Optica Applicata Vol. XXVI, No. 1, 1996

Identification of mechanical parameters of incised cornea by means of finite element method

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The aim of the paper is to examine the correctness of application of the simplifying assumptions describing the deformation of incised cornea, particularly, those which allow the corneal material to be accepted as linear and isotropic. The cornea was modelled as an isotropic material, by use of Finite Element Method, and the changes of the refractive power of the cornea after 8 or 16 radial incisions were calculated. The parameters of assumed isotropic material have been changed in such a way as to get agreement with variations of refractive power after RK, described by surgeons. The results show that it is acceptable to assume the geometrical linearity and that the physical linearity is also admissible in the first approximation. However, the assumption about isotropy of the incised cornea is inadvisable. It means that the use of Young modulus as well as Poisson ratio for description of the corneal deformation should be limited only to intact (not incised) cornea.

1. Introduction

The cornea as the most refractive optical element of an eye plays at least a few important roles in the process of vision and catches more and more attention of scientists representing different branches of natural sciences. However, its two fundamental functions are the optical role as a lens and the mechanical role as a biomechanical shell. The cornea as a biomechanical shell protects the whole eye against external influences or threats and assures the appropriate form of the corneal membrane. The refractive properties of the cornea depend directly on its geometry and on material parameters (optical as well as mechanical). Knowledge of the corneal geometry and its mechanical parameters enables anticipation of the change of its refractive power after surgical intervention.

In papers presenting the elastic properties of the cornea, the mechanical parameters that are most often exposed are the Young modulus and the Poisson ratio [1]–[3]. On the other hand, the examination of ultrastructure of the stroma, which has the major contribution to mechanical and optical properties of the cornea, in an electron microscope shows great anisotropy of its physical properties.

A frequent assumption that one can apply the model of isotropic material in a study of mechanical properties of the cornea demands an accurate argumentation. One may expect that such material as stroma, with great anisotropy, "simulates" during examination different values of Young modulus, depending on a stress state, orientation of the main directions of anisotropy, the form of a sample and its fixation. Such a "reduced" Young modulus can apply only to the state determined by all of the above-mentioned circumstances. For example, the Young modulus assigned for the intact cornea in a membrane state will be different from that determined for the elongated corneal stripe. Frequent use in literature of the Young modulus with reference to the corneal material (*i.e.*, suggesting its isotropy) leant us towards investigation of the correctness of such an assumption.

What strikes the eye when reviewing the publications concerning the Young modulus measurements of the cornea of the human eye (averaged for all the corneal layers) is a great differentiation of the results. Much as the authors agree in the value of the Poisson ratio, and fix it on the level $\nu = 0.5$ (noncompressed medium), the experimentally set values of the Young modulus differ to such a degree that — in our opinion — this cannot be explained in terms of the accuracy of measurements. Table 1 presents some values of the measured Young modulus of the human cornea.

Table 1. Values of Young moduli of the cornea given by different authors

E [MPa]	Material	Authors
57	Strips of the cornea	Andreassen *et al.*
0.4–2.0 Tangent modulus, linearly dependent on stress in the range 0–240 kPa	Strips of the cornea	Nash *et al.*
1.8 Linear approximation in the range 0–40 kPa	Membrane state (Intact cornea)	Woo *et al.*
0.026	Membrane state (Cornea in vivo)	Sjøntoft, Edmund

Interest in the mechanics of the cornea results from the needs of medical practice. The possibilities of the corneal surgery have overtaken in this respect the knowledge about the mechanical consequences of the operation (*e.g.*, about the influence of scars on optical parameters of the cornea). At the initial stage of investigations of mechanical properties of the cornea there has dominated in the 60's an inquiry on the part the individual corneal layers play in carrying the load, and on isolating individual mechanical properties of stroma and Descemet's membrane. At that time, both the technical and methodological possibilities of numerical calculations were significantly limited. Solutions for incised cornea, even by use of all simplifications of the model, were very difficult to obtain. At present the situation is different. A series of very effective numerical methods applied in the mechanics of continuum and the use of computers with a high calculation power make it possible to take into account anisotropy, nonlinear and even rheological phenomena. Therefore one can observe a turn of investigators towards this sphere of study.

A very useful method is presently the Finite Element Method (FEM). A trial study of the eyeball model with the radially incised cornea, by use of FEM is presented by WOO *et al.* [4]. An interesting example of the application of FEM in investigations of the mechanical properties of a stroma is given by PINSKY and DATYE [3]. Independent of classical usage of shell finite elements they proposed an original method of determining anisotropic parameters of the stromal material. This is an example of the new approach to study of the cornea, where one departs from an isotropic model, namely, from the Young modulus.

The assignment of adequate parameters of a material requires an application of more advanced research technique. It would be extremely difficult to implement their measurement in a laboratory, directly on biological specimen. The above-mentioned numerical methods allow us to arrive at a result through such variations of input data (material constants) in order to get the expected result of output data (deformation of the corneal model). For example, in paper [3] one assumes a model of the anisotropic medium with justifiable properties of known microstructure of the stroma. The material parameter (the only one different from zero) has been determined by comparing the deformation of the model of incised cornea with the deformation of identically incised corneas of operated patients.

2. Method

We applied in our study the FEM to verify the assumption about the isotropy of material, and tried to calculate the Young modulus.

Only the human cornea has been considered in our study. According to some experimental settlements we assume that mainly stroma decides about the mechanical properties of the human cornea. Of the other corneral layers only the Descemet's membrane shows its mechanical presence but so imperceptibly that we neglect its influence [5]. Thus, the cornea is considered to be one-layer and homogeneous.

The model of the cornea consists of standard finite elements for 3-D isoparametric, nonlinear solid continuum 20 nodes elements with higher order model. Since the subject of comparison of the corneal model with the real cornea are only displacements, such elements were used without an exception in all volume of the cornea. In calculations, the symmetry of radially incised cornea has been availed by modelling only a wedge with the vertical angle π/n, where n is the number of incisions (8 or 16). On the surface from one side of the wedge, a continuous connection with the next wedge has been simulated by depriving the nodes of the "circumferential" degree of freedom. On the second surface, the incision has been simulated by depriving of "circumferential" degree of freedom only those nodes that were located in the region of the incision. The modelled incisions run from the optical zone to the limbus. As boundary conditions we assumed that all nodes on the corneal border (along the limbus) were fixed. A mesh of finite elements is shown in Fig. 1.

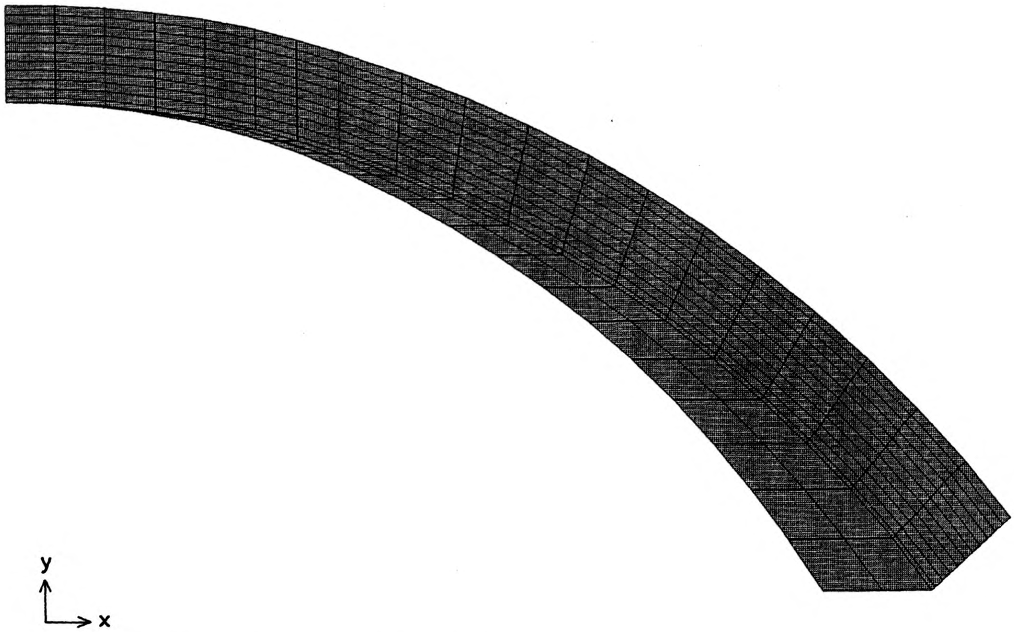

Fig. 1. Mesh of finite elements used for calculations

The procedure was as follows:

1. Creation of isotropic and linearly-elastic mechanical model of radially incised cornea by use of FEM.

2. Comparison of the calculated final configuration of the model of the cornea thus created with the configuration obtained experimentally. Speaking more strictly, a calculated variation of the optical power of the corneal model caused by radial incisions was compared with changes of the optical power of the cornea, observed in surgical practice. The respective experimental values were taken from [3].

The parameters of the model of the cornea taken into consideration:

– outer diameter of the cornea (near the limbus) 11.5 mm,
– radius of the external sphere, R, 7.86 mm,
– central thickness of the cornea, d, 0.52 mm,
– peripheral thickness (near the limbus) 0.65 mm,
– intraocular pressure (IOP) 2.135 kPa (16 mm Hg),
– Poisson ratio, ν, 0.45,
– type of the finite element: 3-D continuum elements.

The refractive power of the cornea in dioptres has been calculated from the formula

$$P = \frac{337.5}{R} \text{ [D]}$$

where R is the central radius of the corneal curvature in mm. The radius of curvature

R was derived from the curvature of the approximating polynomial of 7-th degree. Polynomials were determined from coordinates of 50 nodes by use of the least square method.

The following thesis is submitted for verification. The cornea modelled by means of isotropic material, whose deformations have been calculated by using simplifying assumptions about a linearly elastic material and about geometrically linear model, should show variations of its refractive power (changes of the curvature) caused by radial incisions with given depth and diameter of optical zone – adequate to variations observed in surgical practice.

3. Results

The results, calculated by FEM and measured in surgical practice, concerning variations of the optical power of the cornea after radial keratotomy (RK), are presented in Table 2. Calculated variations of the refractive power are a function of the numbers of incisions (8 or 16), radius of the optical zone (1.50 mm, 1.75 mm and 2.00 mm) and the Young modulus.

Table 2. Changes of the refractive power of the cornea after RK with the depth of incisions equal to 95% of the central corneal thickness

Radius of the optical zone R_{oz} [mm]	Young modulus E [MPa]	Variation of the corneal power in dioptres			
		8 radial incisions		16 radial incisions	
		Calculated	Observed *	Calculated	Observed *
	0.14	−260	−5 to −7	−63.5	−5.5 to −9
	0.76	−5.7		−35	
1.5	2.5	−1.47	aver. −5.5	−6.2	aver. −6
	5	−0.4		−3.1	
1.75	2.5	−0.77	−2.5 to −4.5 aver. −3	−5.43	
2.00	2.5	−0.55	−2.5 to −4 aver. −3		

* Observed variations of the optical power of the cornea after RK are cited from [3].

3.1. Isotropy

For evaluation of the Young modulus, the following parameters were chosen: number of incisions $n = 16$ and the radius of the optical zone $R_{oz} = 1.5$ mm. Calculated changes of the refractive power of the cornea for four values of the Young modulus taken from literature, after 16 radial incisions are presented in the thin line frame of Tab. 2.

One can notice from the table that for 16 incisions the Young modulus $E =$ 2.5 MPa assures the best coincidence of the calculated and measured variations of the refractive power of the cornea (in the bold line frame). In order to confirm the thesis about the correctness of the assumption concerning the isotropy of the corneal material, the same coincidence of the refractive power should be found for other combinations of these two parameters (n and R_{oz}).

In fact, the values contained in Table 2 contradict the thesis about the isotropy of the incised cornea. For example, a model of the cornea with 8 incisions for $R_{oz} =$ 1.5 mm and with the chosen Young modulus $E = 2.5$ MPa varies the refractive power after incisions about 1.47 D, in comparison with the expected variation equal to 5.5 D. As can be seen in this case, the Young modulus is expected to be 0.76 MPa.

Let us notice the fact that assumptions about the noncompressibility of the material ($\nu = 0.5$) and about the applicability of the Hook law are noncongruent; in such a case the relation between the stress and the strain tensors is indeterminate. The calculations performed on the numerical model of the isotropic cornea show small sensitivity of its refractive power to the value of the fraction ν. Thus, accepting $\nu = 0.4$ for use in the calculations presented here makes the solution of the problem possible, although this specific value of ν has no significant influence on the result of calculations.

Being not so severe to attempt rejection of the isotropic model, let us note that the numbers compared are of the same order. Results of measurements of the Young modulus reported in literature attain different values such as $E = 0.026$ MPa [6] and $E = 57$ MPa [1]. Thus, if one accepts the simplifying assumption about the isotropy of the corneal material, the Young modulus according to presented calculations reaches the order of 1 MPa. The range of its variability depends not only on the measurement accuracy, but also on the shape of the sample.

3.2. Geometrical linearity

This problem is associated with the stability of construction and with the qualitative invariableness of its loading. The type of construction considered here as well as its state of stress, that is close to the membrane one, are not threatened with the loss of stability. All the same by the excessive increment of the displacements, the correction of the loading might appear to be necessary, since the loading should be perpendicular to the inner surface of the cornea, even after the deformation. Sticking to the isotropic model, the profile of the outer corneal surface has been calculated for parameters: $E = 2.5$ MPa, $R_{oz} = 1.5$ mm, $n = 16$, and IOP $= 2135$ Pa (16 mm Hg), in the axial section passing through an incision and in the section in the middle between incisions.

Figure 2 presents axial displacements of the intact (curve 1) and the incised (curves 2 and 3) cornea in relation to the profile of the cornea that is not loaded by the IOP (horizontal axis). Curve 3 presents axial displacements of the inner surface of the cornea in the plane of the incision, while curve 2 gives the axial displacement in the plane between incisions. The 3-D axial displacements of the corneal wedge,

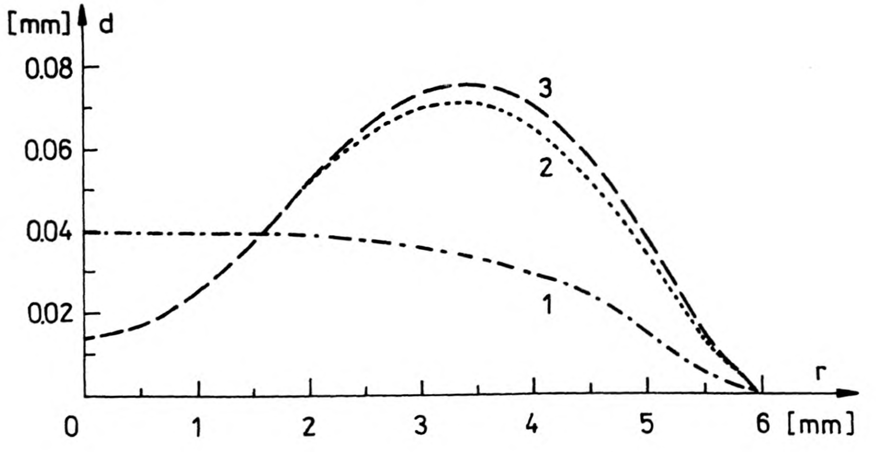

Fig. 2. Axial displacements of the intact and incised cornea in relation to the nonloaded cornea given as the horizontal axis: 1 — intact cornea, 2 — incised cornea in the plane between incisions, 3 — incised cornea in the plane of incisions

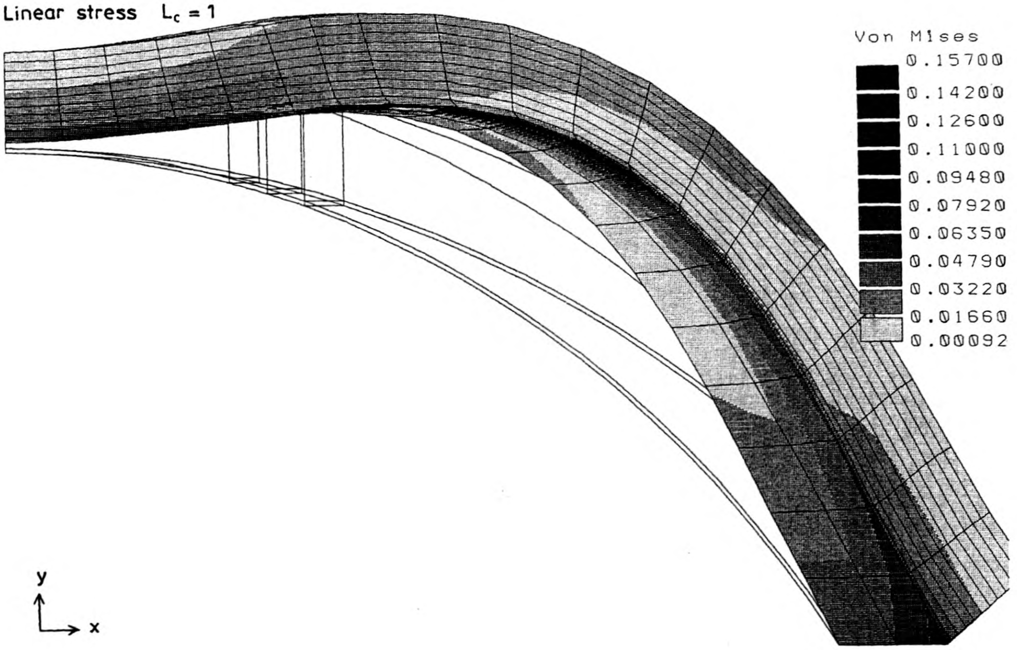

Fig. 3. 3-D axial displacements of the corneal wedge. Values of displacements are magnified 10 times in relation to calculated ones. The gray levels denote the stresses in MPa

limited from both sides by the planes of incisions are shown in Fig. 3. To show clearly the distribution of the displacements, values of displacements are magnified 10 times in relation to calculated ones. The maximal displacement of the cornea caused by incisions is equal to 0.076 mm, and makes about 10% of the maximal corneal thickness. This value coincides with values given in literature [3] and justifies the accepted assumption about the geometrical linearity. The force field which is applied perpendicular to the inner surface of the cornea causes small deformation of the surface, and requires no correction of the perpendicularity.

3.3. Physical linearity

The stress calculated for the discussed model, appearing in the nonincised cornea, is equal to about 17 kPa. The maximal reduced stress according to Mises, calculated for the set of parameters: $E = 2.5$ MPa, $R_{oz} = 1.5$ mm, number of incisions $n = 16$, depth of incisions 95%, IOP = 2135 Pa (16 mm Hg) acts at the top of the incision and amounts to 76 kPa. These two levels of stress are marked on the graph of the elongation of the corneal sample, given in [2], and presented in Fig. 4. As can be seen from the figure, the linear extension of the stress-strain dependence in the range of acting stresses, leads to the error of an order of some scores of percentages. Divergence of this order of the stress calculations refers only to small areas of the medium, in the region of the tops in incisions. On the basis of the results obtained one cannot determine unambiguously the influence of the physical nonlinearity on the refractive power of the corneal model.

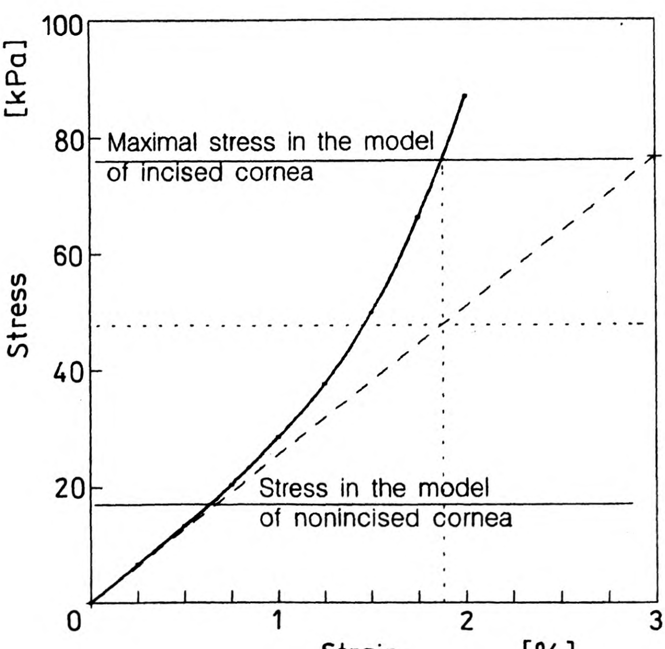

Fig. 4. Graph of elongation of the corneal stripe, according to [2], and its linearization. The assumed Young modulus of the linear part is 2.5 MPa

Before concluding our discussion of the results of calculations one should underline the structural peculiarities of the cornea as a mechanical object. The cornea is a special construction, composed by a material provided with continuous colagen fibrils. Such a structure allows significant theoretical simplifications, up to assumption about the isotropic shell in the membrane state. The damage of the sclera's "armor", spread over the limbus, drastically changes the properties of the corneal shell. After incisions the anisotropy of the corneal material is strongly increased in comparison with the anisotropy of the intact cornea and moreover, the local material parameters (namely in point) become dependent on the geometry of incisions. This implies the different, qualitative approach to the incised and unincised cornea's properties. In other words, the simple mechanical model of the corneal shell can be used as long as the cornea is not incised.

4. Conclusions

1. The corneal material is in fact not isotropic and one may not attribute to it the Young modulus.

2. If in the first approximation one applies the isotropy of the material, then the value of the Young modulus of the incised human cornea, in the cases considered here, is of order 1–3 MPa.

Acknowledgments – This work was supported by Polish Grant (Polish State Committee for Scientific Research, KBN), research project No. 2P30202304.

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Received October 12, 1995