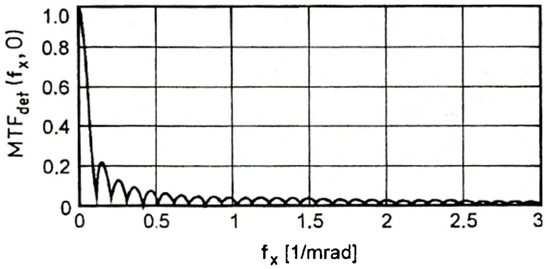

Fig. 7. Section of normalized modulation transfer function of the detector ($MTF_{det}(f_x, f_y)$).

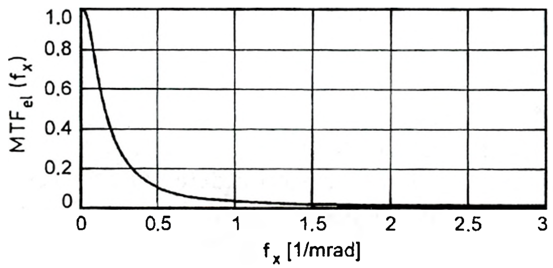

Fig. 8. Normalized modulation transfer function of the electronic subsystem ($MTF_{el}(f_x)$).

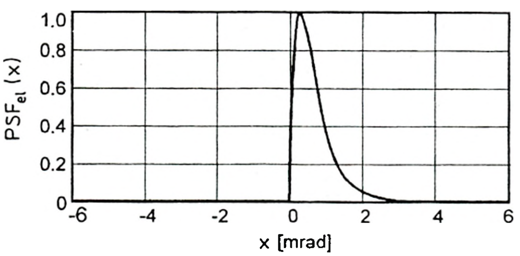

Fig. 9. Normalized point spread function of the electronic subsystem ($PSF_{el}(x)$).

to process it numerically, with the use of the fast Fourier transformation (FFT), its analytic form was determined which was sampled at a constant sampling period. The normalized modulation transfer function of the electronic subsystem obtained ($MTF_{el}(f_x)$) is presented in Fig. 8.

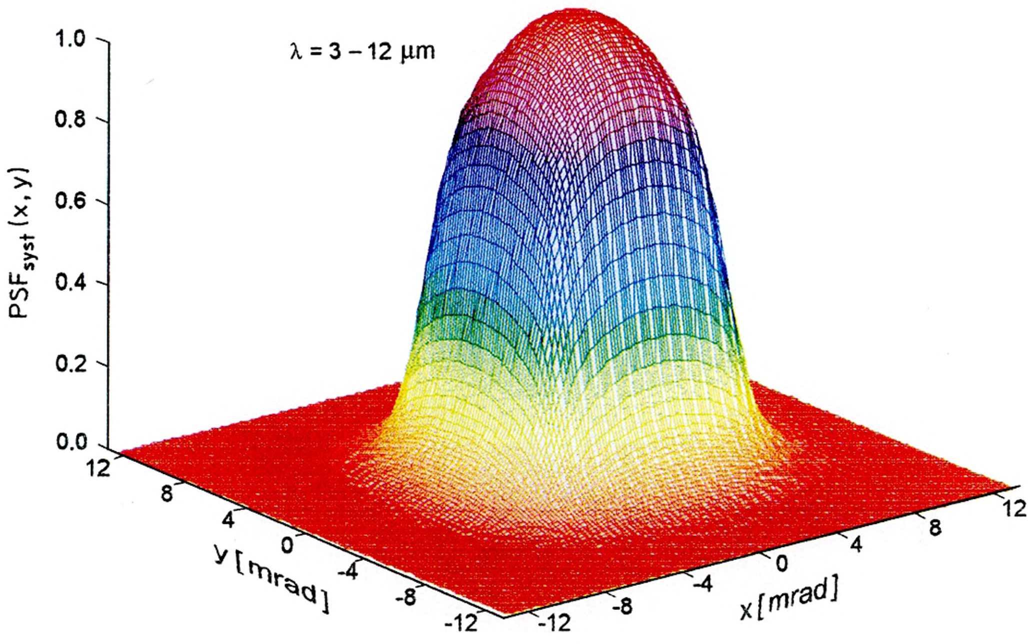

Fig. 10. Normalized point spread function of the thermodetection system ($PSF_{syst}(x,y)$).

The point spread function of electronic subsystem ($\mathrm{PSF}_{el}(x)$) was determined by computing the inverse Fourier transform of $\mathrm{MTF}_{el}(f_x)$ function. The results of computations are presented in Fig. 9.

4. Imaging by the thermodetection system

The optical transfer function of thermodetection system ($\mathrm{OTF}_{syst}(f_x, f_y)$) is a product of the optical transfer functions of individual subsystems. In conditions verifying experimental investigations, the band of spatial frequencies transferred by the objective of thermodetection system modelled is influenced first of all by the optical transfer function of detector ($\mathrm{OTF}_{det}((f_x, f_y)$). Such a situation results most often from the use of a detector of large dimensions. The application of a detector of small volume of the active region enables one to reduce thermal generation of noises and enhance the signal-to-noise ratio. The minimum detector dimensions are limited by the capability of an objective to focus incident radiation on the active surface of detector. This capability is determined by the point spread function of objective ($\mathrm{PSF}^{i}_{ob}(x, y)$). Therefore, the precise model of objective based on the wave optics is of essential importance.

The point spread function of thermodetection system ($\mathrm{PSF}_{syst}(x, y)$) has been determined from the formula

$$\mathrm{PSF}_{syst}(x, y) = \mathrm{FT}^{-1}[\mathrm{OTF}_{syst}(f_x, f_y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{OTF}_{syst}(f_x, f_y) e^{i2\pi(f_x x + f_y y)} \mathrm{d}f_x \mathrm{d}f_y. \tag{3a}$$

The result of computations is illustrated in Fig. 10. One can see the influence of the square shape of the detector, smoothing of function slopes caused by optical transformation by the objective, and broadening of the function in the x-scanning direction resulting from the electronic band-limited subsystem.

Thermal image of a target ($J(x, y)$) emerging in the output of the detection system is dependent on the ideal thermal image of the target ($I(x, y)$) by the relation of two-dimensional convolution with the point spread function of thermodetection

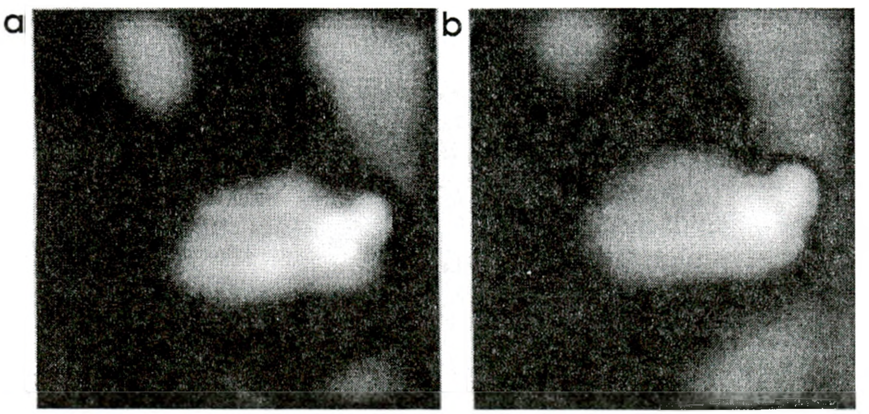

Fig. 11a, b

c

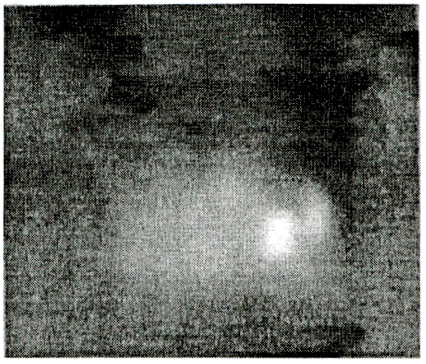

Fig. 11. Comparison of thermal images: **a** — thermovision measuring camera with MTS, **b** — FTTM+MTS, **c** — thermal scanner ST-95.

system ($PSF_{syst}(x, y)$). To verify the model, either a thermogram obtained by means of a measuring termovision camera or an ideal image generated by the faceted thermal target model was assumed as the ideal image (Fig. 11). The results of application of the model to both ideal test images are presented in Fig. 11**a** and **b**. For comparison, Fig. 11**c** illustrates the thermal image obtained fully experimentally.

Quite a good correspondence of the results obtained in comparison with the experiment proves the correctness of the proposed theoretical model of thermodetection system.

5. Summary

A theoretical model of thermodetection system for analysis of thermal imaging in the spectral region of medium (3 – 5 μm) and far (8 – 12 μm) infrared has been proposed. The optical transfer function of objective has been determined based on the wave optics. The point spread function of thermodetection system has been computed for a wide (3 – 12 μm) spectral range. Good correspondence of the theoretical modelling of thermal imaging with the experiment has been proved.

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