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Effective measurements of birefringence properties of nondichroic media using Poincaré sphere

PIOTR KURZYNOWSKI, FLORIAN RATAJCZYK

Institute of Physics, Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

A method for measuring the birefringence properties of nondichronic media using the Poincaré sphere is presented. Simple relations between coordinates of points on the Poincaré sphere representing input and output polarization states of light and the point representing first eigenvector of the medium have been found. From these relations desired polarization parameters of the medium were calculatd.

1. Introduction

There are many methods of determining the birefringence properties of media. Some of them are based on multiple mesurements of the polarization state of light after passing through the birefringent medium [1], [2] and rely on determination elements of the Mueller matrix of the medium and hence polarization parameters of the medium. In the present paper, we propose a simple method of finding polarization properties (the azimuth angle, the ellipticity angle of the first eigenvector of the medium and the phase shift introduced by this medium) of nondichroic medium. Contrary to earlier methods, instead of calculating the elements of the Mueller matrix of the medium we use the Stoke's vector formalism and its representation on the Poincaré sphere. We find simple relations between coordinates of points on the Poincaré sphere representing input and output polarization states of the light and the point representing first eigenvector of the medium. From these relations one can obtain indirectly formulas for the desired polarization parameters of the medium.

2. Measuring method

Let us consider a nondichroic medium with the ellipticity angle ϑ_f of its first eigenvector, placed in a measuring setup at the azimuth angle α_f. The medium introduces the phase shift γ between eigenwaves. In the first step of mesurements known elliptically polarized light with the azimuth α_1 and the ellipticaly angle ϑ_1 enteres the medium and after passing it is analyzed, since the medium changed the polarization state of light giving as a result the output azimuth angle α_1' and the output ellipticity angle ϑ_1'. In the second step the procedure is the same, but the input light has parameters α_2 and ϑ_2, and the medium transforms them to the output parameters α_2' and ϑ_2'. The first eigenvector of the medium $V_f \equiv [M_f, C_f, S_f]$ and parameters $V_i \equiv [M_i, C_i, S_i]$ of the input and $V_i' \equiv [M_i', C_i', S_i']$, $i = 1, 2$ of the output light have their own representation on the Poincaré sphere (Fig. 1), where generally

$$M = \cos 2\alpha \cos 2\vartheta, \tag{1}$$

$$C = \sin 2\alpha \cos 2\vartheta, \tag{2}$$

$$S = \sin 2\vartheta. \tag{3}$$

Graphical representation of light transformation by the medium in the first and second steps is given in Fig. 1. From general rules of this representation it follows

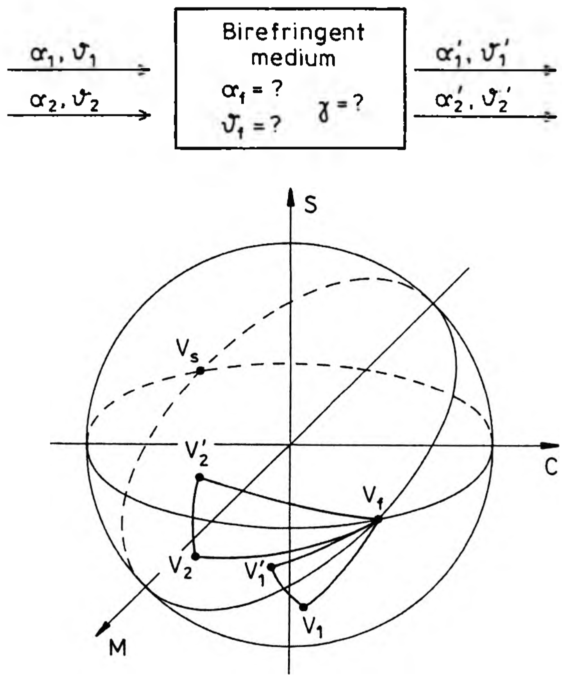

Fig. 1. Transformation on the Poincaré sphere of the polarization state of light by the birefringent medium in the first ($i = 1$) and second ($i = 2$) steps of measurements. V_f – the point representing the first eigenvector of the medium, V_i and $V_i' = 1, 2$ – points representing input and output polarization states of the light, respectively.

immediately that (for the first measurement) the angle between vector V_f representing polarization state of the first eigenvector of the medium and vector V_1 representing polarization state of the input light is the same as the angle between vector V_f and vector V_1' representing polarization state of the output light. The same remark is valid for the second measurement. Then the following scalar products are equal to each other:

$$[M_1, C_1, S_1] \cdot [M_f, C_f, S_f] = [M_1', C_1', S_1'] \cdot [M_f, C_f, S_f], \tag{4}$$

$$[M_2, C_2, S_2] \cdot [M_f, C_f, S_f] = [M_2', C_2', S_2'] \cdot [M_f, C_f, S_f], \tag{5}$$

which can be rewritten as:

$$M_f M_1 + C_f C_1 + S_f S_1 = M_f M_1' + C_f C_1' + S_f S_1', \tag{6}$$

$$M_f M_2 + C_f C_2 + S_f S_2 = M_f M_2' + C_f C_2' + S_f S_2'. \tag{7}$$

This is the system of two equations with two unknowns α_f and ϑ_f since the input parameters α_1, ϑ_1, α_2, ϑ_2 are known from the synthesis of the input light and the output parameters α_1', ϑ_1', α_2', ϑ_2', are known from the analysis of the output light. Substituting Eqs. (1)–(3) containing variables α_f and ϑ_f into Eqs. (6), (7) one can get solutions of the form:

$$\tan 2\alpha_f = -\frac{(M_1-M_1')(S_2-S_2')-(M_2-M_2')(S_1-S_1')}{(C_1-C_1')(S_2-S_2')-(C_2-C_2')(S_1-S_1')}, \tag{8}$$

$$\tan 2\vartheta_f = -\frac{\cos 2\alpha_f(M_1-M_1')+\sin 2\alpha_f(C_1-C_1')}{S_1-S_1'}$$

$$= -\frac{\cos 2\alpha_f(M_2-M_2')+\sin 2\alpha_f(C_2-C_2')}{S_2-S_2'}. \tag{9}$$

The phase shift γ introduced by the medium can be obtained applying identities of the spherical trigonometry. Let us consider the first measurement. If we take into account two great circles the first of which contains points V_f and V_1, the second – points V_f and V_1', then the phase shift γ is an angle between these circles with the center at point V_f. The same remark is valid with regard to the second measurement. It leads to the following equations:

$$\sin\frac{\gamma}{2} = \frac{\sin(A_{11'}/2)}{\sin A_{f1}} = \sin\frac{(A_{22'}/2)}{\sin A_{f2}} \tag{10}$$

where:

$$\cos A_{11'} = M_1 M_1' + C_1 C_1' + S_1 S_1', \tag{11}$$

$$\cos A_{22'} = M_2 M_2' + C_2 C_2' + S_2 S_2', \tag{12}$$

$$\cos A_{f1} = M_f M_1 + C_f C_1 + S_f S_1, \tag{13}$$

$$\cos A_{f2} = M_f M_2 + C_f C_2 + S_f S_2. \tag{14}$$

As one can see from Fig. 1, points V_f, V_s (V_s is the second eigenvector of the medium) representing eigenvectors of the medium lie somewhere on two great circles: the first one containing these points and the point which lies in the middle of the arc V_1V_1' and is perpendicular to this arc, and the second one with points V_f, V_s and the point which lies in the middle of the arc V_2V_2' and is perpendicular to this arc as well. Then points V_f, V_s are cross-sections of these two circles. So there are two solutions of this problem: the first one with parameters (α, ϑ, γ), and the second with ($\alpha+90°$, $-\vartheta$, $360° - \gamma$), which corresponds to the fact that cannot recognize which of the solutions represents the first or second eigenvector of the medium. The right set of results must be chosen using additional messurements of α or γ.

Moreover, due to specific determination of the phase shift γ (function sin in Eq. (10)) we cannot recognize the order of the phase shift and it has to be measured in some other way.

3. Particular cases

In some cases one can reduce the number of measurements from two to one. Below, we consider two cases: the medium with known azimuth angle α_f and the linearly birefringent medium.

3.1. Setup with known azimuth angle of medium

Sometimes the azimuth angle α_f of a medium is known or can be easily measured. From remarks made at the end of Section 2 it follows that one of two desired great circles is known and this is a great circle containing the meridian with the azimuth α_f. So, only one step (measurement) of the above procedure is needed. If this azimuth is known we can orient the medium in the measuring setup with the azimuth $\alpha_f = 0°$. Then the ellipticity angle of the medium can be simply calculated from

$$\tan 2\vartheta_f = -\frac{(M_1 - M_1')}{S_1 - S_1'}, \tag{15}$$

and the phase shift γ should be calculated from Eqs. (10)–(14). Let us note that since the azimuth angle is known the ellipticity angle is determined unambiguously.

3.2. Linearly birefringent medium

In this case the ellipticity angle ϑ_f is equal to zero and one can make measurement only once because we know the orientation of one of the two great circles mentioned above: this in an equator of the Poincaré sphere. The more detailed analysis of this case leads to the conclusion that the setup with input light of the circular polarization state is the simplest way to obtain the polarization parameters of the medium. Then almost trivially one can get that

$$\alpha_f = \alpha_1' - 45°, \tag{16}$$

$$\gamma = N \cdot 360° + 90° - 2\vartheta_1', \tag{17}$$

for the first pair of solutions, and

$$\alpha_f = \alpha_1' + 45°, \tag{18}$$

$$\gamma = N \cdot 360° + 270° + 2\vartheta_1', \tag{19}$$

for the second one, where N is the order of the phase shift.

This can be excellently presented on the Poincaré sphere (Fig. 2): the linearly birefringent medium whose first Stoke's eigenvector lies somewhere on the equator of this sphere transforms the circulary polarized input light whose Stoke's vector is represented by the north pole, along the meridian whose azimuth differs from the azimuth of the first eigenvector of the medium by the value of 90° (let us

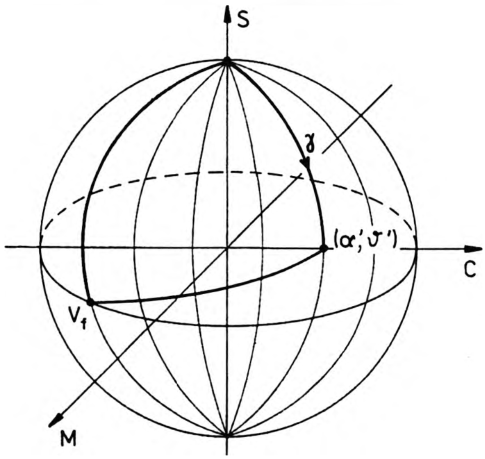

Fig. 2. Transformation on the Poincaré sphere of the polarization state of circularly polarized light by the linearly birefringent medium.

remember that doubled azimuth and elliptically angles are spherical coordinates of points of the Poincaré sphere). Hence we get Eq. (16). Similarly, the length of the arc between the points representing the initial (here circular) and final polarization states of the light is equal to the phase shift introduced by the medium (from the definition) and in this case is equal to the ellipticity change of the light at the same time, which is included in Eq. (17).

4. Conclusions

The presented method is very simple and effective. The accuracy of the method depends, of course, on the choice of polarization parameters of the input light and in some cases could give results with errors being too significant. However, the advantage of this method is that one can simply repeat measurements with other parameters of input light to obtain results with small errors.

References

[1] Ratajczyk F., Urbańczyk W., Optik **79** (1988), 183.
[2] Woźniak W.A., Kurzynowski P., Optik **96** (1994), 147.

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