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The Calculation Method of Filter Stacks for Spectral Correction of Photoelectric Radiation Receivers

The paper presents a new method of the approximation of functions representing the final spectral distribution of an arbitrary colorimetric receiver, spectral sensitivity distribution of a physical radiation receiver, as well as of its correcting colour optical filters whose transmittivity was expressed by Bouguer Law. The computational programme has been proposed for Elliott 803B in the Institute of Electrotechnics. Some results and conclusions concerning the exploitation of the programme, selection of filters and calculation of the stacks of layer filters are given, and the whole problem related to the correction of a trichromatic colorimeter discussed.

1. Introduction

A method of approximation of the logarithms of functions worked out in 1972 is very useful for calculation of the filter stacks to spectral correction of the photoelectric radiation receivers [1]. This simple and interesting method has been published in [2]. It moreover can be adapted to other problems where logarithmic functions occur, for instance to the logarithmic decay of wave quantities*. As both the method of approximation and the problem of spectral correction may be of interest for opticists the following material presents an extensive treatment of the problem.

In the paper [2] an exemplary approximation of the relative spectral photopic light efficiency $V(\lambda)$ by a function of spectral sensitivity of a photoelectric cell $S(\lambda)$ has been calculated the latter being equipped with a set of colour optical filters of internal transmission coefficient $\tau_i(\lambda)$. For the sake of generality the following notation will be here-after accepted: $R_t(\lambda)$ to denote the approximated distribution and $R_d(\lambda)$ to denote the approximating distribution, which do not suggest any particular type of radiation receptor. The photometric and colorimetric receptors of radiation exhibit regular spectral sensitivity characteristics which use of form similar to the bell curve or consist of several such curves. On the other hand characteristics of physical receivers of radiation being distinctly different from one another, are to be adapted to those require in the correcting process.

The fundamentals of the correction problem for radiation receptor were given by Dresler in [3]. The correction is made with the help of colour optical filters, which are positioned in front of the light-sensitive surface of the receiver. The purpose of the filter application is to absorb the radiation in visible spectrum range, where the sensitivity of the receiver is too high. A set of several filter layers (a subtractive or series layer stack according to Fig. 1)

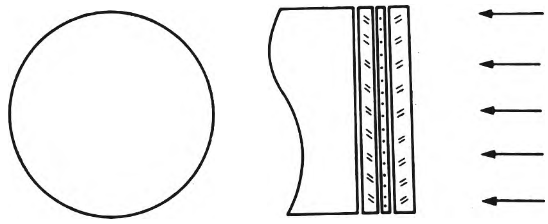

Fig. 1. The stack of layer colour filters for the spectral correction

or several zones (additive or parallel zonal stack according to Fig. 2) or, finally, a combination of both the types of stacks.

The problem of correction is thus to a choice of proper filters (as shown for instance in [4]) followed by calculation of optimum thicknesses of the layer filter in stacks and optimum sur-

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** The respective program, which will be often referred to, was elaborated by Mrs. A. Giembicka for the Centre Institute of Electronics Elliott 9036 computer from the Information Processing.

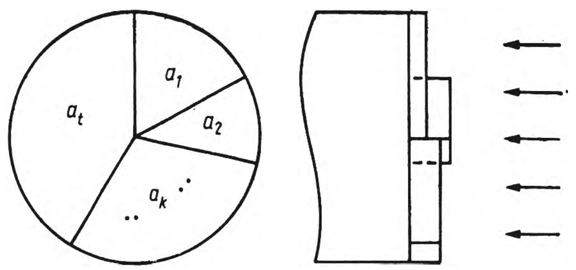

Fig. 2. The stack of zonal colour filter for spectral correction in the form realized in MOBAR device [9]

faces for the zonal stack and finally an evaluation of the correction quality. DAVIES and WYSZECKI [5] proposed a method of rigorous calculation of the zonal stack parameters as well as an approximate method for the layer stack. A very good approximate method for the layer stack was presented by WRIGHT, SANDERS and GIGNAC [6]. In Poland very good results in correcting receptor of radiation have been achieved by A. MAŃK [7].

A rigorous and general solution of the equations for optimal areas and thicknesses of layers is not possible so far, as this means being reduced to the solution of a system of nonlinear equations. This solution, however, is possible in case of receptors whose characteristic consist of separate peaks. Each peak of the response is then approximated by the functions of a separate zone being a multilayer stack.

2. Calculation of multilayer filtering stack

The aim of this paper is to present the whole problem of spectral correction, starting with the recently developed method of optical parameters calculation for a multilayer stack and derived for logarithmic functions of the approximated spectral distributions by the least-squares method. This method [1] consists in determining the minimum of the function

$$\sum_{i=1}^{n} [\ln R_t(\lambda_i) - \ln R_d(\lambda_i)]^2 W(\lambda_i), \tag{1}$$

in which $R_t(\lambda)$ is the given approximated function and

$$R_d(\lambda) = CS(\lambda) \prod_{j=1}^{m} \tau_j(\lambda) \tag{2}$$

is calculated approximating function, being a product of the receptor sensitivity distribution $S(\lambda)$ and the spectral coefficients $\tau_j(\lambda)$ of the internal transmission of m filters, while $W(\lambda)$ is a weighing factor improving the approximation in the spectral regions of a special concern, used for the number n of division points i within the given spectral range.

The parameter C is a scale factor enabling an approximation while using the relative units, accepted for the functions.

The internal transmission coefficients for filters are expressed by the Bouguer Law

$$\tau_j = \exp[-a_j(\lambda)x_j], \tag{3}$$

in which: $a_j(\lambda)$ are the linear absorption indices and x_j denotes the j-th thickness of the filter.

All the functions mentioned should have non-zero and non-negative values defined within the whole spectral range of the given approximated function $R_t(\lambda)$, and at least in that part of the spectral range in which the approximation is being calculated. The optimum filter thickness can be simply found from the minimum condition (1) as the procedure is reduced to solving the system of linear equations with respect unknown filter thicknesses x and the logarithm of the parameter C, while the coefficients at those anknowns depend upon the linear absorption indices $a_j(\lambda)$. The free terms depend on the given approximated function $R_t(\lambda)$ and the spectral distribution of receiver sensitivity $S(\lambda)$.

$$\begin{gathered}
x_1 \sum_{i=1}^{n} a_1^2(\lambda_i) + x_2 \sum_{i=1}^{n} a_2(\lambda_i)a_1(\lambda_i) + \ldots + \\
+ x_m \sum_{i=1}^{n} a_m(\lambda_i)a_1(\lambda_i) - \ln C \sum_{i=1}^{n} a_1(\lambda_i) \\
= \sum_{i=1}^{n} a_1(\lambda_i)[\ln S(\lambda_i) - \ln R_t(\lambda_i)], \\
x_1 \sum_{i=1}^{n} a_1(\lambda_i)a_2(\lambda_i) + x_2 \sum_{i=1}^{n} a_2^2(\lambda_i) + \ldots + \\
+ x_m \sum_{i=1}^{n} a_m(\lambda_i)a_2(\lambda_i) - \ln C \sum_{i=1}^{n} a_2(\lambda_i) \\
= \sum_{i=1}^{n} a_2(\lambda_i)[\ln S(\lambda_i) - \ln R_t(\lambda_i)], \\
\cdot \quad \cdot \quad \cdot \\
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\cdot \quad \cdot \quad \cdot \\
x_1 \sum_{i=1}^{n} a_1(\lambda_i a_m(\lambda_i) + x_2 \sum_{i=1}^{n} a_2(\lambda_i)a_m(\lambda_i) + \ldots +
\end{gathered} \tag{4}$$

$$+x_m \sum_{i=1}^{n} a_m^2(\lambda_i) - \ln C \sum_{i=1}^{n} a_m(\lambda_i)$$

$$= \sum_{i=1}^{n} a_m(\lambda_i)[\ln S(\lambda_i) - \ln R_t(\lambda_i)],$$

$$x_1 \sum_{i=1}^{n} a_1(\lambda_i) + x_2 \sum_{i=1}^{n} a_2(\lambda_i) + \ldots + x_m \sum_{i=1}^{n} a_m(\lambda_i) -$$

$$-n\ln C = \sum_{i=1}^{n} [\ln S(\lambda_i) - \ln R_t(\lambda_i)].$$

To simplify the equations the weighing factors have been omitted by setting $W(\lambda) = 1$, otherwise each term in the system of equations should be multiplied by $W(\lambda_i)$.

In order to perform the calculations a programme for the Elliott 803B computer has been prepared.

A disadvantage of this solution is the necessity of a careful filter selection, since the computer programme should not contain too many details. The advantage of this solution is its extraordinary simplicity, low costs and high accuracy of calculations, especially in the wings of characteristics, where a small difference in the algorithms of the approximated functions signifies a low value of the ratio of those functions as well as small difference in absolute values of these functions. The last feature enables the solution of special tasks, like the measurements of coloured signalling lamps.

The preliminary calculations, not being programmed are reduced to computing the spectral characteristics of the internal transmittivity of filters and the receiver sensitivity. The initial values of the internal spectral transmittivity of filters are obtained directly during the measurement of the filter transmission as compared to the transmission of transparent glass plate its refractive index being close to that of the filter material.

The transmittivity $\tau(\lambda)$ is obtained by measuring the filter transmittivity as referred to nonfiltered radiation beam. The calculation of this internal transmittivity $\tau_i(\lambda)$ may be from the formule

$$\tau(\lambda) = \tau_i(\lambda)(1-\varrho)^2 \cong \tau_i(1-2\varrho), \qquad (5)$$

where ϱ is the coefficient of reflection at each of the two surfaces of the filter. In the above equation the mathematical formalism takes account of elimination of light reflections at the filter surfaces, which occurs when cementing them with the Canadian balsam. The performance of the calculations mentioned above is very advantageous, as it allows to eliminate the unwanted feeding the computer with experimental data suffering from great errors, enables to average the accidental errors, and to perform the necessary interpolations and extrapolations. The labour consumption of the new solution may be determined by the time of calculations. The correction of the receiver with the help of three filters requires five minutes of Elliott 803B computer time for one set of weighing factors.

This procedure if compared with the method operating with functions in the exponential form is an essential simplification, and enables to avoid special methods and to reduce the computer time consumption.

The calculation of the approximation of the characteristics by the method described without using a computer is relatively simple, especially when the filters is not great and the number of experimental points is reduced. To find the spectral characteristics of the linear absorption indices $a_j(\lambda)$ and logarithms of the required theoretical characteristics $R_t(\lambda)$ as well as the receiver sensitivity $S(\lambda)$ sufficiently accurate tables of natural logarithms may be used, while the calculation of the corrected receiver characteristics $R_d(\lambda)$ may be carried out basing on tables of exponential functions. The characteristic of the correct receiver is determined by equations (2) and (3), where x_j are the solutions of the system of linear equations, and $a_j(\lambda)$ those of the logarithmed equation (3). The calculations are reduced to simple algebraic operations. The application of the least-square-and-logarithm method to the functions in logarithmic form with the Bouguer Law coefficients may be employed also in other calculational problems.

The cited thesis [1] beside the new solution (2) contains a discussion of solutions proposed by other authors [3–7]. The following topics are moreover considered: solution of the problem of finding the optimal areas of filter surfaces in the case of zonal stack [8], compilation of solutions for zone and layer stacks, and a complete discussion of matching problem for a set of receptors for tri-chromatic colorimeter varified practically in the MOBAR colour meter, which was constructed in the Institute of Electrotechnics [9]. Hereafter only the new elements of the method developed in the course of further work on spectral correction will be presented.

3. The conclusions from the numerical results obtained in the course of exploiting the programme for calculating the parameters of the multilayer stack of filters according to new method

The results obtained with the help of the program mentioned allow to conclud inter that the characteristics of the elements of corrected photometric systems should fulfil following conditions:

1. The curves of the *internal* spectral transmittivity distribution should be smooth and show mild changes in slope.

2. The composed filters should have different characteristics as for similar responses such as, for instance, those of Schott BG 15 filters the computer priveleges the greater thickness of the worse filter. Moreover, if the characteristics of both the filters differ slightly, the sum of the thicknesses of both filters determined by the computer differs only slightly from the thickness of each of the filters if used to approximation separately. This can be illustrated by calculation, the results which are presented in Fig. 3. The computing concernes a Dr. B. Lan-

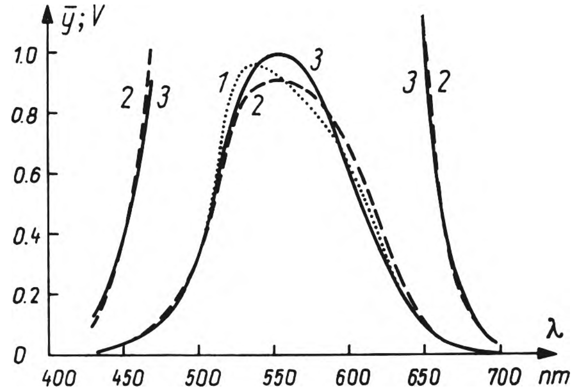

Fig. 3. An example of a two-stage approximation of a selenium cell *1* — with two filters, *2* — with three filters, *3* — approximated function $V(\lambda)$

ge selenium cell which is to be given the spectral sensitivity corresponding to the curve V. If two filters BG15 and OG1 were used the calculated thicknesses amount to 10.1 mm and 0.1 mm, respectively (the dotted curve), the respective thicknesses with three filters BG15, OG1 and BG19 applied, being 0.4, 0.11 and 11 mm (the broken curve). An excellent approximation of things visible at $10\times$ magnification of ordinates should be emphasised. It should be also noticed that in the case of two filters the thickness of BG15 filter amounts to 10.1 mm and is close to the sum (with the negative sign) of the thicknesses of BG15 and BG19 in the case of three filters $(11-0.4) = 10$.

As far as the calculation procedure is concerned, the principle [5] should be assumed. It may be formulated as follows:

To avoid the negative filter thicknesses in the succesive stages of approximation the results of the previous stages should be "frozen" when introducing the new distribution of receiver sensitivity improved by absorption of the filters used at the preceding stage.

From the exploitation of the programme the conclusions concerning the employment of weighing factors during calculation may be also formulated:

1. In the case of photoelectric receivers (cells and multipliers) of sensitivity characteristics strongly dependent upon the wavelength the best results are obtained without using any weighing factors.

2. The weight $bW(\lambda) = R_t(\lambda)$, and in some cases, the weight $cW(\lambda) = 1/\ln^2 R_t(\lambda)$ (for instance for $R_t(\lambda) = \bar{u}(\lambda)$ which goes below and far from the singular value i of the weight c) give good results for the receivers with a flat spectral sensitivity characteristic $S(\lambda)$, for e.g. for thermal receivers.

3. The weight in the form of an arbitrary given function may be exploited in two ways: 1. as a weight

$$F(\lambda) = \Phi_{e\lambda} \tag{6a}$$

taking account of the variability of the spectral distribution of radiation in the measurements of definitive lamps, if the corrected receiver has a flat spectral sensitivity characteristics, and 2. as a weight

$$F(\lambda) = \frac{R_t(\lambda)}{S(\lambda)} \tag{6b}$$

for the correction of arbitrary receivers with correction of approximation of the maxima at the expense of worse correction of the wings. This may be illustrated by an approximation based on experimental data (Fig. 4). This is the correction of a selenium cell produced in Poland, which was obtained by using the stack of four Schott filters: BG18, J_{Dr}, OG1 and GG11 with the respective thicknesses 0.46–0.077 mm; 0.015 mm and 1.096 mm applied to the trichromatic spectral component $\bar{y}(\lambda)$. The approximating curve $R_d(\lambda)$ interlaces with the given curve $\bar{y}(\lambda)$ within the whole spectral range.

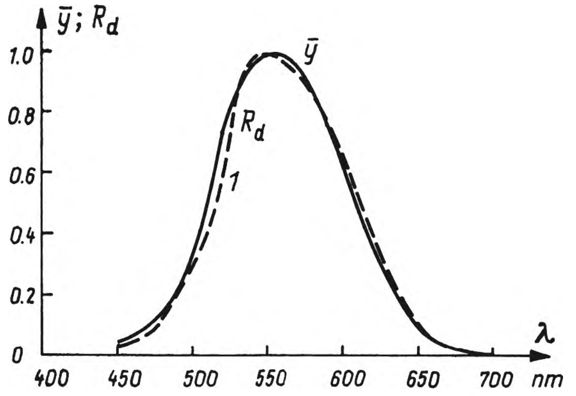

Fig. 4. An example of the selenium cell correction by employing the weighing factors $F(\lambda) = \bar{y}(\lambda)/S(\lambda)$ 1 – the approximating functions, 2 – the approximated function $\bar{y}(\lambda)$

4. The calculation of the zone filter stack for correction a single receptor

Below, a method of zone filter stack calculation will be presented together with the way of employing the method (1) for the case of receiver correction with the filter stack composed of several zones (Fig. 2) by adapting the known method described by W. E. R. DEVIES and G. WYSZECKI [5]. A concise formulation of the sensitivity matching problem for the stack of separatly corrected receptors for a trichromatic colorimeter is also given.

The photocurrent of the receptor is described by the equation

$$i_d = Km \sum_{i=1}^{n} \Phi_{e,\lambda}(\lambda_i) R_d(\lambda) \Delta\lambda \quad (7)$$

in which $\Phi_{e,\lambda}(\lambda_i)$ is the energy distribution in the radiation and $R_d(\lambda)$ denotes the sensitivity distribution of the receiver corrected by filters.

The theoretical value of the given photometric quantity is presented by an integral

$$i_t = Km \sum_{i=1}^{n} \Phi_{e,\lambda}(\lambda_i) R_t(\lambda_i) \Delta\lambda, \quad (8)$$

where $R_t(\lambda)$ is a spectral distribution determined by the given function. If $R_t(\lambda)$ is substituted by the light efficiency function $V(\lambda)$ in (8) then i_t presents the light flux.

The minimum difference of both the expressions evaluated by the least-square method may be put in the following form

$$\sum_{i=1}^{n} \Phi_{e,\lambda}^2(\lambda_i)[R_t(\lambda_i) - R_d(\lambda_i)]^2 = \min \quad (9)$$

assuming for the sake of simplicity is $\Delta\lambda = 1$, this is equivalent to division of the measuring range of photometric quantities into equal intervals of arbitrary value treated during calculations as unity intervals.

The spectral characteristics of the corrected called matched distribution $R_d(\lambda)$ is defined by a function being a product of receiver sensitivity $S(\lambda)$, zone surface areas a_k and a transmission coefficient $\tau_k(\lambda)$ of filters for particular zones k

$$R_d(\lambda) = S(\lambda) \sum_{k=1}^{t} a_k \tau_k(\lambda). \quad (10)$$

In order obtain the value of filter surface area a_k, for which the matching error defined by eq. (9) takes the minimum, the partial derivatives with respect to variables $a_k(k = 1, \ldots, t)$ are calculated. In spite of the formal relation

$$\sum_{k=1}^{t} a_k = \pi r^2$$

all the areas are treated as independent variables because the calculated magnitude of receptor surface area depends upon the accepted relative units, in which the applied photometric quantities have been expressed and may be changed proportionally to all the zones. For the meter with a single receptor the sum of zone areas may be assumed to be equal to the area of the nominal working surface area of the receiver, while the receiver itself should be appropriately calibrated. In order to take account of a set of receptors in further generalization of our considerations, the magnitudes of the working areas of particular receptors are mutually connected and are determined by the ratio of the total sensitivity of particular receptors.

The derivative of (9)

$$\sum_{i=1}^{n} 2\left[R_t(\lambda_i) - R_d(\lambda_i)\Phi_{e,\lambda}^2(\lambda_i)\frac{\partial R_d \lambda_i}{\partial a_k}\right] = 0 \quad (11)$$

is equal to

$$\sum_{i=1}^{n} \{R_t(\lambda_i) - S(\lambda_i)[a_1\tau_1(\lambda_i) + a_2\tau_2(\lambda_i) + \ldots + a_t\tau_t(\lambda_i)]\}\Phi_{e,\lambda}^2(\lambda_i)S(\lambda_i)\tau_k(\lambda_i) = 0. \quad (12)$$

By differentiating with respect to a_k and comparing the derivatives to zero t equations for these variables are obtained

$$\begin{aligned} a_1B_{11}+a_2B_{12}+\ldots+a_tB_{1t} &= B_1,\\ a_1B_{21}+a_2B_{22}+\ldots+a_tB_{2t} &= B_2,\\ &\vdots\\ a_1B_{t1}+a_2B_{t2}+\ldots+a_tB_{tt} &= B_t, \end{aligned} \tag{13}$$

whereas

$$B_{jk} = B_{kj} = \sum_{i=1}^{n} \Phi_{e,\lambda}^2(\lambda_i)S^2(\lambda_i)\tau_k(\lambda_i)\tau_j(\lambda_i) \tag{14a}$$

and

$$B_k = \sum_{i=1}^{n} \Phi_{e,\lambda}^2(\lambda_i)S(\lambda_i)R_t(\lambda_i)\tau_k(\lambda_i). \tag{14b}$$

The solution of these systems of equations gives the sought quantities a_k for zone surfaces.

The universal receivers are often corrected by assuming equal energy radiation distribution. In such a case $\Phi_{e,\lambda}(\lambda_i) = \mathrm{const}$ and being a constant value drops out from the system of equations (13) desappearing from the formulae (14). This results in considerable simplification of calculations.

In the above discussion it is assumed the particular zones of the correcting filter have the spectral distributions of the transmission coefficient determined within the whole wavelength range of the function $R_t(\lambda)$, established in the preceding stage of correction calculations (for instance, with the help of the above programme).

5. Compilation of solutions for layer and zone stacks

So far a general and rigorous solution for optimal layer thicknesses and surface zones of the combined zone-layer stack is not possible. Nevertheless, some special cases are solved exactly, e.g., when two maxima of the required theoretical characteristics are two separate not overlapping peaks, then each of them may be corrected by a separate zone composed of one or several layers (the case of receptor x, i.e. a spectral trichromatic component $\bar{x}(\lambda)$ of a normal XYZ CIE 1931 colorimetric system). In practical realization, constituent elements of the requested theoretical characteristics may be forseen, and a zone composed of optimal layers calculated for each element separately. Next, optimal zone surfaces may be computed. This practical procedure gives an approximate solution.

To find the parameters of zone stack it has to be assumed that the particular zones of the stack exhibit spectral characteristics of the transmission coefficient determined within the whole required wavelength range of the theoretical function, determined in the previous stage of correction according to section 2 in [1]. Next, the calculated layer stacks may be treated as zones of combined zone-layer stack and the method presented in section 3 in [1] may be employed to find the zone surface areas.

The solution of the layer stack described in section 2 in [1] may be exploited, generally speaking, to calculate the correction filters transforming in a specified way the spectral composition of the radiation beam in other words to calculate the so called "ideal" filter or, finally, to correct a colour standard, an illuminant or a light receiver. According to the method presented in [1] in place of characteristics denoted by symbol R the characteristics marked by symbol T should be accepted, which denote an approximation of the ideal filter characteristics

$$T_t(\lambda) = R_t(\lambda):\ S(\lambda) \tag{15}$$

and

$$T_d(\lambda) = C\prod_{j=1}^{m} \exp[-a_j(\lambda)x_j].$$

In this way the optimum thicknesses of correction layer stacks may be computed for separate component elements of the requested theoretical characteristics theseupon the optimum zone surface areas of the combined zone-layer stack for the total characteristics may be found as indicated in section [3].

An example is shown in Fig. 5 for a zone-layer stack composed of two zones, of which one (86°) is empty, while its complement is a layer stack of two filters GG6 $\neq 2$ and VG6 $\neq 2$.

The area under the transmission curve of this filter is represented in the form of a sum of areas under the line parallel to the abscissa (for the empty zone) and that under the curves (for the zones with filters, respectively). The solution (1) is applied to each part separately. A similar procedure may be applied for colorimetric receptors i.e. for $R_t(\lambda)$ defined by equation (2).

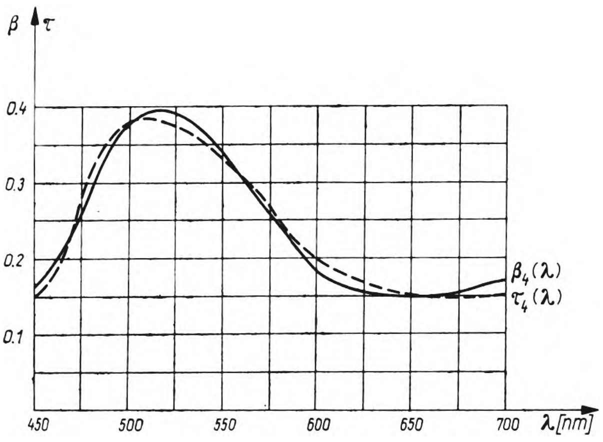

Fig. 5. A comparison of spectral luminance coefficient $\beta(\lambda)$ of colour 2,5 G6/6 and a spectral transmission coefficient $\tau(\lambda)$ of a zone-layer filter stack in the MOBAR 70 device [9]

6. Correction of the receptor system for trichromatic colorimeter

Calculation of the stack of separately corrected receptors becomes simple, if either spectral characteristics of all the receptors can be measured simultaneously under identical experimental conditions, or characteristics of single receptors are measured separately but the coefficients of proportionality between the ordinate of these characteristics are determined under identical experimental conditions.

To perform a simultaneous measurement of the characteristics of all the receptors we must employ either a photometric sphere of the designed colorimeter, or a special photometric sphere. In the latter the receptors directed into the sphere and located symmetrically with respect to beam emerging from the monochromator. Then they do not receive directly the incident beam but that the scattered one. Such a sphere creates the same illuminating conditions for all the receptors.

An alternative measurement of the separate spectral characteristics of particular receptors requires that the sensitivity of all the receptors be compared for a given wavelength under identical experimental conditions. For a proper comparative measurement, the monochromatic flux on the surface of the particular receptors should be kept constant. The wavelength should be chosen so that all the receptors have considerable sensitivity of order of half the maximum value.

In the case of trichromatc x, y, z colorimeter, chosen to show all the essential solutions determining the matching of the receptor set in a possibility form we have

$$\text{(a)} \qquad a'_{x1} = a_{x1}\left(\frac{K}{k_{p,x1}}\right)\frac{\sum\limits_{\lambda=\lambda_0}^{\lambda_1}\bar{x}(\lambda)}{\sum\limits_{\lambda=\lambda_0}^{\lambda_1}x_m(\lambda)},$$

$$\text{(b)} \qquad a'_{x2} = a_{x2}\left(\frac{K}{k_{p,x2}}\right)\frac{\sum\limits_{\lambda=\lambda_1}^{\lambda_2}\bar{x}(\lambda)}{\sum\limits_{\lambda=\lambda_1}^{\lambda_2}x_m(\lambda)}, \qquad (17)$$

$$\text{(c)} \qquad a'_{z} = a_{z}\left(\frac{K}{k_{p,z}}\right)\frac{\sum\limits_{\lambda}\bar{z}(\lambda)}{\sum\limits_{\lambda}z_m(\lambda)},$$

where

$$K = k_{p,y}\frac{a'_y}{a_y}\frac{\sum\limits_{\lambda}y_m(\lambda)}{\sum\limits_{\lambda}\bar{y}(\lambda)}. \qquad (18)$$

In these formulae: $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ are trichromatic spectral sensitivity distributions of receptors corrected separately, k_p are the coefficients transforming the relative distributions to the comparable quantities (i.e. those measured under identical experimental conditions [8], a_{x1}, a_{x2}, a_y, a_z denote the working areas of the receptors at the moment of comparative distribution measurement, a'_{x1}, a'_{x2}, a'_y, a'_z are the corrected working areas of receptors in the stack matched as a whole, and λ is a wavelength from the visible range. The spectral range $\lambda_0 < \lambda < \lambda_1$ corresponds to the part x_1, and $\lambda_1 < \lambda < \lambda_2$ corresponds to the part x_2 of the receptor x. If the relative distribution measurement for the receptor x is made for the parts x_1 and x_2, separately, then the coefficient k_p desintegrates into $k_{p,x1}$ and $k_{p,x2}$. On the other hand if all the relative distributions can be measured under comparable conditions then the coefficients k_p become equal to unity.

The corrected surfaces of receptors are directly proportional to: the receptor surface areas at the moment the comparative characteristics are being measured; the magnification of the reference receptor area (i.e. the ratio a'_y: a_y), and the ratio of sums of trichromatic spectral components of the given receptor to that of the reference receptor. Simultaneously, it is inversely proportional to the ratio of the coefficients k_p and the ratio of sums of the relative characteristics.

In the equations (17) and (18) the receptor y is assumed to be a reference receptor. The choise of the reference receptor area depends on the real magnitude of the receptor surface area; the value of the latter should not exceed the area of the active surface of the least sensitive receptor. This condition being satisfied for initial areas of receptors surfaces a_x, a_y and a_z we may assume $a'_y = a_y$, which leads to a further simplification of the formulae.

A possible alternative procedure is to exploit maximally the receptor area by choosing the area of least sensitive receptor as a maximal admissive area restricted however by the magnitude of the working receptor area.

The equations (17) and (18) derived for the case of equal-energy radiation, i.e. for the correction of universal meter receptor do not contain any spectral distribution $\Phi_{e,\lambda}$ of radiation. In the case of receptor correction of a meter for definite type of lamps the respective spectral flux distribution of the radiation must be introduced to all the sums of spectral trichromatic components in the equations and the relative distributions in the form of a second factor.

7. An analysis of approximation quality and errors

So far, the criterion according to which the measure of spectral correction error could be accepted has not been precisely defined. The leading world laboratories achieve an approximation not exceeding 20% of the ordinate [10] in the short wavelength wing of the relative spectral light efficiency curve within the wavelength range 400–450 mm. Despite such a great approximation error of the receptor sensitivity the inaccuracy of the photometric measurement does not exceed 15%, because of a small of those regions of the spectrum in the measured photometric quantity [10]. The closer the edges of the visual range the greater the approximation errors, especially in corrections achieved by using modest means available in smaller laboratories. The difficulties in error estimations result from the inaccuracies in the wings of the curves, which are not represented in the general error value. Nevertheless, the following errors may be defined:

Maximum error

$$B_{\max} = \max|R_t(\lambda) - R_d(\lambda)|. \qquad (19)$$

Average absolute error

$$B_a = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} |R_t(\lambda) - R_d(\lambda)| d\lambda \qquad (19a)$$

or in approximate form

$$B_a = \frac{1}{n} \sum_{\lambda=\lambda_1}^{\lambda_2} |R_t(\lambda) - R_d(\lambda)|. \qquad (19b)$$

Mean-square error

$$B_k = \left\{ \frac{1}{n} \sum_{\lambda=\lambda_1}^{\lambda_2} [R_t(\lambda) - R_d(\lambda)]^2 \right\}^{1/2}. \qquad (19c)$$

The above errors are improper in the problems refering to correlations of a great eye sensitivity to colours at the wings with the relative spectral light efficiency $V(\lambda)$. For instance this sensitivity in the region of red or blue colour influences considerably the whole perception of light stimulus. Moreover, the excellent eye adaptability to the luminance of the percepted view field in some cases speaks for the relative errors to be used, they can be defined as follows:

Maximum relative error

$$B_{\max,w} = \max \frac{|R_t(\lambda) - R_d(\lambda)|}{R_t(\lambda)}. \qquad (20a)$$

Average relative error

$$B_{a,w} = \frac{1}{n} \sum_{\lambda=\lambda_1}^{\lambda_2} \frac{|R_t(\lambda) - R_d(\lambda)|}{R_t(\lambda)}. \qquad (20b)$$

Relative mean-square error

$$B_{k,w} = \left\{ \frac{1}{n} \sum_{\lambda=\lambda_1}^{\lambda_2} \left(\frac{R_t(\lambda) - R_d(\lambda)}{R_t(\lambda)} \right)^2 \right\}^{1/2}. \qquad (20c)$$

For the above definitions (20) of relative errors the poor corrections are characterized by great inaccuracies due to small value of the function ordinates of $R_t(\lambda)$ at the ends of the range.

This follows from the form of the formulae (20). Such definition of the error is useful in the photometric problems connected with the radiation of narrow spectral ranges at the ends of the measurement range, where $V(\lambda)$ takes small values. This in the case, for instance, in light signalling problems. However, in many other problems, the great contribution of measured quantity error coming from the ends of the sensitivity curve would not give any reliable evaluation of the total measured photometric quantity.

The situation is worsened by a wide application of the excharging sources with strong spectral lines, exceeding the intensity of the continuous part of spectrum by three orders of magnitude. If the approximation in the line position is poor, then the whole correction of the receptor should be classified as unsatisfactory. Fortunately enough, absolute errors can be defined so, that the intensity of the measured radiation be taken into account. In this case a weighing function in the form of a second factor, multiplying the differences of the approximated distribution, is introduced to the formulae (19); the energetic spectral flux distribution $\Phi_{e,\lambda}$ being as the weighing function. The absolute errors defined in this way, represent the differences between the real and measured photometric quantities.

In the case of relative errors (20) an introduction of the radiation flux distribution is not possible, because it is eliminated in the product of factors. An attempt to overcome this difficulty proposed by the author lies in defining the approximation with application of the weights $W(\lambda) = \Phi_{e,\lambda}$. An analogous weight may be also introduced to the definitions of the relative error determined so far by (20). This increases the calculational value of the errors for great values of the weighing function $W(\lambda)$.

The definition of the approximation error may be completed additional conditions for the factor C of the approximation scale. These conditions have been proposed, among others, by G. WYSZECKI [11, 14]. Although the Wyszecki's elaboration concerns the realization of normal sources corresponding to illuminants his definitions may be adapted to spectral correction of radiation receptors. In approximation of trichromatic spectral components in colorimetric receptors another method based on Nimeroff index [11, 14] can be also used.

Finally, total errors of the photometric quantities may be also defined. The approximation error may be approximated by the following definitions:

The total approximate error

$$B = \sum_{\lambda=\lambda_1}^{\lambda_2} R_t(\lambda) - \sum_{\lambda=\lambda_1}^{\lambda_2} R_d(\lambda). \quad (21a)$$

The total approximate relative error

$$B_w = p = \frac{\sum_{\lambda=\lambda_1}^{\lambda_2} R_t(\lambda) - \sum_{\lambda=\lambda_i}^{\lambda_2} R_d(\lambda)}{\sum_{\lambda=\lambda_1}^{\lambda_2} R_t(\lambda)}, \quad (21b)$$

in which B denotes the difference, and B_w denotes the relative difference of areas under the approximated curves representing the spectral sensitivity distribution of the corrected receptor.

A precise definition of total errors of photometric quantities may be given by the equations:

$$B_f = K_m\Big[\sum_{\lambda=\lambda_1}^{\lambda_2} R_t(\lambda)\Phi_{e,\lambda}(\lambda) - \sum_{\lambda=\lambda_1}^{\lambda_2} R_d(\lambda)\Phi_{e,\lambda}\Big] \quad (22a)$$

for the total error, and

$$B_{f,w} = p_\varphi = \frac{\sum_{\lambda=\lambda_1}^{\lambda_2} R_t(\lambda)\Phi_{e,\lambda}(\lambda) - \sum_{\lambda=\lambda_1}^{\lambda_2} R_d(\lambda)\Phi_{e,\lambda}(\lambda)}{\sum_{\lambda=\lambda_1}^{\lambda_2} R_t(\lambda)\Phi_{e,\lambda}(\lambda)} \quad (22b)$$

for the relative total error.

These errors have a simple interpretation as an absolute error and a relative one of the measured photometric quantity, but their evaluation for the purpose of approximation is tedious unless the automatized computation technique is applied. Besides, the errors (22) should not be used as the measure of approximation, because they may be too small for particular approximations with definite spectral radiation distribution on one hand, and give too bad corrections in other applications, on the other one.

To overcome this difficulty, the sum of the squares of several relative differences (those real and measured by a corrected receptor) obtained for the given value of the photometric quantities can be employed for several different spectral radiation distributions. This procedure given by WRIGHT, SANDERS and GIGNAC [6] is based on the formulae

$$P = \sum_{\varphi} p_\varphi^2, \quad (23)$$

in which the values p_φ are identical with the total relative error $B_{f,w}$ (22b). The relative differences p_φ are defined for selected types of lamps of different spectral radiation distributions and although some difference may be accidently small despite considerable deviations of approximation, their sum for lamps of different spectral radiation distribution is representative for the whole spectral range.

Of the two approximation programmes prepared by the authors mentioned above a more accurate one consists in minimizing the sum P and allows to reach the accuracy within the limits $\Delta_{p_\varphi} = 0.1^0/_0$.

In this country good results of experimental correction are achieved by A. MAŃK, who applies this own methods, and proposes as a measure of error

$$E_{\max} = \max \int_{\lambda_i}^{\lambda_k} \frac{[R_t(\lambda) - R_d(\lambda)]W^z(\lambda)}{R_t(\lambda)|\lambda_b - \lambda_a|} d\lambda, \qquad (24)$$

where λ_i and λ_k denote the origine and the end of the considered spectral range and λ_a and λ_b are taken from this range.

However

$$W^z(\lambda) = \frac{|\lambda_b - \lambda_a| \int_{\lambda_a}^{\lambda_b} \Phi_{e,\lambda}(\lambda) d\lambda}{\int_{\lambda_a}^{\lambda_b} \Phi_{e,\lambda}(\lambda) R_t(\lambda) d\lambda}. \qquad (25)$$

The error (24) is transformed into a maximum relative error $B_{\max,w}$ (20a) for a small wavelength interval. The error (24) defined for the needs of experimental evaluation of quality incorporates the features of the relative and absolute error in the weighed spectral range by taking account of spectral radiation distribution. A. MAŃK in [13] gives the error in the form of maximum error $B_{f,w}$ (22b) and defines it for different radiations emitted by natural artificial sources.

8. The application of the computational program to the evaluation of approximation quality and errors

According to the programme already mentioned and prepared in the Institute of Electrotechnics, the Elliott 803B computer calculates the following quantities: the values of ordinates and areas under the curves $R_t(\lambda)$ and $R_d(\lambda)$, the relative area difference p, according to formula (21b), and the ratio q of areas under those curves, the matching function

$$D(\lambda) = \frac{R_d(\lambda)}{R_t(\lambda)} \qquad (26)$$

and its average value. They all allow to conclude about the approximation quality. The value of the area ratio q serves in the programme to calculating the corrected matched distribution $R_d(\lambda)$ of area equal to that under the theoretical curve $R_t(\lambda)$. According to the author's opinion the above quantities should be completed by the calculation of relative errors, using formulae (20b) and (20c) taking eventually into account the measured spectral radiation distribution (the value of error (20a) follows from the analysis of data obtained from the computer in the said programme). The calculation of the relative errors is realized by simple computational operations over the ordinates of distributions and the number of experimental points, the knowledge of which is necessary in each approximation method.

The above conclusion may be justified by the discussion of numerical data (Table) for

The values of the approximation errors of the relative spectral light efficiency $V(\lambda)$ by a spectral distribution of the EMI 9529B photoelectric multiplier corrected with the Schott GG16 BG18 and FGR4 filters shown in Fig. 6

Error definition	Approximation value			
	Two filters		Three filters	
Value of $R_t(\lambda)$	$\lambda = 490$	0.208	$\lambda = 480$	0.139
$R_d(\lambda)$		0.412		0.162
$D(\lambda)$		1.98		1.17
$B_{\max,w}(\lambda)$		0.98		0.17
Sum $R_t(\lambda)$		10.64		10.64
Sum $R_d(\lambda)$		10.44		10.46
Value of p	$\lambda = 490$	0.018	$\lambda = 480$	0.017
q		1.018		1.017
r		1.064		1.005
$R'_d(\lambda) = qR_d(\lambda)$		0.420		0.165
Sum of relative differences		7.94		2.03
Value of $B_{a,w}$		0.305		0.078
Sum of the relative difference of squares		4.35		0.237
Value of $B_{k,w}$		0.41		0.095

The number of points of the spectral range division $n=26$.

a typical case of a EMI 952B photoelectric multiplier correction with two Schott filters (of thicknesses in mm) GG16/1.8 and BG18/0.49 and three filters GG16/0.9, BG18/0.83 and FGR4/4.46, respectively. The approximation has been shown in Fig. 6. The analysis of the results for this approximation proves that in two succesive stages of calculation (for two and three filters) the maximum value of the matching function $D(\lambda)$ decreases from 1.98 to 1.17, while the maximal relative error decreases six times and the area under the curve of

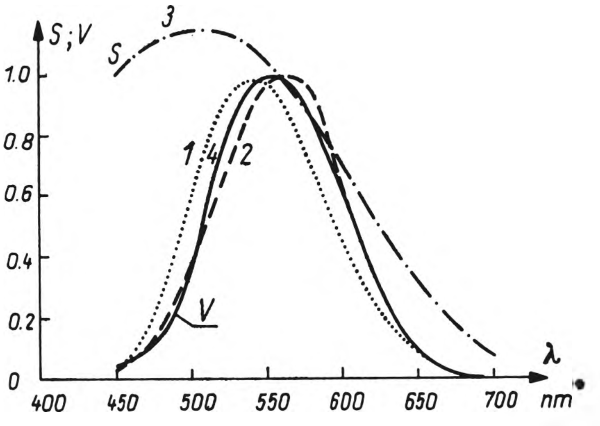

Fig. 6. An example of a two-stage approximation of a photomultiplier; *1* — with two filters, *2* — with three filters, *3* — without filters, *4* — approximated function $V(\lambda)$

the corrected receiver approaches slightly the value of area under the given approximation curve. The value of relative area difference p and the area ratio q behave in a similar way. On the other hand, the average value of the matching function r being a sensitive index quickly approaches the unity. The sums of relative differences of ordinates and the average relative error diminish four times, while the sum of the squares of relative differences drops by factor 1/20, which results in a four time decrease of the mean-square error.

From the above analysis it follows, that it is possible to evaluate in the presented way also the correction achieved with modest means by applying an useful method and modern calculation technique.

In order to establish the set of error definitions necessary for correction evaluation further research works an preparation of physical receivers of optical radiation should performed.

9. An application of the spectral correction method

Finally, it should be explained that the new method of function approximation have been elaborated for a spectral correction of colorimetric elements of the MOBAR meter realized in the Light Technique Section of the Institute of Electrotechnique [9]. The unwanted deviations of the meter characteristics from the requested form are spealing for the initial rather than for the present state difficulties.

It should be noticed that the spectral corrections of the receptors and of the colour patterns for MOBAR were realized by using the zone-layer filter stacks, thus the proper operation of the meter proves the applicability of the methods discussed. They seem to be a further step leading to a relatively simple and cheap spectral correction. By the new method an exact spectral correction can be quickly adrived provided however that of the transmission characteristics of optical filters and the sensibility characteristics of photoelectric radiation receivers are well known.

Satisfactory results of logarithmic approximation of functions suggest moreover its applicability to other problems, which can be represented in the logarithmic form, for instance, an exponential decay of charge or currents of fields.

Other works which either facilitate the application of the solutions given in this article (like [15] which present the subject, phenomena, methods, terminology and application of colorimetry) or present fundamentals of these solutions [16] should be mentioned.

Метод расчета наборов фильтров для спектральной коррекции фотоэлектрических приемников излучения

Изложен новый метод аппроксимации логарифмов функции, представляющих целевое распределение по спектру произвольного колориметрического приемника и распределение по спектру чувствительности физического приемника излучения, а также корретирующих его цветных оптических фильтров, прозрачность которых выражена законом Буге. Собраны результаты и сделаны выводы из эксплуатации программы для ЭЦВМ Эллиот 803B, разработанной в Институте электротехники, касающиеся подбора фильтров и методики расчета слоевых фильтрующих составов. Описана совокупность проблем коррекции трехцветного колориметра.

Поскольку форма распределений по спектру колориметрических приемников близка к колоколообразной кривой, то ввиду специфики аппроксимации логарифмов лучше всего приспособлены крылья кривых (относительные отклонения приблизительно везде одинаковы, а абсолютные наиболее значительны в максимумах кривых). Для более эффективной аппроксимации в макзимумах кривых применяются расчетные весовые функции. Во избежание расчета отрицательных толщин фильтров следует подбирать возможно разные и гладкие их начальные характеристики, а также замораживать очередные результаты расчета и рассматривать полученные характеристики как исходные

для дальнейшего расчета. Сверх того, нельзя неограниченно пользоваться объемом изменчивости всех применяемых функций, значения которых должны быть опреленные и положительные во всей области спектра или, по крайней мере, в той ее части, где рассчитывается аппроксимация.

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Received, October 14, 1975