Optica Applicata, Vol. XXVI, No. 4, 1996

Localization of light in dielectric media

M. RUSEK

Institute of Physics, Polish Academy of Sciences, al. Lotników 32/46, 02–668 Warszawa, Poland.

Recently random dielectric structures with typical length scale matching the wavelength of electromagnetic radiation have attracted a great deal of attention, both in the microwave and in the optical part of the spectrum. Propagation of electromagnetic waves in these structures resembles, to some extent, the properties of electrons in disordered semiconductors. Therefore, many ideas concerning transport properties of light and microwaves in such media exploit the theoretical methods and concepts of solid-state physics that have been developed over many decades. One of these concepts is electron localization in noncrystalline systems such as amorphous semiconductors or disordered insulators. As shown by ANDERSON [1], in a sufficiently disordered *infinite* material an entire band of electronic states can be spatially localized. Thus, for any energy from this band, the stationary solution of the Schrödinger equation is localized for almost any realization of the random potential. Prior to the work due to Anderson, it was believed that electronic states in infinite media are either extended, by analogy with the Bloch picture for crystalline solids, or are localized around *isolated* spatial regions such as surfaces and impurities [2].

It is commonly believed that the Anderson localization is completely based on the interference effects in multiple elastic scattering. However, interference is a common property of all wave phenomena. No wonder, therefore, that many generalizations of electron localization to electromagnetic waves have been proposed [3]–[7]. So-called weak localization of electromagnetic waves manifesting itself as enhanced coherent backscattering is presently relatively well understood theoretically [8]–[10] and established experimentally [11]–[13] beyond any doubts. The question is whether interference effects in random dielectric media can lead to *strong* localization. Despite some reasonable indications that strong localization could be possible in 3D random dielectric structures (mainly some suspensions of TiO_2 spheres in air or in some low-refractive-index substances [14]–[18] have been considered) the convincing experimental demonstration has been given only for 2D [19]. In this case, the strongly-scattering medium has been provided by a set of dielectric cylinders randomly placed between two parallel aluminium plates on half the sites of a square lattice.

Despite the huge amount of existing literature about Anderson localization of electromagnetic waves, there still is lack of sound theoretical models concerning this interesting effect. To be realistic, such models should be based directly on the

Maxwell equations. On the other hand, they should be simple enough to provide calculations without too many too-crude approximations. The main purpose of our paper is to construct such a model for the 2D localization and to elaborate in detail its major consequences. We restrict ourselves to the study of the properties of the stationary solutions $\vec{E}(\vec{r}, t) = \mathrm{Re}[\vec{\mathscr{E}}(\vec{r})e^{-i\omega t}]$ of the Maxwell equations. Consequently, the polarization of the medium is considered to be the oscillatory function of time $\vec{P}(\vec{r}, t) = \mathrm{R}[\vec{\mathscr{P}}(\vec{r})e^{-i\omega t}]$. By investigating the region of frequencies ω corresponding to the band of localized waves, we expect to provide some deeper insight into the existing experimental results.

We believe that what really counts for localization is the scattering cross-section and not the geometrical shape and real size of the scatterer. Therefore we will represent the dielectric cylinders located at the points $\vec{\rho}_a$ by 2D dipoles

$$\vec{\mathscr{P}}(\vec{r}) = \sum_a \vec{p}_a \delta^{(2)}(\vec{\rho} - \vec{\rho}_a). \tag{1}$$

Since the polarization of our system varies only at a certain plane, we have introduced cylindric coordinates $\vec{r} = (\vec{\rho}, z)$ in the above formula. In principle this approximation is justified only when the wavelength is much larger than the diameter of the dielectric cylinders under consideration and simultaneously much smaller than their height.

It is known that the theory of multiple scattering of electromagnetic waves by dielectric particles is tremendously simplified in the limit of point scatterers. Many multiple-scattering effects have been obtained qualitatively for coupled electrical dipoles. Examples are: universal conductance fluctuations [20], enhanced backscattering [21], and dependent scattering [22]. But, on the other hand, several mathematical problems emerge in the formulation of interactions of point-like particles with electromagnetic waves [22]–[24]. It turns out that to use safely the point scatterer approximation it is essential to use a representation for the scatterers that fulfils the optical theorem rigorously and conserves energy in the scattering processes [25]

$$i\pi k^2 \vec{p}_a = \frac{1}{2}(e^{i\varphi} - 1)\vec{\mathscr{E}}'(\vec{\rho}_a) \tag{2}$$

where $k = \omega/c$ is the wavenumber in vacuum. The field acting on the a-th cylinder

$$\vec{\mathscr{E}}'(\vec{\rho}_a) = \vec{\mathscr{E}}^{(0)}(\vec{\rho}_a) + \sum_{b \neq a} \vec{\mathscr{E}}_b(\vec{\rho}_a) \tag{3}$$

is the sum of some free field $\vec{\mathscr{E}}^{(0)}(\vec{\rho})$, which obeys the Maxwell equations in vacuum, and waves scattered by all *other* cylinders [25]

$$\vec{\mathscr{E}}_a(\vec{\rho}) = 2k^2 K_0(-ik|\vec{\rho} - \vec{\rho}_a|)\vec{p}_a \tag{4}$$

where K_0 denotes the modified Bessel function of the second kind.

Inserting Equation (4) into (3), and using (2), we obtain the system of linear equations

$$\sum_b M_{ab}\vec{\mathscr{E}}'(\vec{\rho}_b) = \vec{\mathscr{E}}^{(0)}(\vec{\rho}_a), \tag{5}$$

determining the field acting on each cylinder $\vec{\mathscr{E}}'(\vec{\rho}_a)$ for a given field of the free wave incident on the system $\vec{\mathscr{E}}^{(0)}(\vec{\rho}_a)$. If we solve it and use again Eqs. (2) and (4) then we are able to find the electromagnetic field everywhere in space (outside the cylinders)

$$\vec{\mathscr{E}}(\vec{\rho}) = \vec{\mathscr{E}}^{(0)}(\vec{\rho}) + \sum_a \vec{\mathscr{E}}_a(\vec{\rho}) \quad \text{for} \quad \vec{\rho} \neq \vec{\rho}_a. \tag{6}$$

Similar integral equations relating the stationary outgoing wave to the stationary incoming wave are known in the general scattering theory as the Lippmann–Schwinger equation [26]. In that formalism localized waves correspond to nonzero solutions of these equations (in our case Eqs. (5)) for the incoming wave equal to zero [27] (for different arguments based on the analysis of the behaviour of the energy density of the field, see [25]).

It is convenient to study eigenvectors

$$\sum_b G_{ab}\vec{\mathscr{E}}_j'(\vec{\rho}_b) = \lambda_j \vec{\mathscr{E}}_j'(\vec{\rho}_a), \tag{7}$$

of the matrix

$$i\pi G_{ab} = \begin{cases} 2K_0(-ik|\vec{\rho}_a - \vec{\rho}_b|) & \text{for} \quad a \neq b, \\ 0 & \text{for} \quad a = b, \end{cases} \tag{8}$$

which depends only on the positions of the cylindres $k\vec{\rho}_a$ scaled in wavelengths. It turns out that there exists a one-to-one mapping between electromagnetic waves localized in the system of cylinders (1) and eigenvectors $\vec{\mathscr{E}}_j'(\vec{\rho}_a)$ corresponding to eigenvalues λ_j obeying the condition

$$\mathrm{Re}\lambda_j = -1. \tag{9}$$

To prove this statement let us observe that, if we take

$$\Phi = \arg(1+\lambda_j/2) - \arg(\lambda_j/2), \tag{10}$$

then those eigenvectors are simultaneously eigenvectors of the M matrix from Eq. (5) corresponding to the eigenvalues $\Lambda_j(\Phi) = (1+\lambda_j/2) - e^{i\Phi}(\lambda_j/2) = 0$.

Let us stress that, for given position of the cylinders $\vec{\rho}_a$, the localization condition (9) may be fulfilled only at *discrete* frequencies ω (or wavenumbers k). According to Eq. (10) each of these frequencies is assigned a *single* value of the parameter Φ describing the scattering properties of the cylinders. This means that localized electromagnetic waves existing in systems (1) consisting of well separated dielectric cylindres correspond to *discrete* points on the plane $\{\omega, \Phi\}$. Similarly, localized states in quantum mechanics always appear only at discrete energies. However, in the case of a *disordered* and *unbounded* system, the countable set of energies corresponding to localized states becomes dense in some finite interval, in the same way as the rational numbers are dense in the real numbers [28]. But it is always difficult to separate energies allowed to the electron from energies which may be arbitrarily near, and by convention the spectrum is always a coarse-grained object [28].

Physically speaking, there exists an entire *continuous* band of spatially localized electronic states. This happens when Anderson localization occurs in solid state physics. It turns out that in the case of *random* and *infinite* systems of cylinders (1) there exists an analogous *continuous* region on the plane $\{\omega, \Phi\}$. After choosing a point (ω,Φ) from this region a localized wave of frequency ω exists in almost any (*i.e.*, out of a set of zero measure) random distribution of the cylinders described by the scattering properties Φ.

To illustrate this statement we have diagonalized numerically the G matrix (8) for 10^3 different distributions $\vec{\rho}_a$ of *finite* number of N cylinders placed randomly inside a square, with the uniform density $n = 1$ cylinder per wavelength squared. Then we have found the minimal rectangle on the complex plane containing all those eigenvalues, divided it into 100×100 small regions and counted the *total* number of eigenvalues λ_j inside each of these regions. The surface plot of the resulting probability distribution $P_N(\lambda)$ for $N = 300$ is presented in Fig. 1. In addition, in Fig. 2 we have prepared contour plots of $P_N(\lambda)$ for different numbers of cylinders

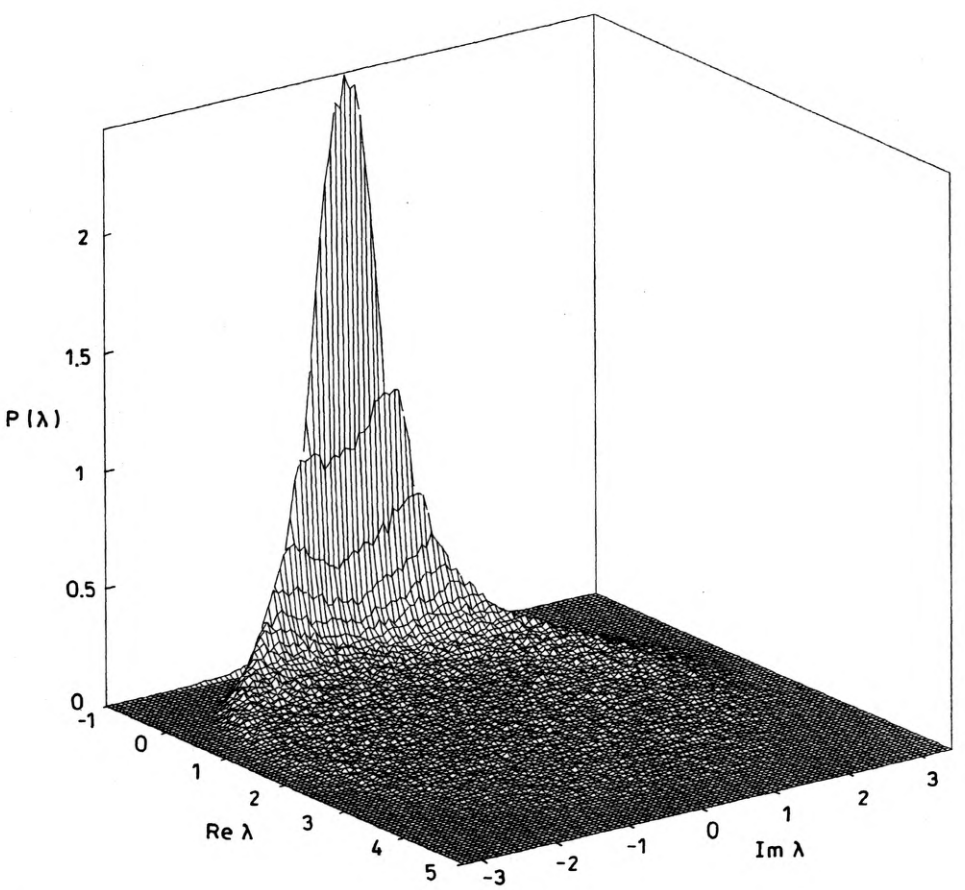

Fig. 1.

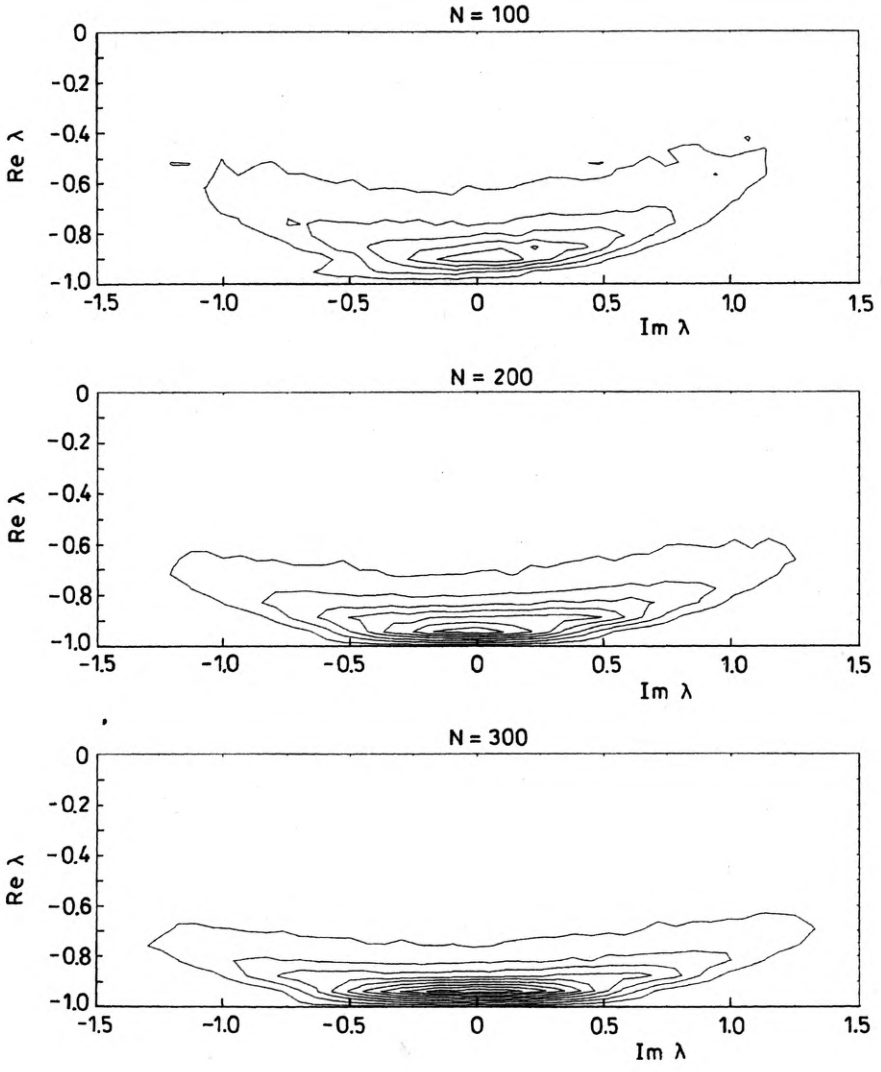

Fig. 2.

$N = 100, 200, 300$. It follows from inspection of Fig. 2 that for increasing values of N, the probability distribution $P_N(\lambda)$ moves towards the $\mathrm{Re}\lambda = -1$ axis and simultaneously its variance along the $\mathrm{Im}\lambda = \text{const.}$ axes decreases. Our numerical investigations indicate that in the limit of an infinite medium, the probability distribution under consideration will tend to

$$\lim_{N\to\infty} P_N(\lambda) = \delta(\mathrm{Re}\lambda + 1) f(\mathrm{Im}\lambda). \tag{11}$$

This means that for almost any random distributions of the cylinders $\vec{\rho}_a$, the condition (9) holds, and thus a localized wave with frequency ω (determined by the

condition $n = 1$) exists. However, as follows from inspection of Eq. (11), this happens only if $f(\mathrm{Im}\lambda) \neq 0$. This requirement together with Eqs. (10) and (9) determines the values of Φ suitable for localization. This region of Φ may be regarded as the section of continuous region corresponding to localized waves on the $\{\omega, \Phi\}$ plane along a certain $\omega = \text{const.}$ axis.

In summary, we have presented a novel theoretical approach to localization of electromagnetic waves in 2D dielectric media. The most important, and deeply new, step taken here is the willingness to deal with probability distributions, not averages. Generally speaking, most physicists immediately apply averaging procedure as soon as "disorder" is introduced in their model. When the scattered intensity is averaged over some random variable, a transport theory emerges [30]. But, as pointed out by Anderson, "there is a very important and fundamental truth about random systems we must always keep in mind: no real atom is an average atom, nor is an experiment done on an ensemble of samples" [31].

References

[1] ANDERSON P. W., Phys. Rev. **109** (1958), 1492.
[2] RAMAKRISHNAN T. V., in ref. [29], pp. 213–304.
[3] JOHN S., Phys. Rev. Lett. **53** (1984), 2169.
[4] ANDERSON P. W., Philos. Mag. B **52** (1985), 505.
[5] JOHN S., Phys. Rev. Lett. **58** (1987), 2486.
[6] *Analogies in Optics and Micro Electronics*, [Ed.] W. van Haeringen, D. Lenstra, Kluwer, Dordrecht 1990.
[7] [In] *Photonic Band Gaps and Localization*, NATO ASI Series, [Ed.] C. M. Soukoulis, Plenum Press, New York 1993.
[8] AKKERMANS E., WOLF P. E., MAYNARD R., Phys. Rev. Lett. **56** (1986), 1471.
[9] STEPHEN M. J., CWILLICH G., Phys. Rev. B **34** (1986), 7564.
[10] MACKINTOSH F. C., JOHN S., Phys. Rev. B **37** (1988), 1884.
[11] KUGA Y., ISHIMARU A., J. Opt. Soc. Am. A **1** (1984), 831.
[12] ALBADA M. P. V., LAGENDIJK E., Phys. Rev. Lett. **55** (1985), 2692.
[13] WOLF P.-E., MARET G., Phys. Rev. Lett. **55** (1985), 2696.
[14] VAN ALBADA M. P., LAGENDIJK A., VAN DER MARK M. B., in ref. [6], pp. 85–103.
[15] WATSON G. H., Jr., FLEURY P. A., MCCALL S. L., Phys. Rev. Lett. **58** (1987), 945.
[16] GENACK A. Z., Phys. Rev. Lett. **58** (1987), 2043.
[17] DRAKE J. M., GENACK A. Z., Phys. Rev. Lett. **63** (1989), 259.
[18] GENACK A. Z., GARCIA N., Phys. Rev. Lett. **66** (1991), 2064.
[19] DALICHAOUCH R., *et al.*, Nature **354** (1991), 53.
[20] LEE P. A., STONE A. D., Phys. Rev. Lett. **55** (1985), 1622.
[21] VAN DER MARK M. B., VAN ALBADA M. P., LAGENDIJK A., Phys. Rev. B **37** (1988), 3575.
[22] VAN TIGGELEN B. A., LAGENDIJK A., TIP A., J. Phys. C **2** (1990), 7653.
[23] VAN TIGGELEN B. A., Ph.D. Thesis, University of Amsterdam, Amsterdam 1992.
[24] NIEUWENHUIZEN T. M., LAGENDIJK A., VAN TIGGELEN B. A., Phys. Lett. A **169** (1992), 191.
[25] RUSEK M., ORŁOWSKI A., Phys. Rev. E **51** (1995), R2763.
[26] LIPPMANN B. A., SCHWINGER J., Phys. Rev. **79** (1950), 469.
[27] TAYLOR J. R., *Scattering Theory. The Quantum Theory of Nonrelativistic Collisions*, Wiley, New York 1972.
[28] SOUILLARD B., in ref. [29], pp. 305–382.

[29] *Chance and Matter*, No. XLVI, [In] *Les Houches*, [Ed.] J. Souletie, J. Vannimenus, R. Stora, North-Holland, Amsterdam 1987.
[30] LAGENDIJK A., VAN TIGGELEN B. A., Phys. Rep. **270** (1996), 143.
[31] ANDERSON P. W., Rev. Mod. Phys. **50** (1978), 191.

Received November 27, 1996