Optica Applicata, Vol. XXXII, No. 3, 2002

Method of lines — the new vectorial approach to optical phenomena in diode lasers

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A comprehensive vectorial optical approach to optical phenomena in diode lasers, namely the method of lines (MoL), has been compared with a scalar approach, *i.e.*, the effective index method (EIM). Both methods have been applied to model optical behaviour of a modern oxide-confined 1.3-μm quantum-dot (InGa)As/GaAs edge-emitting diode laser. As expected, the scalar EIM approach gives accurate results for diode lasers of standard constructions, *i.e.*, laser structures with relatively wide active regions and slow (not abrupt) changes in the index of refraction. The vectorial MoL, on the other hand, enables us to investigate optical fields in more complicated diode laser structures (for example, in microresonator lasers) for which the above restrictions are not valid.

1. Introduction

Operation of a diode laser may be theoretically simulated using methods of computer physics. Then all physical phenomena crucial for this operation are described mathematically, usually in a form of differential equations. It has been confirmed many times that the physics of semiconductor lasers is very complicated, not only as a result of their complex structures but also because of many (often unexpected) nonlinearities and various (often mutual) interactions between individual physical phenomena. Therefore, a proper description of these interactions is very important for simulation exactness. Even a seemingly negligible interaction should be taken into consideration (if possible) because in some special cases its influence may turn out to be very essential as a result of complex network of many usually nonlinear interactions.

Such comprehensive models of a diode laser operation enable their structure optimization, as well as simulation of their anticipated performance characteristics using the so-called computer experiment. With the aid of such an experiment, an impact of any change in structure construction or in the numerical value of any physical coefficient may be easily investigated. Additionally, physics of a diode laser under consideration may be more deeply examined in a full complexity of many interrelated physical phenomena.

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Comprehensive models of diode lasers are composed of at least four parts: the electrical model, the optical model, the thermal model, and the gain model. The optical model should provide intensity profiles of various radiation modes. Besides, photon numbers belonging to each mode should be found from the rate equations describing mutual interactions between photon fields and carriers.

Just over their lasing thresholds, standard diode lasers are proved to emit linearly TE polarized light in the plane perpendicular to the direction of laser emission [1]. This light is switched to the orthogonal linear polarization when the operating current is increased [2]. So, with an exception of an intermediate case, emission of standard diode lasers may be considered as linearly polarized. Then relatively simple scalar models may be used to describe their optical properties. Their common advantage is obviously their speed.

Recently, transverse sizes of active regions in modern diode lasers are often considerably reduced, especially in the case of vertical-cavity surface-emitting lasers (VCSELs). In such lasers, known as microresonator diode lasers, scalar models may lead to inaccurate results. Then fully vectorial approach to optical phenomena should be applied.

The main goal of this work is to compare the scalar optical approaches in modelling the optical phenomena in diode lasers with vectorial ones, using the effective-index method (EIM) and the method of lines (MoL) as their examples. In particular, limits of usability of simple scalar approaches are determined.

2. Assumptions

The examplary laser structure to be considered is the modern oxide-confined 1.3-μm quantum-dot (InGa)As/GaAs edge-emitting (in plane) diode laser [3] shown in Fig. 1. Let us introduce the co-ordinate system (see Fig. 2) with the z-axis along the direction of light propagation, the yz plane parallel to the p-n junction plane, and the x-axis perpendicular to it.

The current flow is funneled by high-electrical-resistivity Al_xO_y oxide layers located beyond central active region stripe (in the y direction) on both sides of the active region (in the x direction, see Fig. 1). Oxide layers influence also the heat-flux extraction process and the waveguiding phenomenon which is taken into account in the model.

3. Effective index method

The common assumption used in all scalar optical approaches is that the electric field of an electromagnetic wave is polarized in one direction. Then all real polarization pieces of information are lost. In standard diode-laser structures it was usually unimportant. It often becomes crucial in modern diode-laser designs for which optical fields have relatively large transverse components, *e.g.*, in the case of higher-order transverse modes and/or smaller (microresonator) devices. Then more advanced vectorial optical models should be used.

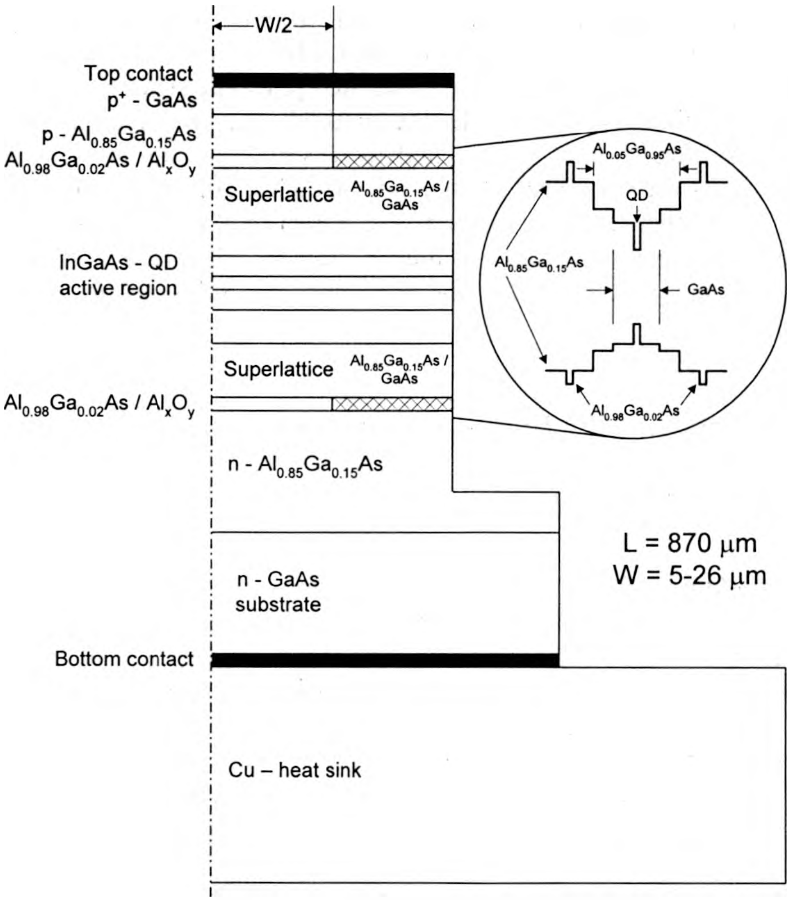

Fig. 1. Structure of 1.3-μm quantum-dot (QD) (InGa)As/GaAs edge-emitting diode laser. L and W — length and width, respectively, of the active region.

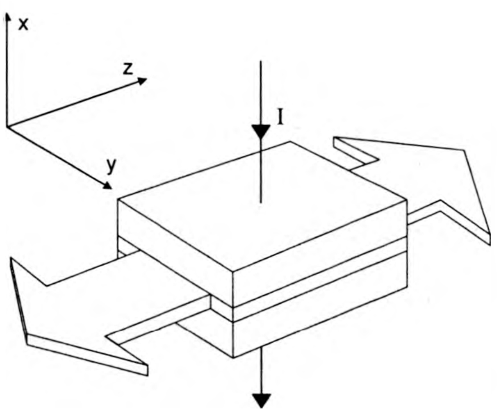

Fig. 2. Co-ordinate system (I — operation current).

Among all known scalar approaches used in the simulation of optical phenomena in diode lasers, the EIM is undoubtedly the most efficient one [4]. It gives surprisingly satisfactory results even for very sophisticated photonic structures with complex distributions of refractive indices. In the method, analyzed regions are divided into narrow subregions for which analytical solutions are found. In inhomogeneous layers, refractive indices should not manifest abrupt changes in directions perpendicular to the wave propagation. The influence of temperature and carrier concentration on changes of refractive indices may be taken into account. Then the optical field Ψ may be expressed as the product of two solutions obtained for perpendicular directions

$$\Psi(x,y) = \Psi_x(x)\Psi_{xy}(x,y), \tag{1}$$

so the two-dimensional problem may be reduced to two one-dimensional ones [4]

$$\frac{\partial^2 \Psi_{xy}}{\partial y^2} + [k_0^2 n^2(x,y) - n_{\rm eff}^2(x)]\Psi_{xy} = 0, \tag{2a}$$

$$\frac{\partial^2 \Psi_x}{\partial x^2} + [k_0^2 n_{\rm eff}^2(x) - \beta^2]\Psi_x = 0, \tag{2b}$$

which are solved using standard mathematical methods. For each x value, the effective index $n_{\rm eff}$ is calculated as a field-weighted average of all refractive indices corresponding to successive layers. The general solution of the wave equation (2) for the field corresponding to the m-th mode in the j-th layer may be expressed in the form of superposition of two waves running in the opposite directions

$$\Psi_{j,m} = A_{j,m}\exp[\gamma_{j,m}(x-x_j)] + B_{j,m}\exp[-\gamma_{j,m}(x-x_j)] \tag{3}$$

where $\gamma_{j,m}$ is the complex propagation constant for the m-th mode in the j-th layer and x_j stands for the x-coordinate of the interface between the j-th layer and the $(j+1)$-th layer.

4. Method of lines

In order to properly model the operation of modern diode laser structures, it is imperative to be able to describe optical, electrical, thermal, and gain phenomena crucial for this operation with much better exactness than in earlier, less detailed models. In particular, more exact, comprehensive optical models of diode lasers should be vectorial ones, *i.e.*, without any artificial restrictions regarding directions of the electric and magnetic fields.

The MoL [5] is such a semianalytical fully vectorial optical approach. The laser structure is divided into layers of constant material parameters along the x direction. In the simulation of edge-emitting devices, MoL allows lateral (along the y axis) and longitudinal (along the z axis) dependences of the refractive index, carrier density, optical gain, and temperature to be included. Analogous radial and azimuthal changes are acceptable in VCSEL modelling. In the method discretization is performed only as far as it is necessary, whereas an analytical procedure is used elsewhere. This enables obtaining accurate results with less computational effort than in case of other fully vectorial techniques such as the finite element method or the finite difference method. What is more, the MoL approach has no problems with the relative convergence behaviour. Non-physical or spurious modes do not appear in MoL. It makes possible to analyze small size structures.

The solution is looked for in an analytical form along the x direction (perpendicular to the p-n junction plane), whereas the structure is discretized along the y direction. The y derivatives in the wave equation are replaced with appropriate difference operators [5], which yield the set of coupled ordinary differential equations. The above equations have to be orthogonalized. The general solution is assumed in a form of a superposition of exponential functions. The optical gain and optical losses are introduced as imaginary parts of the complex refractive indices of successive structure layers. Finally, for the optical field of the m-th mode within the j-th layer, one obtains the following decoupled equation system:

$$\frac{d^2}{dx}\bar{\Psi}_{j,m} - k_{x,j,m}^2\bar{\Psi}_{j,m} = 0, \tag{4}$$

with the analytical solutions

$$\bar{\Psi}_{j,m} = A_{j,m}\cosh\left[k_{x,j,m}(x-x_j)\right] + B_{j,m}\sinh\left[k_{x,j,m}(x-x_j)\right] \tag{5}$$

where $\bar{\Psi}_{j,m}$ must satisfy the Helmholz equation and the Sturm–Lieuville differential equation [6] and where $\bar{\Psi}_{j,m}$ stands for the optical field after transformation to principle axis. The original field is obtained by an inverse transformation [7]. More details about the solving algorithm may be found in papers [5]–[7].

5. Results

Usability of both optical approaches in diode-laser modelling, *i.e.*, the EIM and the MoL, are compared for a modern oxide-confined 1.3-μm quantum-dot edge -emitting diode laser [3] (see Fig. 1). In two figures (Fig. 3 and Fig. 4), values of the effective refractive index n_{eff} determined using both the methods are plotted as functions of some construction parameters, *i.e.*, the aperture width S left for the current flow in the high-electrical-resistivity oxidation layer (being very close to the width W of the stripe active region) and the thickness d_{ox} of this layer. All properties of an optical field can be determined using the n_{eff} value. As expected, the most distinct differences between results obtained with the aid of both the methods can be noticed in the case of small apertures or abrupt changes in refractive indices.

As one can see in Fig. 3, results of both the methods are in quite a good agreement for relatively wide apertures ($S \gg \lambda$), whereas they become divergent for aperture widths S comparable with the emission wavelength λ. Higher values of the

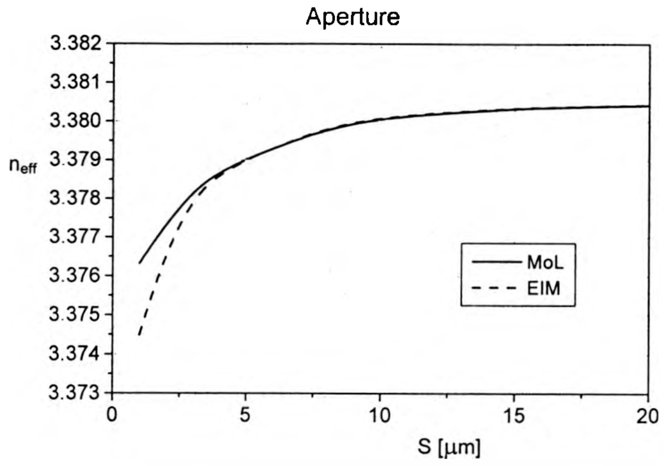

Fig. 3. Impact of the aperture width W on the effective index n_{eff} with calculated the aid of the MoL and EIM.

effective index of refraction obtained using the vectorial MoL approach mean that in this case the EIM scalar approach underestimates concentration of an optical field in the active region, *i.e.*, in the central area between the low-index-of-refraction oxide layers.

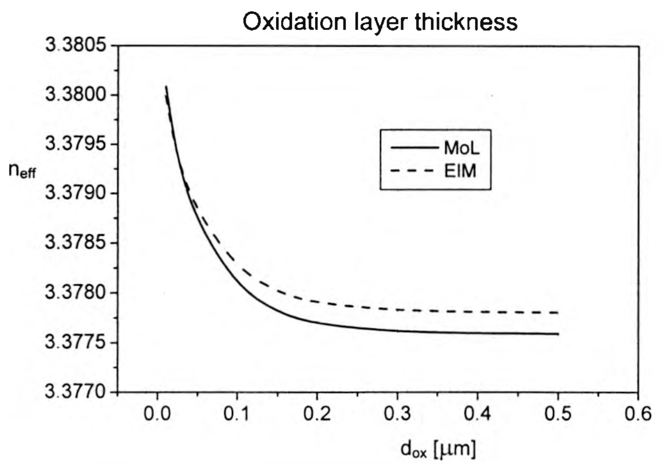

Fig. 4. Impact of the aperture width d_{ox} on the effective index n_{eff} calculated with the aid of the MoL and EIM.

Figure 4 illustrates an influence on n_{eff} of the thickness d_{ox} of the Al_xO_y oxidized layer. The thinner this layer is the less disturbance in the distribution of the refractive index in the y direction parallel to the p-n junction plane is induced and the assumptions introduced in the scalar EIM approach become more exact.

Therefore both the scalar and the vectorial solutions are very close to each other for a thin oxidized layer. With an increase in thickness of this layer, differences between both the above solutions become more distinct.

6. Conclusions

In the present work, usability of scalar and vectorial optical approaches has been compared using the effective index method and the method of lines as their examples. As expected, the scalar EIM approach has been found to give accurate results for standard edge-emitting diode lasers with relatively wide active regions and slow (not abrupt) changes in distributions of an index of refraction. The vectorial MoL approach enables us to emphasize more subtle behaviour of laser optical fields, which turns out, however, to be sometimes crucial for performance analysis of more sophisticated laser structures.

Acknowledgments – This work was supported by the State Committee for Scientific Research (KBN), Poland, grants Nos. 7-T711B-045-21 and 7-T11B-073-21.

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Received May 13, 2002