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The Effect of Spherical Aberration on an Electron Beam with Gaussian Current Density Distribution

The effect of spherical aberration on the characteristic parameters of the beam, such as half-radius, characteristic radii and maximal current density has been examined by calculating the beam current density distribution for the best focusing plane. The results are given in the form of diagrams and approximate analytical relations.

1. Introduction

An essential problem in designing electron-optical systems is the estimation of the effect of aberrations on the electron beam diameter and current density. To solve this problem, it is necessary to take account of spherical aberration, considered as the error restricting resolving power of electron-optic devices, the effective compensation of which is as yet not possible.

Many ways of combining electron-optical aberrations have been proposed; for example the beam radius in the image plane and the radii of the aberration spots may be summed either linearly or in quadrature (geometrical composition) [1]. So far, however, these propositions have not been adequately tested and for many other reasons seem to be unacceptable. The problem of combining electron-optical aberrations has also been discussed by K. I. Harte [2]. He has shown that if an electron beam is distorted by electron-optical aberrations and its current density is distributed arbitrarily, then the mean square deviation (moment of the second order) of the current density at the image is the geometric sum of the mean square deviations for the distributions of the aberration-free beam (S_0) and of the aberration scattering spots ($S_1, S_2, \ldots$), according to the formula

$$S^2 = S_0^2+S_1^2+S_2^2+\ldots . \quad (1)$$

The theorem presented is of a great cognitive importance, because of its general character. In practice, however, it can hardly ever be applied. On the one hand, mean square deviation cannot be calculated for all current density distributions (e.g. for a distribution of the type $j(r) = A(l+r^2)^2$ with a finite current $I = \pi A$, mean square deviation $S = \alpha$). On the other hand, this parameter does not give satisfactory information about physical properties of electron beam. Although the reconstruction of the beam current density distribution based on a sequence of moments of higher order is possible, even the calculation of the moment of fourth order raises serious difficulties; moreover, the composition principles of these moments are not known.

The best solution, which allows the effect of electron-optical aberrations on electron beam to be taken into account, is to determine the current density distribution of the final beam. This distribution can be determined numerically [3]. Being, however, rather complicated and time consumming this process is not much use for practical purposes. In many cases, sufficient information about the character of the current density distribution of the beam can be obtained from knowledge of the distribution in several points, i.e. by the knowledge of several radii (distances) for which the current density of the beam assumes defined values. For a given current density distribution of the aberration-free beam and for a particular value of the electron-optical aberration, the dependence of such characteristic radii on error value can be presented graphically or in the form of approximate analytical relations. Although such a convenient form cannot be used to illustrate the effects of all electron-optical aberrations (and their combinations) on all the possible types of current density distributions of the beam, nevertheless, the discussion of a few basic cases can be of practical importance. In the subsequent parts of the present paper, an attempt has been made to illustrate the effect of spherical aberration on an electron beam with Gaussian distribution of current density.

2. Calculation method

In order to determine the current density distribution of the final beam, a numerical method [3] has been used. This method resembles that employed by Harte [2] in initial assumptions.

The current density of an ideal (aberration-free) beam in the image plane, at a point with radial co-

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ordinate ϱ, is a sum of current densities of elementary current streams reaching this point. With the assumption that the beam has an axial symmetry the current density of such stream is given by the relation

$$dI_1 = B(\varrho)d\omega = B(\varrho)\alpha d\psi d\alpha, \tag{2}$$

where:

B — electron brightness,

α — convergence angle,

ψ — azimuth angle,

ω — solid angle.

In the case of electron-optical aberrations, the point of final image with coordinate r is reached by elementary streams intended for different points of the ideal image with different coordinates ϱ. These points are contained within the figure determined by the error Δ of the electron-optical aberration in question (Fig. 1). The coordinate ϱ of the ideal image

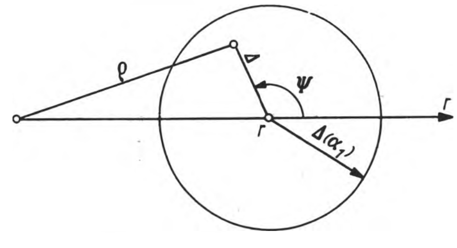

Fig. 1. Scattering region of electron beam

point transmitting one of current streams to the final image point of the coordinate r, can be calculated from the Carnot's formula

$$\varrho^2 = r^2+\Delta^2+2r\Delta\cos\psi. \tag{3}$$

If we assume that the electron brightness distribution in the ideal image is a Gaussian one, then in view of (2) and (3) the current density distribution in the final image can be described by the formula

$$I_{1S} = \int_0^{\alpha_1}\int_0^{2\pi} B(r,\alpha,\psi)\alpha d\psi d\alpha = \int_0^{\alpha_1}\int_0^{2\pi} B_0\exp\left[-\frac{1}{a^2}(r^2+\Delta^2+2r\Delta\cos\psi\right]\alpha d\psi d\alpha, \tag{4}$$

where

B_0 — maximal brightness,

a — parameter of Gaussian distribution,

α_1 — aperture angle of the beam on the image side.

From now on, only spherical aberration is taken into account. The value of this error at the best focusing plane is given by the equation

$$\Delta = C_S\alpha^3+Z_p\alpha, \tag{5}$$

where the distance Z_p of the best focusing plane from the Gaussian plane is given by the relation

$$Z_p = -\frac{3}{4}C_S\alpha_1^2, \tag{6}$$

and C_S is the spherical aberration constant.

Some numerical calculations based on the relations (4), (5) and (6) and performed by means of a computer allowed us to determine fourteen current density distributions of the beam in the best focusing plane for spherical aberration errors ranging from $\Delta_S = 0.5a$ to $\Delta_S = 100a$. The maximal value of the error referred to the Gaussian imaging plane, denoted by Δ_S, was determined from the relation

$$\Delta_S = C_S\alpha_1^3. \tag{7}$$

The results of several calculations are illustrated in Fig. 2. These curves represent reduced current density distributions of the final beam in the best focusing plane, for the given values of the spherical aberration error Δ_S. The radial coordinate r of the distribution was referred to the half-radius r_h, that is, the value of radial coordinate r, within which one-half of the total value of beam current I is contained, i.e.

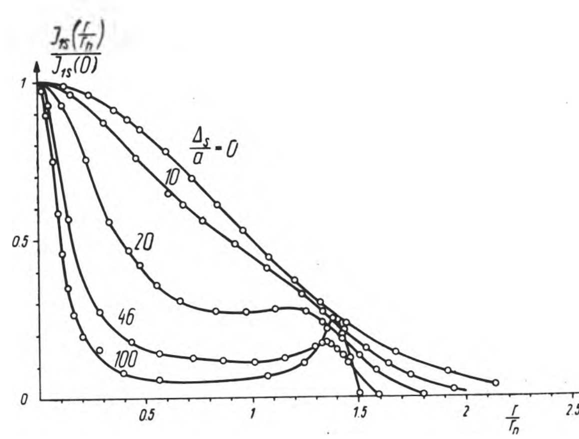

Fig. 2. Reduced distributions of electron beam current density

$$\int_0^{r_h} I_{1S}(r)2\pi r dr = \frac{1}{2}\int_0^{\infty} I_{1S}(r)2\pi r dr = \frac{I}{2}. \tag{8}$$

3. Parameters of the beam current density distribution

For the current density distributions of the beam, determined numerically, a number of parameters such as half-radius, characteristic radii and maximal beam current density, constituting a simplified characterization of these distributions, have been assumed. Further parts of the paper present the dependences of these parameters on the value of spherical aberration error.

A. Half-radius

The current density distribution within the spherical aberration spot can be determined in the following way:

The current dI of an elementary current stream contained within the element of solid angle, is given by the equation

$$dI = id\omega = i\,ad\psi da, \tag{9}$$

where i is angular current density of the beam.

This elementary current stream falls on the appropriate element of the surface dF of the aberration spot, satisfying the relation

$$dI = J_S dF = J_S \Delta d\Delta d\psi, \tag{10}$$

where J_s is a surface current density of the beam.

The total current of the beam is thus given by the following expression

$$I = \int_0^{a_1}\int_0^{2\pi} i\,ad\psi da = \pi i a_1^2. \tag{11}$$

In the best focusing plane, the current flowing through the element of aberration spot surface is the sum of the currents of three current streams, the inclination angles a of which are the roots of the error equation (5). Thus, the beam current contained within a half-radius r_{hs} results from the equation

$$\frac{I}{2} = 2\pi i\left(\int_0^{a_{1h}} ada + \int_{a_{2h}}^{a_0} ada + \int_{a_0}^{a_{3h}} ada\right)$$
$$= \pi i(a_{1h}^2 - a_{2h}^2 + a_{3h}^2) \tag{12}$$

where a_{1h}, a_{2h}, a_{3h} are the roots of the error equation (5) with the assumption that $\Delta = r_{hs}$, and a_0 is inclination angle of the ray intersecting the origin of the system of coordinates. To determine the half-radius r_{hs} it is necessary to calculate only one of the three roots of the error equation. Using Vieta's formulae and equations (11) and (12) we get

$$a_{2h} = \sqrt{-\frac{Z_p}{C_s} - \frac{a_1^2}{4}} = \frac{a_1}{\sqrt{2}}. \tag{13}$$

Finally, the half-radius of the spherical aberration spot in the plane of best focus is given by the relation

$$r_{hs} = C_s a_1^3\left(-\frac{1}{4\sqrt{2}}\right) = -\frac{\Delta s}{4\sqrt{2}}. \tag{14}$$

The density distribution of the current beam within the aberration spot is obtained by comparing relations (9) and (10) and taking account of the error equation in the form

$$J_s = \frac{iada}{\Delta d\Delta}$$
$$= \frac{i}{(C_s a^2 + Z_p)(3C_s a^2 + Z_p)}. \tag{15}$$

Using the relations (5), (6) and (11), the mean square deviation S_s for the above distribution may be written:

$$S_s^2 = \frac{1}{I}\int_0^1 \Delta^2 J_s 2\pi\Delta d\Delta = \frac{1}{\pi i a_1^2}\int_0^{\Delta_1} \Delta^2 \frac{iada}{\Delta d\Delta} 2\pi\Delta d\Delta = \frac{2}{a_1^2}\int_0^{a_1}(C_s d^3 + Z_p a)^2 ada = \frac{1}{32}C_s^2 a_1^6 = \frac{\Delta_s^2}{32}. \tag{16}$$

Comparison of (14) and (16) leads to the conclusion that for an aberration spot in the best focusing plane, the absolute values of the half-radius r_{hs} and of the mean square deviation S_s are equal. For a Gaussian distribution, the respective values of the half-radius ϱ_h and the mean square deviation S_0 amount to

$$\varrho_h = a\sqrt{\ln 2} = 0.833a, \quad S_0 = a. \tag{17}$$

The difference between them is thus relatively small. Hence, it may be assumed with some approximation, that, in conformity with equation (1), the half-radii of a beam with a Gaussian current distribution ϱ_h and of an aberration spot r_{hs} add geometrically in the best focusing plane. Thus, the half-radius of final beam r_h results from the relation

$$r_h \simeq \sqrt{\varrho_h^2 + r_{hs}^2} = a\sqrt{\ln 2 + \frac{\Delta_s^2}{32a^2}}. \tag{18}$$

Numerical calculations of half-radii r_h performed for several current density distributions — determined from the relation (4) — have shown that formula (18) leads errors not exceeding a few per cent.

B. Characteristic radii

Three characteristic radii denoted by $r_{2/3}$, $r_{1/2}$, and $r_{1/e}$ have been considered. These radii describe the position of distribution points for which the relative values of current density amount to 2/3, 1/2, and $1/e$, respectively, according to the formulae:

$$\frac{I_{1S}(r_{2/3})}{I_{1S}(0)} = \frac{2}{3}, \quad \frac{I_{1S}(r_{1/2})}{I_{1S}(0)} = \frac{1}{2}, \quad \frac{I_{1S}(r_{1/e})}{I_{1S}(0)} = \frac{1}{e}. \tag{19}$$

General rules describing the effect of spherical aberration on characteristic radii are difficult to obtain analytically. The relationship is presented graphically in Fig. 3, using a number of current density distributions determined numerically. The values of consecutive characteristic radii were referred to the successive values of half-radius r_h. These curves show that the dependence of the characteristic radii on spherical aberration is complicated and cannot be presented in the form of a simple relation, holding within the

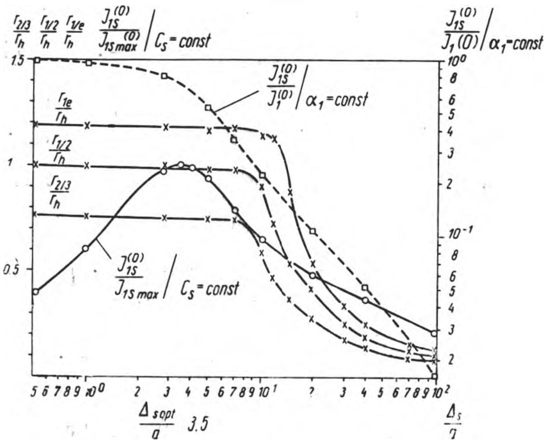

Fig. 3. Reduced parameters of electron beam current density distribution vs the error of spherical aberration

whole range of errors considered by us. Nevertheless, it can be observed that for small errors, $\Delta_s \leqslant 7a$ the final current density distributions differ little from a Gaussian, and the characteristic radii keep a constant proportion with respect to the half-radius. Consequently, the following relations can be used (for $\Delta_s \leqslant 7a$) with an error less than 3%,

$$r_{2/3} \simeq 0.766\, r_h,$$

$$r_{1/2} \simeq r_h,$$

$$r_{1/e} \simeq 1.2\, r_h,$$

where r_h is calculated from (18). However, within the error range mentioned above, the most convenient way is to use a Gaussian distribution and to correct the distribution parameter, according to the relation

$$a_s \simeq 1.2\, r_h \quad \text{for} \quad \Delta_s \leqslant 7a. \tag{21}$$

C. Maximal current density

The effect of spherical aberration on maximal beam current density was already discussed analytically in [3], using a series expansion of (4). Accurate numerical calculation allow us to make precise the results obtained previously.

The effect of spherical aberration on axial (i.e. for $r = 0$) density of beam current in the best focusing plane is illustrated by two curves presented in Fig. 3. The first curve represents the axial density of the final beam referred to the axial current density of an aberration-free beam with the same aperture angle ($\alpha_1 = \text{const}$). Within the range $\Delta_s \leqslant 10a$ this curve can be approximated, with an error smaller than 4%, by the relation

$$\left.\frac{I_{1s}(0)}{I_1(0)}\right|_{\alpha_1=\text{const}} \simeq \left(\frac{\varrho h}{r_h}\right)^{7/4}, \tag{22}$$

in which the effect of spherical aberration has been taken into account by considering the half-radius r_h.

The second curve describes the axial current density of the final beam I_{1s} (0), referred to the maximal value of this density $I_{1s\max}$ (0) in conditions when the constant of spherical aberration is invariant ($C_s = \text{const}$). The maximum value of axial current density is obtained when

$$\alpha_{1\,\text{opt}} = \left(\frac{3.5a}{C_s}\right)^{1/3} \tag{23}$$

and for the resulting optimal value of spherical aberration:

$$\Delta_{s\,\text{opt}} = 3.5a. \tag{24}$$

4. Conclusions

The basic purpose of the paper was to analyse the effect of spherical aberration on the electron beam current distribution and to present the results in a form convenient for practical applications. The assumed Gaussian current density distribution of an aberration-free beam is most similar to the distributions observed in the case of beams with a small perveance. The range of errors, within which its effect on the distribution parameters was successfully represented by simple analytical relations, is sufficiently wide for practical applications, since its exceeds twice the error value recognized to be optimal.

Влияние сферической аберрации
на электронный пучок с гауссовым распределением
плотности тока

Исследовано влияние сферической аберрации на характеристические параметры пучка, такие как половиночный луч, характеристические и максимальные лучи, плотность тока, причем вычислено распределение плотности тока пучка для наилучшей плоскости фокусирования. Результаты приведены в форме диаграмм и приближенных аналитических связей.

References

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Received, October 30, 1976