*Florian Ratajczyk**

On Influence of the Triangle Striae on the Strehl Definition of the Aberrational Optical System

In this article a way of determining the changes in Strehl definition in real optical systems is presented, which include N arbitrarily oriented striae in the pupil plane. The calculations have been performed under assumption that the striae cause a triangle deformation of the wavefront. The maximal values of those deformations may be different for different striae but must be constant along the stria lengths.

1. Introduction

In the course of last years the influence of the extended heterogeneities, like striae and birefringence on the imaging quality of perfect optical systems have been analysed in the literature in a relatively detailed way. Among those papers only one disscussed the influence of the striae on the Strehl definition in the aberrated systems, under a simplifying assumption that the striae cause a restangular deformation of the incident wavefront. The present paper is a first attempt to estimate the influence of the real striae producing a triangular deformation of the wavefront on the Strehl definition in real optical systems.

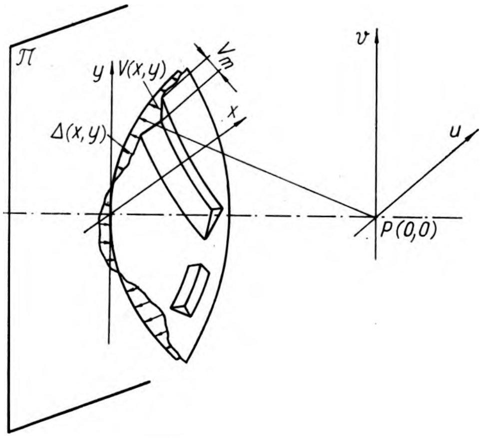

Fig. 1. A deformation of the aberrational wavefront in the exit pupil caused by the presence of striae

2. An Influence of the Triangle Striae Located in the Pupil of an Aberrated Optical System on the Strehl Definition

Let us assume that plane π is given in the exist pupil together with a Gauss sphere tangent to π with the centre at the point $P(0, 0)$ (Fig. 1). The wave aberration $V(x, y)$ measured along the Gauss sphere radius from the real wave front to the said sphere is an algebraic sum of the own aberrations $\Delta(x, y)$ of the system, and the aberration $V_s(x, y)$ introduced by the striae i.e.

$$V(x, y) = \Delta(x, y) + V_s(x, y). \qquad (1)$$

The coordinates x and y are the so-called dimensionless coordinates of a Cartesian coordinate systems located in the exit pupil plane. Let as assume that the pupil is circular, of the diameter $2a$, and its centre coicides with the x, y-coordinate system origin. When assuming the radius of the pupil as a reference value we have

$$x = \frac{x'}{a}, \; y = \frac{y'}{a},$$

where x', y' — dimensional coordinates.

As it is well-known the Strehl definition in the case of small aberrations of the system is defined by the formula

$$J = 1 - K^2 [\langle\langle V^2(x, y)\rangle\rangle - (\langle\langle V(x, y)\rangle\rangle)^2], \qquad (2)$$

the averaging being performed over the exit pupil area

$$\langle\langle V^2(x, y)\rangle\rangle = \frac{1}{\pi} \iint_S V^2(x, y) dxdy,$$

$$\langle\langle V(x, y)\rangle\rangle = \frac{1}{\pi} \iint_S V(x, y) dxdy,$$

where $S = \dfrac{S'}{a^2} = \pi$ denotes its normed surface.

By taking account of the fact that the wave aberration is a sum of the system own aberration and those

* Institute of Physics, Wrocław Technical University, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

introduced by the striae (1) the average values occuring in (2) may be represented in the form:

$$\langle\!\langle V^2(x,y)\rangle\!\rangle = \langle\!\langle(\Delta(x,y)+V_s(x,y))^2\rangle\!\rangle$$
$$= \langle\!\langle\Delta^2(x,y)\rangle\!\rangle + 2\langle\!\langle\Delta(x,y)V_s(x,y)\rangle\!\rangle + \langle\!\langle V_s^2(x,y)\rangle\!\rangle (\langle\!\langle V(x,y)\rangle\!\rangle)^2 = (\langle\!\langle\Delta(x,y)\rangle\!\rangle + \langle\!\langle V_s(x,y)\rangle\!\rangle)^2$$
$$= (\langle\!\langle\Delta(x,y)\rangle\!\rangle)^2 + 2\langle\!\langle\Delta(x,y)\rangle\!\rangle\langle\!\langle V_s(x,y)\rangle\!\rangle + (\langle\!\langle V_s(x,y)\rangle\!\rangle)^2. \quad (3)$$

Let us assume further that N triangle striae of $2b_j$ width $2h_j$ lengths are located in the pupil causing maximum deformation of the wavefront V_{mj}(Fig. 2). The quantities b and h are normed.

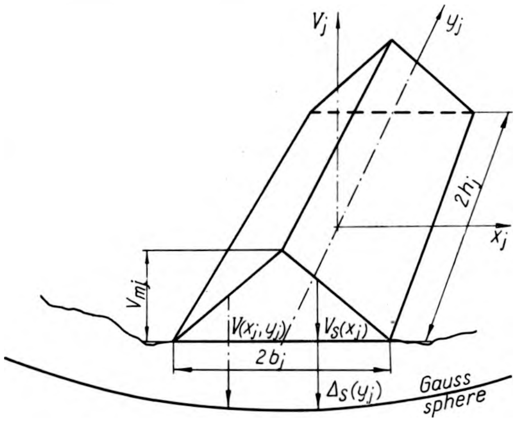

Fig. 2. The geometrical magnitudes describing wavefront aberration caused by a triangle stria

The average quantities occurring in the formula (3) and including aberrations caused by striae may by easily estimated for j-th striae, by assuming a x_j, y_j-coordinate system, the axes of which being respectivly perpendicular and parallel to the striae. Then we have for j-th striae:

$$\langle\!\langle V_{sj}(x_j, y_j)\rangle\!\rangle = \frac{S_{sj}}{2\pi} V_{mj},$$

$$\langle\!\langle V_{sj}^2(x_j, y_j)\rangle\!\rangle = \frac{S_{sj}}{3\pi} V_{mj}^2,$$

where S_{sj} presents a normed area of the j-th striae.

Some troubles are connected with the estimation of the average product expression «$\Delta(x_j, y_j)V_{sj}(x_j, y_j)$». It has to be noted that the product is different from zero only in the region of striae. Outside the striae $V_{s_j}(x_j, y_j) = 0$.

The striae in the glass are typically of small width. The aberration $\Delta(x_j, y_j)$ may be considered as a constant quantity along the striae width, and thus, it is a function of one variable y_j. In order to emphasize that this is only the wave aberration of this part of the wave surface, which is covered by deformation caused by striae, we introduce an index s and write:

$$\Delta_{sj}(x_j, y_j) = 2b\Delta_{sj}(y_j).$$

From Fig. 2 it is clear that the function $V_{s_j}(x_j y_j)$ is one-dimensional, i.e.

$$V_{sj}(x_j, y_j) = 2hV_{sj}(x_j).$$

Hence

$$\langle\!\langle V_{sj}(x_j, y_j)\Delta_{sj}(x_j, y_j)\rangle\!\rangle = \frac{1}{\pi}\int_{-b}^{+b}\int_{-h}^{+h} V_{sj}(x_j)\cdot\Delta_{sj}(y_j)dx_j dy_j = \frac{V_{mj}}{2\pi} 2bj \int_{-h}^{+h}\Delta_{sj}(y_j)dy_j = \frac{V_{mj}S_{sj}}{2\pi}\bar{\Delta}_{sj}.$$

where $\bar{\Delta}_{s_j}$ denotes an average value of the own aberrations of the system within the j-th striae area.

After substituting the obtained averaged values to the formule (3) we obtain for the N striae

$$\langle\!\langle V^2(x,y)\rangle\!\rangle = \langle\!\langle\Delta^2(x,y)\rangle\!\rangle + \sum_{j=1}^{N} V_{mj}\bar{\Delta}_{sj}\frac{S_{sj}}{\pi} + \frac{1}{3}\sum_{j=1}^{N} V_{mj}^2\frac{S_{sj}}{\pi}\,(\langle\!\langle V(x,y)\rangle\!\rangle)^2$$
$$= (\langle\!\langle\Delta(x,y)\rangle\!\rangle)^2 + \langle\!\langle\Delta(x,y)\rangle\!\rangle\sum_{j=1}^{N} V_{m_j}\frac{S_{sj}}{\pi} + \frac{1}{4}\Big(\sum_{j=1}^{N} V_{mj}\frac{S_{sj}}{\pi}\Big)^2.$$

By substituting these expressions, in turn, to the formula (2) the final form of this formula is obtained; it determines the Strehl definition of the aberrated system including N striae, each producing a triangular deformation of wavefront:

$$J = J_0 - K^2\Big[\sum_{j=1}^{N} V_{mj}\bar{\Delta}_{sj}S_{wj} - \langle\!\langle\Delta(x,y)\rangle\!\rangle\sum_{j=1}^{N} V_{mj}S_{wj} - \frac{1}{4}\Big(\sum_{j=1}^{N} V_{mj}S_{wj}\Big)^2 + \frac{1}{3}\sum_{j=1}^{N} V_{mj}^2 S_{wj}\Big], \quad (4)$$

where J_0 — denotes the Strehl definition of the aberrated system without striae, S_{wj} — is the relative area of the j-th stria (the ratio of the j-th stria area to that of the pupil).

In the case of a single stria the formula (4) is simplified to the form:

$$J = J_0 - K^2 V_m^2 \left[\frac{1}{3} - \frac{S_w}{4} + \frac{\bar{\Delta}_s - \langle\langle \Delta(x,y) \rangle\rangle}{V_m} \right] S_w. \quad (5)$$

If it is additionally assumed that the striae area is not large as compared to the beam cross-section area the formula (5) may be written in a more compact form

$$J = J_0 - K^2 V_m^2 \left(\frac{1}{3} + \frac{L}{V_m} \right) S_w, \quad (6)$$

where L denotes a difference between the average wave aberration of the region of striae and the average wave aberration of the system in the pupil area.

3. Discussion of results

Form (6) it may the easily determined which striae are disturbing in the optical system. It may be concluded that the Strehl definition I will be lowered if

$$\frac{L}{V_m} > -\frac{1}{3}.$$

It is interesting that the striae may improve the Strehl definition of the system. This occurs if

$$\frac{L}{V_m} < -\frac{1}{3}.$$

The formula (5) determining the influence of the striae on the Strehl definition is an approximate one. The approximation may be estimated by comparing J calculated from (5) with that evaluated from non--simplified formula.

The values given in the table have been calculated on the basis of both the formulae. The percentage error is also presented.

V_m	$\lambda/4$			$\lambda/8$		
S_w	Approximate result	Accurate result	Error	Approximate result	Accurate result	Error
0.1	0.924	0.933	1	0.981	0.981	0
0.05	0.959	0.965	0.6	0.990	0.990	0

From the table it follows that for really possible values of S_w and the wave aberrations introduced by the striae the approximation is absolutetly satisfactory for practical applications. Formula (5) may be used for tolerating the striae in the glass in the optical systems of small own aberrations.

Влияние треугольных полос на светлоту Штреля аберрационных оптических систем

В статье представлен способ определения изменения светлоты Штреля действительных оптических систем, содержащих в зрачке произвольно ориентированных полос. Расчет производился при предположении, что полосы вызывают треугольную деформацию фронта волны. Максимальные значения этих деформаций могут быть различны для разных полос, но они должны быть постоянными на всей их длине.

References

[1] Hofmann Ch., Reichardt I., Experimentalle Technik der Physik XXIII, 1975, copy 5, p. 513–523.

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