

Detection of Cherenkov radiation: the inverse problem to generation of nondiffracting beams

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Nondiffracting beams have been in the center of interest for the last decade. These beams, however, can be realized in practice only approximately. The simplest scheme for generating such a beam is to use a sufficiently narrow annular source placed in the back focal plane of a lens. Behind the lens, the beam is pseudo-nondiffracting in some length. The inverse problem is detection of Cherenkov radiation. The trace of a charged particle travelling through a dielectric medium faster than light emits this radiation at a certain angle. Generated conical wave front creates in the back focal plane of a lens a diffraction pattern in the form of a ring. A theoretical analysis of both problems is given in the paper.

Keywords: Cherenkov radiation, nondiffracting beam, conical wavefront.

1. Introduction

The first to draw attention to nondiffracting beams were DURNIN and co-workers [1], [2]. They formulated a concept of this kind of beams and analyzed their fundamental properties. Naturally, this gave rise to a question of how to generate such beams. Although one of the first suggestion was to make use of holographic generation [3], in the main method an axicon lens was employed [4]. Actual methods of generation cannot produce perfect diffraction-free beams. Such beams are called a pseudo-nondiffracting beams [5], [6], which have complicated angular spectra [7]. In our analysis, the pseudo-nondiffracting beams are considered as being generated by use of an annular source located in the back focal plane of a collimator lens. Only an infinite narrow slit of the source is considered, which means that the beam is described by the Bessel function of zero order. On the other hand, if a reversal propagation of radiation is considered, a problem of description of the Cherenkov radiation detection arises. The purpose of this paper is to analyze the problem of detecting Cherenkov radiation and to show the dependence of the annular spot width on the line source length.

2. Reciprocal beams

2.1. Generation of pseudo-nondiffracting beams

The pseudo-nondiffracting beam can be generated by collimating the beam of a coherent circular slit source. With the slit narrowing the beam approaches the Bessel beam. Only the infinite narrow slit gives the perfect Bessel beam. Figure 1 shows the necessary optical set. The plane of the source with coordinates x, y is in the front focal plane of the lens with coordinates ξ, η . At an arbitrary distance ζ from the lens a screen for a spot of the beam is placed. Coordinates of the diffraction pattern are x', y' . The radius of the circular source is r . The outgoing ray from an arbitrary point Q of the source transmits the lens at the point Π and continues to the point Q' on the screen. The diffraction pattern can be supposed as rotationally symmetric depending on the radial coordinate r' only.

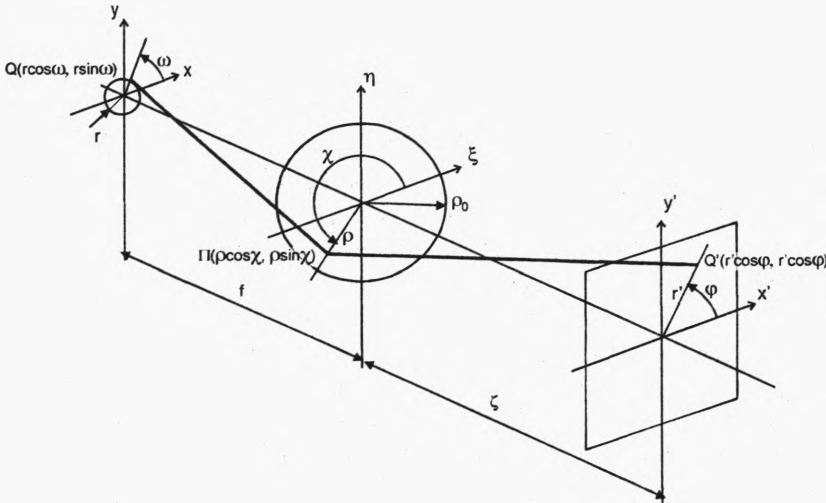


Fig. 1. Scheme of the optical arrangement for generation of pseudo-nondiffracting beams.

The light diffraction on the path from the source to the screen is given by the relation

$$\begin{aligned}
 u\{x', y', \zeta\} = & \int_{\chi=0}^{2\pi} \int_{\rho=0}^{\rho_0} \left[\int_{\omega=0}^{2\pi} \exp\left\{ ik\sqrt{f^2 + (\rho \cos \chi - r \cos \omega)^2 + (\rho \sin \chi - r \sin \omega)^2} \right\} r d\omega \right] \\
 & \times \exp\left\{ -ik\frac{\rho^2}{2f} \right\} \exp\left\{ ik\sqrt{\zeta^2 + (r' \cos \phi - \rho \cos \chi)^2 + (r' \sin \phi - \rho \sin \chi)^2} \right\} d\chi \rho d\rho
 \end{aligned} \tag{1}$$

where the first integral is the light propagation from the source to the entrance plane of the lens, the exponential term describes the transmission through the lens, and the second integral gives the light propagation from the lens to the screen; k denotes the angular repetency $k = 2\pi/\lambda$. The points are given by the radial coordinates. The angles are denoted by Greek characters ω , χ , and φ . Provided that the longitudinal coordinates are much longer than the lateral coordinates, $f \gg r$, ρ ; $\zeta \gg \rho$, r' , the terms under the square roots can be approximated by:

$$:\sqrt{1} \approx f + \frac{\rho^2}{2f} + \frac{r^2}{2f} - \frac{\rho r}{f} (\cos \chi \cos \omega + \sin \chi \sin \omega) = f + \frac{\rho^2}{2f} + \frac{r^2}{2f} - \frac{\rho r}{f} \cos(\chi - \omega),$$

$$:\sqrt{2} \approx \zeta + \frac{r'^2}{2\zeta} + \frac{\rho^2}{2\zeta} - \frac{r'\rho}{\zeta} \cos(\varphi - \chi),$$

and Eq. (1) can be written as

$$\begin{aligned}
 u\{x', y', \zeta\} &= \\
 &= \int_{\omega=0}^{2\pi} \int_{\chi=0}^{2\pi} \int_{\rho=0}^{\rho_0} \exp\left\{ ik \left[f + \frac{\rho^2}{2f} + \frac{r^2}{2f} + \zeta + \frac{r'^2}{2\zeta} + \frac{\rho^2}{2\zeta} \right. \right. \\
 &\quad \left. \left. - \frac{\rho r}{f} \cos(\chi - \omega) - \frac{r'\rho}{\zeta} \cos(\varphi - \chi) - \frac{\rho^2}{2f} \right] \right\} r d\omega d\chi \rho d\rho. \tag{2}
 \end{aligned}$$

After rearranging the right-hand side the last relation will have the form

$$u\{x', y', \zeta\} = A''' \iiint \exp\left\{ ik \left[\frac{\rho^2}{2\zeta} - \frac{\rho r}{f} \cos(\chi - \omega) - \frac{r'\rho}{\zeta} \cos(\varphi - \chi) \right] \right\} d\omega d\chi \rho d\rho \tag{3}$$

where the coefficient A''' involves all what is independent of the integral variables.

Using twice the relation

$$2\pi J_0\{a\} = \int_0^{2\pi} \exp\{-ia \cos(\theta - \phi)\} d\theta$$

(where J_0 is the Bessel function of the first kind and zero order), Eq. (3) can be rewritten as

$$u\{x', y', \zeta\} = A''' \int_{\rho=0}^{\rho_0} \exp\left(ik \frac{\rho^2}{2\zeta} \right) J_0\left(k \frac{\rho r}{f} \right) J_0\left(k \frac{r'\rho}{\zeta} \right) \rho d\rho. \tag{4}$$

To solve this equation one should apply the relation

$$\int_0^\infty \exp(-a^2 x^2) J_\nu(bx) J_\nu(cx) x dx = BI_\nu\left(\frac{bc}{2a^2}\right),$$

in which J_ν denotes the Bessel function of the first kind and ν -th order, and I_ν denotes the Bessel function of the second kind. The result of application is

$$u\{x', y', \zeta\} = A'' I_\nu\left(\frac{k r r'}{-i f}\right). \tag{5}$$

As can be seen here the result is independent of the distance ζ . The beam is diffraction-free. The final relation can be obtained using

$$I_\nu(x) = \exp\left(-i\frac{\pi}{2}\nu\right) J_\nu(ix)$$

and the light amplitude has the form

$$u\{x', y', \zeta\} = A J_0\left(2\pi \frac{r}{\lambda} \frac{r'}{f}\right). \tag{6}$$

The final Eq. (6) represents an expression for the lateral distribution of the amplitude. The intensity in the beam is described by the square of the zero-order Bessel function. The argument of the function involves the product of the circular source radius in the wavelength units and the radial coordinate of the spot point in the focal length units. The diameter of the beam corn increases with a decrease of the circular

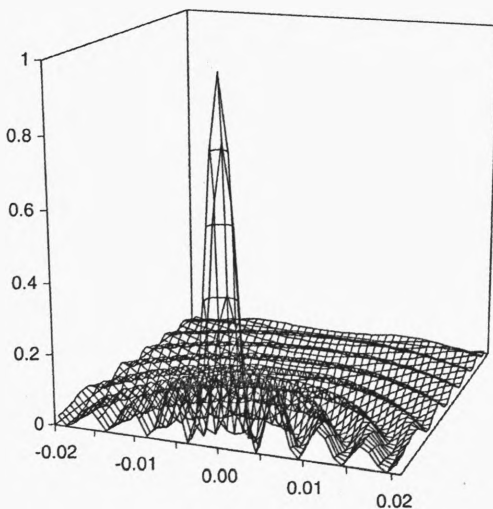


Fig. 2. Distribution of intensity in the Bessel beam.

source radius. Figure 2 shows the relative lateral intensity distribution for the source radius $r = 100\lambda$.

2.2. Detection of the Cherenkov radiation

As was mentioned previously, the Cherenkov radiation could be shown as a reciprocal problem to the one of the diffraction-free Bessel beam. For generation of the Cherenkov beam an optical scheme represented in Fig. 3 can be used. This optical scheme has an arrangement reversed with respect to the arrangement of the Bessel beam generation. Along the arrangement axis the line coherent source Q is distributed. This line source emits radiation with the conical wave front. Rays of this radiation make the constant angle θ_0 with the axis. The conical wave front impinges on the entrance plane of the lens L . This lens transforms the wave front and the beam produces a diffraction pattern on the screen, which is located in the back focal plane of the lens. The diffraction pattern should have the rotational symmetry. Provided that the source is infinite, which means that its length and the lens are of infinite size, the spot must be a circular line. In other cases the diffraction pattern will be more complicated.

The light diffraction on the path from the source to the screen is given by the relation

$$u'\{r'\} = \int_{\rho_0 - \Delta\rho/2}^{\rho_0 + \Delta\rho/2} \exp(ik\rho\sin\theta_0) \exp\left(-i\frac{\rho^2}{2f}\right) \exp\left(ik\sqrt{(r' - \rho)^2 + \zeta^2}\right) d\rho \quad (7)$$

where the limits of integration are given by the circular ring of the width $\Delta\rho$ on the lens. The first exponential term represents propagation from the source to the lens, the second term describes transmission through the lens, and the third term expresses

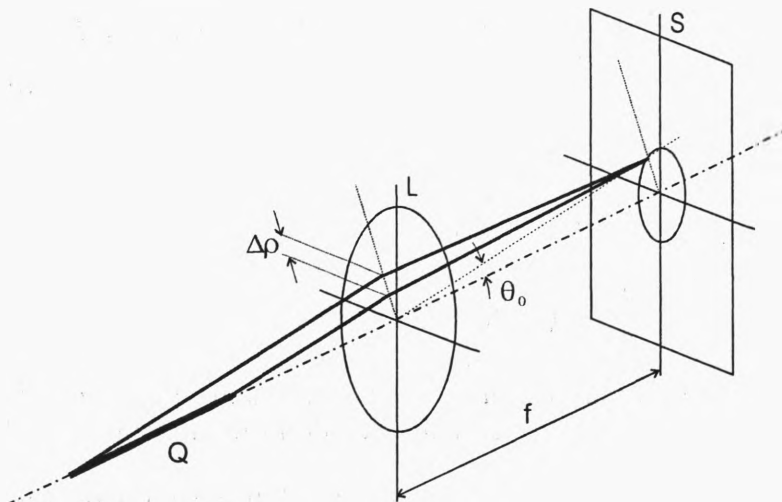


Fig. 3. Scheme of optical arrangement for detection of the Cherenkov radiation.

propagation from the lens to the screen. Provided that longitudinal coordinates are much longer than lateral coordinates, $f, \zeta \gg \rho, r'$, the term under the square root can be approximated by

$$\sqrt{\dots} = \zeta + \frac{r'^2}{2\zeta} + \frac{\rho^2}{2\zeta} - \frac{\rho r'}{\zeta}$$

and Eq. (7) can be written as

$$u'\{r'\} = \exp\left\{ik\left(\zeta + \frac{r'^2}{2\zeta}\right)\right\} \int_{\rho_0 - \Delta\rho/2}^{\rho_0 + \Delta\rho/2} \exp\left\{ik\left(\rho \sin\theta_0 - \frac{\rho^2}{2f} + \frac{\rho^2}{2\zeta} - \frac{\rho r'}{\zeta}\right)\right\} d\rho. \quad (8)$$

If the screen is situated in the back focal plane, $\zeta = f$, Eq. (8) will simplify to the form

$$u'\{r'\} = \exp\left\{ik\left(f + \frac{r'^2}{2f}\right)\right\} \int_{\rho_0 - \Delta\rho/2}^{\rho_0 + \Delta\rho/2} \exp\left\{ik\left(\sin\theta_0 - \frac{r'}{f}\right)\rho\right\} d\rho. \quad (9)$$

It is reasonable to introduce the relative coordinate in units of the focal length

$$\frac{r'}{f} = \sin\theta.$$

After carrying out the integration of Eq. (9) the distribution of the amplitude will be

$$u'\{\sin\theta\} = \exp\left\{ik\left[f\left(1 + \frac{\sin^2\theta}{2}\right) + (\sin\theta_0 - \sin\theta)\rho_0\right]\right\} \frac{2i \sin\left\{k(\sin\theta_0 - \sin\theta)\frac{\Delta\rho}{2}\right\}}{ik(\sin\theta_0 - \sin\theta)} \quad (10)$$

and for the intensity it holds

$$I\{\sin\theta\} = (\Delta\rho)^2 \frac{\sin^2\left\{k\frac{\Delta\rho}{2}(\sin\theta_0 - \sin\theta)\right\}}{\left(k\frac{\Delta\rho}{2}\right)^2(\sin\theta_0 - \sin\theta)^2}. \quad (11)$$

The final Eq. (11) shows that the intensity distribution is similar to diffraction on the straight slit. However, in this case the slit is curved and the diffraction pattern is curved, too.

Figure 4 shows distribution of relative intensity in the diffraction pattern of the Cherenkov beam. The diagram is calculated for the angle of propagation of the

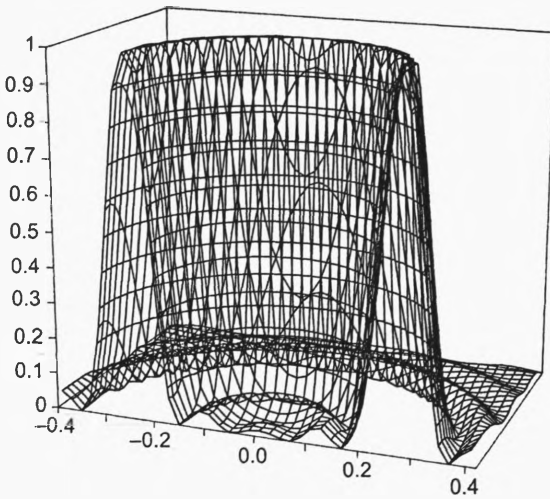


Fig. 4. Distribution of intensity in the detection directions of the Cherenkov detector.

Cherenkov rays $\theta_0 = 15^\circ$ and for the width of the circular ring on the lens $\Delta\rho = 10\lambda$. This means that the length of the line source is $l = \Delta\rho/\sin\theta_0 \approx 40\lambda$.

3. Conclusions

The diffraction-free beams are considered as being generated in the optical scheme where the annular radiation source is located in the back focal plane of the collimator lens. Provided that the source has infinitely narrow width the nondiffracting Bessel beam arises behind the lens. The diameter of the beam depends on the radius of the source ring. For the inverse problem of the Cherenkov radiation detection the width of the annular beam spot on the screen located in the front focal plane depends on the length of the line source along the optical axis.

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References

- [1] DURNIN J., MICELI, J.J., JR., EBERLY J.H., *Phys. Rev. Lett.* **58** (1987), 1499.
- [2] DURNIN J., *J. Opt. Soc. Am. A* **4** (1987), 651.
- [3] TURUNEN J., VASARA A., FRIBERG A.T., *Appl. Opt.* **27** (1988), 3959.
- [4] HERMAN R.M., WIGGINS T.A., *J. Opt. Soc. Am. A* **8** (1991), 932.
- [5] ROSEN J., SALIK B., YARIV A., *J. Opt. Soc. Am. A* **12** (1995), 2446.
- [6] LIU R., DONG B.-Z., YANG G.-Z., GU B.-Y., *J. Opt. Soc. Am. A* **15** (1998), 144.
- [7] BOUCHAL Z., *Opt. Commun.* **197** (2001), 23.