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Letters to the Editor

Polariscopic measurement of the optical path difference using the spectral analysis method

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A method of measurement of the optical path difference introduced by the elliptically birefringent medium is presented. The method is based on the spectral analysis of the white light passing through the medium. It offers a wide range of measured path differences and possibility of being easily applied in automated measurement setup.

1. Introduction

The optical path difference introduced by the birefrigent medium is one of the most important parameters which are measured in the anisotropy media optics. There are many method of measuring this value. Some of them are direct methods based on compensation of the phase difference introduced by examined medium using elements introducing known phase difference (for example, a Wollastone prism). There are also indirect methods, where changes of the polarization state of light after passing through the medium are measured and hence the optical path difference could be calculated. A method based on the spectral analysis of light in a polariscopic setup for linearly and circularly birefringent media is described in [1]. In the present paper, generalization of this method to elliptically birefringent media is presented.

2. Principle of measurement

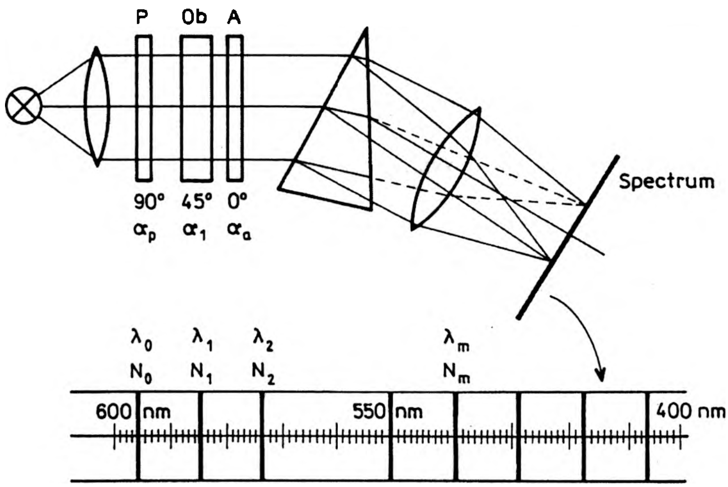
The idea of the measurement is shown in the Figure. White light passes through the polarizer, birefringent medium, analyzer and is analyzed spectrally using, for example, the prism. The polarizer and the analyzer are crossed and the azimuth angle of the first eigenvector of the medium differs from the azimuth angle of the polarizer by the angle $-45°$. The spectral intensity I of light at the output of the setup takes the form

$$I = I_{\max} \sin^2\left(\frac{\pi R}{\lambda}\right) \tag{1}$$

where $I_{\max}$ is the intensity of light when $\pi R/\lambda = (2n-1)\pi/2$, $n = 1, 2 \ldots$, R is an optical path difference and λ is the light wavelength. Since the spectrum of light is continuous and the medium is homogeneous some of the waves are extinguished, namely, those for which the wavelenghts fulfill the following condition:

$$\frac{R}{\lambda} = N, N = 0,\ 1,\ 2\ \ldots, \tag{2}$$

which can be observed as dark lines in the spectrum of the light.



Scheme of the measurement of optical path difference using spectral analysis method.

The optical path difference R_0 for the wavelength λ_0 corresponding to some dark line in the spectrum is then given by

$$R_0 = \lambda_0 N_0, \tag{3}$$

and for another wavelength λ_m corresponding to another dark line in the spectrum

$$R_m = \lambda_m N_m \tag{4}$$

where

$$N_m = N_0 + m. \tag{5}$$

From the definition the optical path difference R is given by

$$R = d|n_s - n_f| \tag{6}$$

where d is a geometrical path of light in the medium, n_s and n_f are the refractive indices of slow and fast eigenwaves of the medium, accordingly. Applying this definition to Eqs. (3) and (4) one can obtain the equation for the optical path difference R_0 for the wavelength λ_0

$$R_0 = m\left|\frac{\lambda_0 \lambda_m}{D\lambda_0 - \lambda_m}\right|, \tag{7}$$

where

$$D = \frac{(n_s - n_f)_m}{(n_s - n_f)_0}. \tag{8}$$

The refractive index difference in the nominator in Eq. (8) in related to the wavelength λ_m, while in the denominator – to the wavelength λ_0. So, in order to find the optical path difference R_0 one should measure at least two wavelengths of light the intensity of which is zero and know spectral properties of the medium (in order to calculate the value of D).

3. Determination of the correction factor D

3.1. Linearly birefringent medium

From Eqation (8) it follows that one should know the nature of the medium birefringence and hence the refractive index difference $(n_s - n_f)$ of eigenwaves of the medium in a given derection. For linearly birefringent media this difference depends on principal refractive indices (n_o, n_e for uniaxial ones and n_x, n_z for biaxial ones) of the medium and the angle φ between the direction of wave propagation and the optical axis of uniaxial crystals of angles φ_1 and φ_2 between the direction of wave propagation and the optical axes of biaxial crystals. For uniaxial crystals this approximated relation is as follows [2]:

$$|n_s - n_f| = |n_o - n_e| \sin^2\varphi, \tag{9a}$$

while for biaxial crystals

$$|n_s - n_f| = |n_z - n_x| \sin\varphi_1 \sin\varphi_2. \tag{9b}$$

In most cases principal refractive indices of crystals are determined so that when angles described above are known it is easy to calculate the refractive indices of eigenwaves. Fortunately, this knowledge is not necessary to calculate the correction factor D because it does not depend on these angles, which is easy to show by substituting Eq. (9a) or Eq. (9b) into Eq. (8).

Then

$$D = \frac{(n_s - n_f)_m}{(n_s - n_f)_0} = \frac{(n_o - n_e)_m}{(n_o - n_e)_0} \tag{10a}$$

for uniaxial crystals, and

$$D = \frac{(n_s - n_f)_m}{(n_s - n_f)_0} = \frac{(n_x - n_z)}{(n_x - n_z)_0} \tag{10b}$$

for biaxial ones.

3.2. Circularly birefringent medium

Polarization properties of circularly birefringent media are described by the twist angle Γ of the polarization plane rather than the optical path difference R. These values are correlated with each other by the following equation:

$$\Gamma = \frac{\pi R}{\lambda}. \tag{11}$$

On the other hand, the twist angle Γ depends on the geometrical path d of the light in the medium and the material constant Γ_s called a specific twist angle

$$\Gamma = \Gamma_s d. \tag{12}$$

Applying Eq. (11) and Eq. (12) to formulae describing the desired optical path difference R one should obtain

$$D = \frac{\Gamma_{s,m}\,\lambda_m}{\Gamma_{s,0}\,\lambda_0}, \tag{13}$$

where $\Gamma_{s,m}$ an d $\Gamma_{s,0}$ are specific twist angles for the wavelengths λ_m and λ_0, respectively.

3.3. Elliptically birefringent uniaxial medium

In the case of the elliptically birefringent uniaxial medium calculation of the refractive difference of eigenwaves is more complicated. We solve this problem using the superposition rule which is presented in detail in [3]–[5]. This rule says that the phenomenon of elliptical birefringence of the medium could be treated as a simultaneous superposition of linear and circular properties of the medium, which could be mathematically represented by the followinfg three equations:

$$\Delta n^2 = \Delta n_l^2 + \Delta n_c^2, \tag{14}$$

$$\Delta n_l = \Delta n \cos 2\vartheta, \tag{15}$$

$$\Delta n_c = \Delta n \sin 2\vartheta \tag{16}$$

where Δn is the refractive index difference of the elliptically birefringent medium, Δn_l and Δn_c are its linear and circular components, respectively, and ϑ is an ellipticity angle of the first eigenvector of the medium. Let us emphasize the fact that these quantities are strictly related with a given direction of the light propagation.

These linear Δn_l and circular Δn_c components depend on the angle φ between the direction of light propagation and the optical axis of the medium. The linear component Δn_l is equal to the value described by Eq. (9a)

$$\Delta n_l = |n_o - n_e| \sin^2\varphi \tag{17a}$$

while circular component Δn_c is done by the following equation [3]:

$$\Delta n_c = \frac{[(G \cdot \vec{s}) \cdot \vec{s}]^2}{n_o n_e}, \tag{17b}$$

where G is a gyration tensor and $\vec{s}$ is a unitary vector of the wave propagation. Let us note that the angle φ is included in Eq. (17b) in non-evident form, in scalar product of $G \cdot \vec{s}$ and $\vec{s}$. Formally, the angles ϑ and φ are correlated with each other.

The dependence of the ellipticity angle ϑ on the angle φ is different for different crystallographic classes of crystals since forms of gyration tensors are different. This formula could be found by dividing Eq. (16) by Eq. (15) and substituting Δn_l and Δn_c from Eqs. (17). For example, for quartz this dependence is of the type

$$\tan 2\vartheta \propto \frac{g_{11}\sin^2\varphi + g_{33}\cos^2\varphi}{\sin\varphi}, \tag{18}$$

and for the wavelength $\lambda = 510$ nm, $g_{11} = 5.82 \cdot 10^{-5}$ and $g_{33} = -12.96 \cdot 10^{-5}$ [5]. It is just from this equation that the well known angle $\varphi = 56.2°$ for which the natural activity of quartz disappeared could be calculated.

Using relations between the phase shift and the optical path one can present the final form of correction factor D as follows:

$$D = \frac{[\Delta n_l^2 + \Delta n_c^2]_m}{[\Delta n_l^2 + \Delta n_c^2]_0} \tag{19}$$

where indices m and 0 mean that the nominator should be calculated for the wavelength λ_m, while the denominator should be calculated for the wavelength λ_0.

4. Conclusions

A method for measuring the optical path difference introduced by the elliptically birefringent medium is presented. The method is based on the spectral analysis of the white light passing through the medium. It is easy and simple, although it requires knowledge about the properties of the medium such as a kind of crystal and its orientation with regard to the direction of light propagation. A wide range of the measured path differences is one of the advantages of this method. Moreover, it can be easily applied in automated measurement setup.

References

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