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Counter-balance of linear and nonlinear optical activity for soliton-like pulses

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The problem of temporal soliton propagation in a nonlinear Kerr medium with natural optical activity is considered. The evolution of the state of polarization of the soliton is described using the Stokes parameters. It is shown that due to the balance between the nonlinear optical activity and linear gyrotropy a special class of stationary elliptically polarized solitons appears.

The problem of soliton propagation in nonlinear media with natural [1], [2] and induced gyrotropy [3] in both the optical and phonon [2] frequency regions has attracted much attention in recent years. Our investigation is related to the particular problem of the evolution of the polarization state of soliton-like pulses in a naturally gyrotropic Kerr medium, which has important applications and has not been considered before. The approach we have introduced in this paper can be applied to the investigation of the self-guided beams as well. Spatial solitons with evolving polarization in Kerr media without natural gyrotropy were discussed by Snyder *et al.* [4].

It is known that the plane of polarization of linearly polarized light is rotating during the propagation through the linear gyrotropic medium (*linear optical activity*) [4]. In the case of elliptically polarized light the natural gyrotropy leads to the rotation of polarization ellipse. On the other hand, the ellipse of polarization of an eliptically polarized strong wave is also rotating in Kerr nonlinear media [5]. We shall call this effect *nonlinear optical activity*. Clearly, if these two effects exist simultaneously in the same medium, the evolution of the state of polarization becomes more complicated. In this work we show that in the case of elliptically polarized light propagating in the nonlinear Kerr medium it is possible to arrange special conditions such that the rotation of the polarization ellipse due to the nonlinearity (*nonlinear optical activity*) is compensated by the presence of natural gyrotropy (*linear optical activity*). Note that we do not consider the effect of the nonlinear gyration appearing due to the spatial dispersion of the third-order susceptibility [3].

The pulse propagation in a nonlinear Kerr medium with natural optical activity in the plane wave approximation can be described by the system of two coupled nonlinear Schrödinger equations for the complex amplitudes U and V of two circularly polarized components of an electric field:

$$iU_\xi+\beta U+\frac{1}{2}U_{\tau\tau}+(|U|^2+A|V|^2)U=0,$$
$$iV_\xi-\beta V+\frac{1}{2}V_{\tau\tau}+(A|U|^2+|V|^2)V=0 \tag{1}$$

where ξ is the normalized longitudinal coordinate z, τ is the normalized retarded time, β and A are the parameters of the medium which are responsible for the *linear* and *nonlinear optical activity*, respectively. The value of *nonlinear factor* (which determines the magnitude of the cross-phase modulation term) for the isotropic medium is $A=2$. The *linear gyrotropic factor* $\beta \sim g\sqrt{\varepsilon^{(0)}}$ is determined by the relation between the electric field $\mathbf{E}$ and the linear part of the electric flux density $\mathbf{D}$ in the isotropic medium with natural optical activity: $\mathbf{D}=\varepsilon^{(0)}\mathbf{E}+ig\sqrt{\varepsilon^{(0)}}|\mathbf{E}\mathbf{e}_z|$, where $\varepsilon^{(0)}$ is the dielectric constant, g is the component of gyration vector [5] and $\mathbf{e}_z$ is the unit vector along z axis.

Let us suppose the existence of the pulse-like solutions of (1) in the separable form:

$$U=u(\tau,q_1,q_2)\exp(iq_1\xi),\quad V=v(\tau,q_2,q_2)\exp(iq_2\xi) \tag{2}$$

where the parameters q_1 and q_2 are the intensity-dependent shifts of the wavenumber resulting from the nonlinear coupling between the two components of the electric field as well as from the gyrotropic effect. Due to the absence of the linear coupling [7] there is no energy exchange between the two polarization components. Hence, the two components of the soliton-like pulse do not change their shapes ($u(\tau,q_1,q_2)$ and $v(\tau,q_1,q_2)$) upon propagation along the ξ-axis. For these stationary profiles of solitary waves we have the set of two second-order ordinary differential equations:

$$\frac{1}{2}u_{\tau\tau}-(q_1-\beta)u+(|u|^2+A|u|^2)u=0,$$
$$\frac{1}{2}v_{\tau\tau}-(q_2+\beta)v+(A|u|^2+|v|^2)v=0 \tag{3}$$

where u and v are real functions subject to the boundary conditions: u, $v\to 0$, and $u_\tau v_\tau \to 0$ when $\tau\to\pm\infty$.

Equations (3) have simple exact solutions representing bright solitons with right- and left-hand circular polarization. The stationary profiles for these two solitons have the forms:

$$u=\sqrt{2(q_1-\beta)}\operatorname{sech}(\sqrt{2(q_1-\beta)}\tau),\quad v=0;$$
$$u=0,\quad v=\sqrt{2(q_2+\beta)}\operatorname{sech}(\sqrt{2(q_2+\beta)}\tau). \tag{4}$$

We can also write down the solution for a linearly polarized wave for which $q_1-q_2=2\beta$

$$u = \frac{\sqrt{2(q_1-\beta)}}{\sqrt{1+A}}\operatorname{sech}(\sqrt{2(q_1-\beta)}\tau), \quad v = u. \tag{5}$$

The exact stationary solutions with u and $v \neq 0$ can be found only numerically. However, using the methods of the perturbation theory we can provide the analytical description of the elliptically polarized solutions which are close to the circularly polarized ones (4). In absence of the natural gyrotropy ($\beta = 0$) solutions of this type and their stability have been analysed in [8]–[11]. In our case ($\beta \neq$) they are:

$$u \approx \frac{\sqrt{2(q_1-\beta)}}{\cosh(\sqrt{2(q_1-\beta)}\tau)}, \quad v \approx \frac{v_0}{\cosh^{\nu}(\sqrt{2(q_1-\beta)}\tau)}, \tag{6}$$

and

$$u \approx \frac{u_0}{\cosh^{1/\nu}(\sqrt{2(q_2+\beta)}\tau)}, \quad v \approx \frac{\sqrt{2(q_2+\beta)}}{\cosh(\sqrt{2(q_2+\beta)}\tau)} \tag{7}$$

where:

$$\nu = \sqrt{\frac{q_2+\beta}{q_1-\beta}}, \quad \nu(\nu+1) = 2A. \tag{8}$$

The amplitudes u_0 and v_0 can be easily obtained using the representation of the Eqs. (3) as the equations of motion for the particle in the two-dimensional potential well [12]. Note that the terms with β in Eqs. (1) can be removed using the transformation $U = U'\exp(i\beta\xi)$, $U = U'\exp(-i\beta\xi)$ and the system can be reduced to the simpler one considered in [8]–[11]. This simplified set of equations allows to find the profiles of the soliton components and the relation between their amplitudes but the main effect we are interested in, namely the evolution of the state of polarization, in this case will be lost.

The state of polarization of the pulse-like solution of (1) can be represented by a point on the *Poincaré sphere* in the space of real Stokes parameters [13] which are defined as follows:

$$\begin{aligned} &s_0 = 2(|U|^2+|V|^2), \quad s_1 = 2(U^*V+UV^*), \\ &s_2 = -2i(U^*V-UV^*), \quad s_3 = 2(|U|^2-|V|^2). \end{aligned} \tag{9}$$

Taking these definitions into account, and using the substitution (2), we can rewrite (1) in terms of the Stokes parameters [7]:

$$\begin{aligned} &\frac{d}{d\xi}s_0 = 0, \quad \frac{d}{d\xi}s_1 = \omega s_2, \\ &\frac{d}{d\xi}s_2 = -\omega s_1, \quad \frac{d}{d\xi}s_3 = 0 \end{aligned} \tag{10}$$

where $\omega = q_1 - q_2$. The evolution of the state of polarization of any soliton-like solution of (1) during the propagation can then be qualitatively analyzed as a motion of the Stokes vector $\mathbf{s}_0 = \{s_1, s_2, s_3\}$ for the centre of the pulse ($\tau = 0$) on the Poincaré sphere. As the profiles of two components of the field are different, the state of polarization changes from the central part of the pulse to its tail. However, it is fixed at any point τ. From practical point of view, it is convenient to follow the evolution at the centre of the pulse ($\tau = 0$), where the intensity is maximal. This evolution is governed by the solutions of (10) which have the form:

$$s_1 = s_0 \cos\theta \cos(\omega\xi + \Phi),$$

$$s_2 = -s_0 \cos\theta \sin(\omega\xi + \Phi), \ s_3 = a_0 \sin\theta. \tag{11}$$

The quantities s_0 and s_3 are conserved during the propagation, and the pulse evolution manifests itself as a rotation of the Stokes vector around the s_3 axis on the Poincaré sphere. The quantity θ is the angle formed between $\mathbf{s}_0$ and the plane (s_1, s_2), which does not change with ξ and defines *the degree of ellipticity* for the centre of the pulse. The constant angle Φ defines the initial phase of the rotation.

Depending on the initial conditions and the values of parameters, the polarization of the pulse exhibits qualitatively different types of evolution. Circularly polarized initial pulses ($\theta = \pm\pi/2$) are represented by the stationary points on the Poincaré sphere $\{0, 0, \pm s_0\}$. The shapes of the pulses in this case are given by Eqs. (4). The rate of polarization rotation ω for the pulses with *nearly circular polarization* can be obtained using the perturbation method. The profiles of these solitons are presented above (6), (7). The conditions of their existence (8) lead us to the following expression for the frequency:

$$\omega = (v^{\pm 2} - 1)(q_2 + \beta) + 2\beta, \quad (\theta = \pm\pi/2) \tag{12}$$

where $v \approx 1.56$. The values of ω found from (12) (with the fixed $q_2 + \beta = 1$) are shown in Fig. 1.

In general, for $-\pi/2 < \theta < \pi/2$ the state of polarization always remains elliptic or linear. The frequency of the polarization rotation can be written in the form

$$\omega = \frac{1}{2}\left(\left.\frac{d^2u}{d\tau^2}\right|_{\tau=0} u_0^{-1} - \left.\frac{d^2v}{d\tau^2}\right|_{\tau=0} v_0^{-1}\right) + (1 + A)(u_0^2 - v_0^2) + 2\beta \tag{13}$$

where: $u_0 = u(\tau = 0)$, $v_0 = v(\tau = 0)$. We can see from expression (13) that the Stokes vector of a *linearly polarized* initial pulse ($\theta = 0$, $u_0 = v_0$), rotates along the equatorial line of the Poincaré sphere. The frequency of this rotation is $\omega = 2\beta$ (see Fig. 1). In the absence of the *linear optical activity* ($\beta = 0$) the plane of polarization does not rotate. The shape of stationary linearly polarized solution is given by expression (5). For $-\pi/2 < \theta < 0$ and $0 < \theta < \pi/2$ the evolution of the polarization is described as a rotation of the Stokes vector along one of the circles parallel to the equatorial line on the Poincaré sphere. In absence of the linear optical activity ($\beta = 0$), this rotation is due to the nonlinearity ($A \neq 1$) only (the effect of *nonlinear optical activity*).

Finally, we consider the case of existence of both the nonlinear and linear gyration simultaneously ($\beta \neq 0$ and $A \neq 1$). In this case, the expression of ω (13) leads

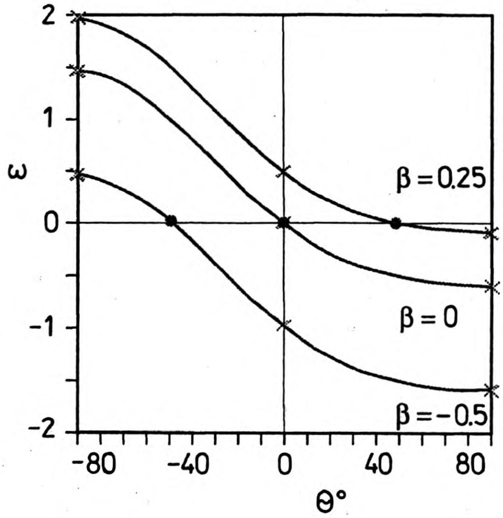

Fig. 1. Rotation frequency ω of the polarization ellipse vs. ellipticity θ for the centre of the pulse. The values indicated by crosses are obtained analytically. The parameter $q_2+\beta=1$ is fixed

to the remarkable conclusion. Suppose we have found the solution of Eq. (1) with certain degree of ellipticity $\theta_0(u_0,v_0)$ of the centre of the pulse. If ω then occurs to be equal to zero for this value of θ_0, *the rotation of the polarization ellipse due to the effect of nonlinear gyration is completely compensated by the natural activity*. Solutions of this type with arbitrary initial phase Φ and a certain θ_0 can be represented by the set of *stationary points* on the Poincaré sphere which form a circle parallel to the equatorial line. Each *stationary point* of this set corresponds to the *stationary pulse-like solutions* of Eq. (1) with an elliptical polarization and with a *fixed direction of the axis of the polarization ellipse* at the pulse centre. The directions of rotation of polarization ellipse are opposite above and below this circle.

The frequency of the rotation of the polarization ellipse depends on the degree of ellipticity through the amplitudes u_0 and v_0. This dependence for different given values of the linear gyrotropic factor β can be obtained by numerical simulations (see Fig. 1). All simulations were carried out with the fixed parameter $q_2+\beta=1$. As we can see, depending on the value of β, the condition $\omega=0$ becomes valid for different degrees of ellipticity θ_0 of the initial pulse. The values of β which make the frequency vanish in the case of linear and nearly circular polarization of the pulse can be found analytically, using the solutions (5) and (6), (7). The trajectories of the Stokes vector with the fixed and rotating polarization on the Poincaré sphere for three particular cases: $\theta_0=0$, $\theta_0=\pi/2$ and $0<\theta_0<\pi/2$, are given in Fig. 2. In the case when $\theta_0\to\pm\pi/2$, the circle of stationary solutions converges into the stationary point located on the pole (see Fig. 2c).

It is essential in our problem that for fixed parameters of the medium β and A, there are two parameters of the solutions of Eq. (1): q_1 and q_2. Hence, the general solution of the set (1) is the *continuous two-parameter family*. This family of solutions

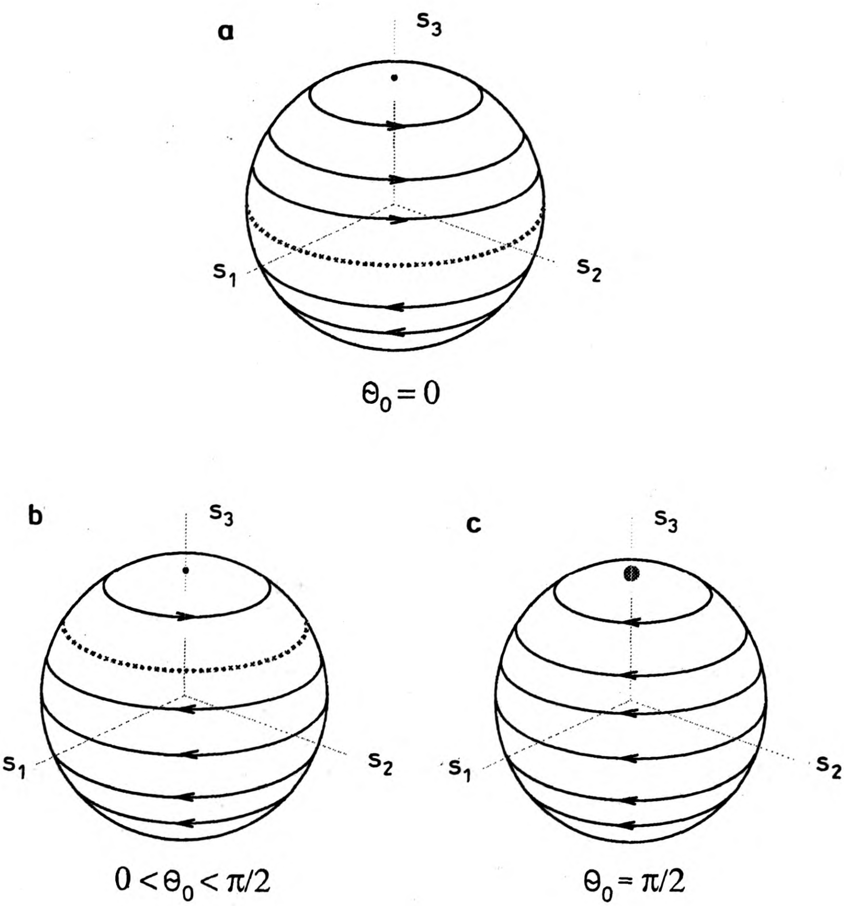

Fig. 2. Three different sets of the stationary solutions on the Poincaré sphere with (a) linear, (b) elliptic, and (c) circular polarization. The trajectories with rotating polarization ellipse are shown by solid lines. The arrows indicate the direction of the rotation

can be presented in terms of two other parameters: the total energy of the soliton-like pulse $Q = \int(|u|^2 + |v|^2)d\tau$ and the degree of ellipticity θ, which are experimentally measurable quantities. The continuity of the family of the solutions then have a simple physical meaning. Namely, the pulse of given degree of ellipticity θ can have any value of the total energy Q, and vice versa, the pulse of given energy can be of any degree of ellipticity from $\theta = -\pi/2$ to $\theta = \pi/2$. In this problem there is no way to reduce the number of the parameters of the solution. The choice $q_2 + \beta = 1$ which we made in numerical simulations enables us to investigate the whole family of solutions varying the parameter β instead of q_2.

It is known that the solitons which we are studying are stable relative to the perturbation of their profiles [11] as well as to perturbations of their relative positions [14]. The question remains about polarization instabilities. According to the classification of the theory of dynamical systems, each stationary point on the

Poincaré sphere is an elliptic point, which means that all of them are stable. However, the value of the growth rate of the perturbation δ is purely imaginary ($\delta = i\omega$) in the case of the circularly polarized solitons, and is equal to zero ($\delta = 0$) in the case of the stationary solitons with the elliptical polarization. Hence, circularly polarized solitons are stable and the stationary elliptically polarized soliton-like pulses are *neutrally stable.*

In conclusion, we have shown that linear rotation of the polarization ellipse caused by the Kerr nonlinearity can be cancelled due to the presence of natural (linear) optical activity. This phenomenon is of practical importance in the design of novel fast nonlinear all-optical switches.

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References

[1] Karpman V I., Phys. Rev. Lett. **74** (1995), 2455.
[2] Primatarova M. T., Stoichev K. T., Phys. Rev. B **49** (1994), 6634.
[3] Makarov V. A., Petrov K. P., *Quantum Electron.* **23** (1993), 880.
[4] Snyder A. W., Hewlett S. J., Mitchell D. J., Phys. Rev. Lett. **72** (1994), 1012.
[5] Maker P. D., Terhune R. W., Savage C. H., Phys. Rev. Lett. **12** (1964), 507.
[6] Landau L. D., Lifshitz E. M., *Electrodynamics of Continuous Media*, Pergamon Press, Oxford 1983, Chap. 12, p. 497.
[7] Akhmediev N. N., Soto-Crespo J. M., Phys. Rev. E **49** (1994), 5742.
[8] Inoue Y., J. Plasma Physics **16** (1976), 439.
[9] Afanasjev V. V., Grudinin A. B., Sov. Lightwave Commun. **3** (1993), 77.
[10] Haelterman M., Sheppard A., Phys. Rev. E **49** (1994), 3376.
[11] Silberger Y., Barad Y., Opt. Lett. **20** (1995), 246.
[12] Akhmediev N., Ankiewicz A., Phys. Rev. Lett. **70** (1993), 2395.
[13] Born M., Wolf E., *Principles of Optics*, Pergamon Press, New York 1959, Chap. 4, p. 49.
[14] Menyuk C. R., J. Opt. Soc. Am. B **5** (1988), 392.

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