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Hybrid modes in nonlinear Kerr media

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In the paper the hybrid modes in the isotropic nonlinear Kerr media are analyzed. The simple but approximate analytical description of the second order modes in the unbounded nonlinear medium is suggested. The accuracy of the approximation is discussed. The exact analytical solution describing the balanced field is found. The continuity conditions in the five layered planar structure are solved and the corresponding mode equation is obtained.

1. Introduction

The nonlinear hybrid modes are guided waves containing all six components of electromagnetic field propagating with the same velocity in the same direction and coupled between them by means of the interaction with the medium [1], [2]. Since Kerr dielectrics [3], [4] with positive nonlinearity focus the propagating fields [5], the hybrid modes can appear even in the unbounded (or semi-unbounded) media. Their fields can possess many maxima in space but the ratio of TE to TM components (polarization of the mode) is strictly determined for every particular mode [1], [2]. The structure of the modes depends significantly of the nature of nonlinear processes, for instance, the first order, soliton-like mode cannot appear for the isotropic mechanism of nonlinearity [1]. The first and second order hybrid modes have approximate, analytical description [2]. In the presented paper, the above description shall be applied to the isotropic (electrostrictive or thermal) mechanism of nonlinearity.

The Kerr nonlinearity of isotropic type enables us to obtain an analytical, exact solution of the hybrid modes equations. This solution (so-called balanced) corresponds to the situation in which the nonlinear dielectric permittivity generated by propagating field remains constant across the medium. Naturally, such distribution of the refractive index can result only in a nonlinear slab sandwiched between two linear media. By now this solution has been applied successfully to the pure TM case only [6]. Since the balanced solution exists for the hybrid cases too, we can try to fit the balanced fields into many layered nonlinear structure.

2. Asymptotic solutions in unbounded isotropic Kerr media

The hybrid wave of propagation constant β travelling along the z axis in the isotropic, nonlinear Kerr medium is described by electromagnetic field components of the form [1], [2]: $\vec{E}(x,y,z) = (E_x(x),E_y(x),iE_z(x))\exp(i(\beta z-\omega t))$, $\vec{H}(x,y,z) = (H_x(x),H_y(x),-iH_z(x))\exp(i(\beta z-\omega t))$, with $\pm i$ term in z components symbolizing the phase shift $\pi/2$ between them and the other components [4], [7]. In the Kerr dielectric of the electrostrictive or thermal type all electric components are involved in the material equation $\varepsilon = \varepsilon_L+\alpha(E_x^2+E_y^2+E_z^2)$, where $\alpha > 0$. The equation describing the hybrid fields attains the simplest form if we introduce the rescaled, dimensionless variables expressing fields: $u = \alpha E_y^2/\varepsilon_L$, $v = \mu_0^2c^2\alpha H_y^2/\varepsilon_L^2$, their derivatives: $R = c^2\alpha/\omega^2\varepsilon_L^2(dE_y/dx)^2$, $Q = \mu_0^2c^4\alpha/\omega^2\varepsilon_L^3(dH_y/dx)^2$, distance $\xi = \omega\sqrt{\varepsilon_K}x/c$, propagation constant $b = c^2\beta^2/\omega^2\varepsilon_L$ and permittivity $e = \varepsilon/\varepsilon_L$ [1], [2]:

$$\left(\frac{du}{d\xi}\right)^2 = 2uR,$$

$$\left(\frac{dv}{d\xi}\right)^2 = 4vQ, \tag{1}$$

$$R(u,v,e) = bu+\left(\frac{2b}{e}-1\right)v-\frac{e^2-1}{2}+C,$$

$$Q(u,v,e) = e^2(e-1-u)-bv. \tag{2}$$

The last equation, written as the partial differential equation satisfied by $e(u,v)$ is [2]

$$\left(\frac{\partial e}{\partial u}-\frac{e^3}{2bv+e^3}\right)^2 uR = \left(\frac{\partial e}{\partial v}-\frac{e(2b-e)}{2bv+e^3}\right)^2 vQ. \tag{3}$$

To solve Equation (3) and to determine $e(u,v)$ we only need the algebraic equations (2), so it is the natural way to the set (1)–(3). But any solution of the differential equation depends on constant of integration. In the first of Eq. (2) (it is the first integral of the system of Maxwell's equations [1]–[5]) this integration constant C vanishes in the infinite or semi-infinite Kerr medium [3], [7]. For the fields in the unbounded Kerr dielectric we shall apply two types of initial conditions. The first one $e(0,0) = 1$ or $R(0,0) = 0$ and $Q(0,0) = 0$ is appropriate to all physical solutions (vanishing for $\xi \to \pm\infty$), while the second one $e(0,v_0) = e_0$ or $R(0,v_0) = R_0 > 0$ and $Q(0,v_0) = 0$ describes antisymmetric E_y and symmetric H_y (this is the symmetry of the second order hybrid mode in the unbounded Kerr medium (2]).

The set of Equations (2), (3) has no exact, analytical solutions. Nevertheless, assuming the small field approximation [2] we can determine the analytical solution as Taylor's series with respect to u and v. Keeping only the most significant terms we obtain the following functions $R(u,v)$ and $Q(u,v)$ satisfying the initial conditions of the first type:

$$R(u,v) = \Omega^2u+\ldots,$$

$$Q(u,v) = \Omega^2v+\ldots \tag{4}$$

where $\Omega^2 = b-1$. Using series (4) we can try to solve differential Eq. (1). In this way we arrive at the functions $u(\xi)$ and $v(\xi)$ expressing approximate profiles of the hybrid fields. Unfortunately, for the isotropic Kerr medium we have no solutions describing the first-order soliton-like modes [1]–[3]. To determine the solution of the second-order we should consider the symmetry of the field. The symmetry requires that the functions satisfy Eqs. (1)–(3) and the initial conditions of both types. Thus, the approximate solution of the differential Eq. (3) and the initial condition of the secondy type are:

$$R = \Omega^2 v_0 - \frac{b^2 v_0^2}{2} + \frac{b^2 v_0 u}{2} + \Omega^2 (v - v_0) + \ldots,$$
$$Q = \left(\Omega^2 - \frac{b(4-b)v_0}{2}\right) u + \ldots . \quad (5)$$

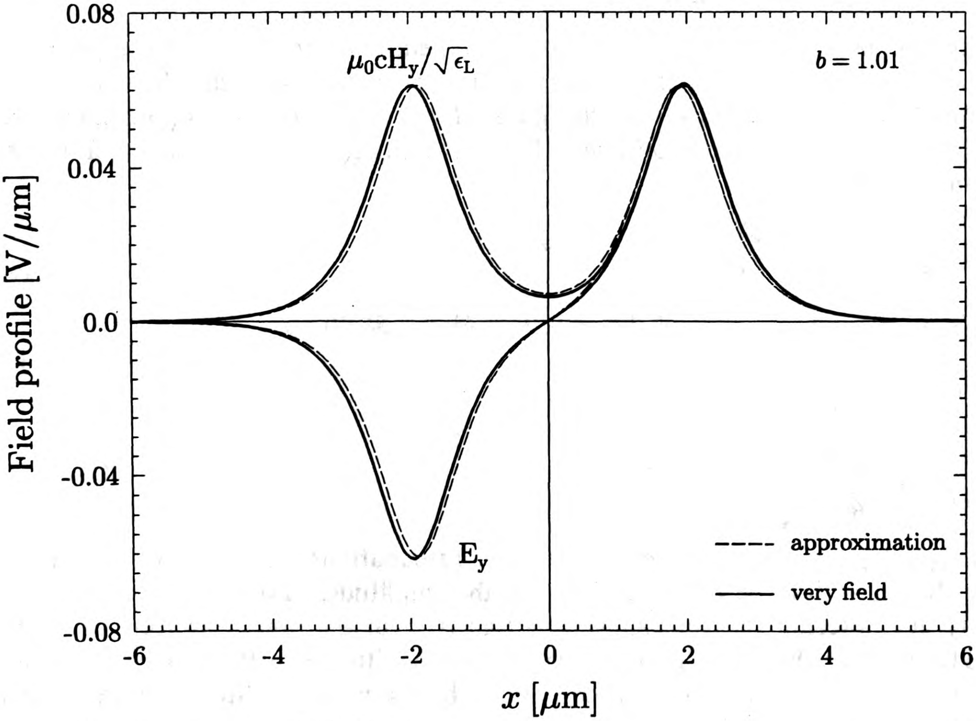

Fig. 1. Second order hybrid mode in the unbounded Kerr medium. Comparison of the exact, numerical solution and the analytical approximation (6). $\varepsilon_L = 2.3716$, $\alpha = 6.4 \cdot 10^{-12}$ m²/V².

Two solutions (4) and (5) can agree in the limit $b \to 1$ ($\Omega \to 0$) only if $v_0 \sim \Omega^4$ and $u \to v$ for small fields. Therefore the second-order solution satisfying the symmetry condition has both fields asymptotically (that is for $\xi \to \pm\infty$) equal in normalized units. But the functions that describe spatial profiles of soliton-like waves with antisymmetric E_y and symmetric H_y are more complicated than Sech-like solution corresponding to the first-order modes [2]. Such functions satisfying all Equations (1)–(5) in the limit $\xi \to \pm\infty$ and $\Omega \to 0$ have the form:

$$u(\xi) = \frac{12\sinh^2\Omega\xi}{(\cosh^2\Omega\xi + 3/\Omega^2)^2},$$
$$v(\xi) = \frac{12\cosh^2\Omega\xi}{(\cosh^2\Omega\xi + 3/\Omega^2)^2}. \tag{6}$$

In Figure 1, we compare two field diestributions of the second-order hybrid mode obtained for $b = 1.01$ ($\Omega = 0.1$) – the first one describing the exact, numerical solution and the second, given by analytical functions (6). Nevertheless, the presented method estimates rather the exact fields (the coefficient in Eqs. (6) determined with accuracy to the most significant terms), while the similarity of both profiles is amazing.

3. Hybrid balanced solution

In the nonlinear Kerr slab an electromagnetic field of a specific shape can generate the constant value of the nonlinear permittivity across the whole slab. This so-called balanced field has been fitted to the pure TM case [6]. But analogous balanced solution also exists for the hybrid fields [1]. Assuming $\varepsilon(x) = \text{const} > \varepsilon_L$ we obtain in original units:

$$E_y = E_{y0}\sin(kx - \Phi),$$
$$H_y = H_{y0}\cos(kx - \Phi) \tag{7}$$

where $k^2 = \omega^2\varepsilon/c^2 - \beta^2$, while the amplitudes are given by:

$$H_{y0} = \frac{\omega\varepsilon}{\mu_0 c^2 \beta}\sqrt{\frac{\varepsilon - \varepsilon_L}{\alpha}},$$
$$E_{y0} = \frac{\sqrt{\beta^2 - k^2}}{\beta}\sqrt{\frac{\varepsilon - \varepsilon_L}{\alpha}}. \tag{8}$$

Using (7), (8) we can conclude from Maxwell's equations that E_x is shifted in the phase by $\pi/2$ with respect to E_y and E_z and the amplitudes of the fields are chosen in a way guaranteeing the constancy of permittivity in the nonlinear Kerr medium.

The exact analytical solution (7), (8) cannot be fitted into three-layered system, but we can satisfy all continuity conditions by assuming additional linear layers between the nonlinear film and linear cover and substrate. For simplicity let us consider the symmetric structure containing the nonlinear film of thickness h sandwiched between two linear buffers of thickness h_b and permittivity $\varepsilon_b < \varepsilon_L$ and linear covers of permittivity ε_s. The fields in the buffer $-h_b < x < 0$ and in the cover $x < -h_b$ are described by the functions:

$$E_y = \begin{cases} E_b\sin(k_b x - \Phi_E), & -h_b < x < 0, \\ E_c\exp(\varkappa_c x + h_b), & x < -h_b \end{cases}$$

$$H_y = \begin{cases} H_b\cos(k_b x - \Phi_H), & -h_b < x < 0, \\ H_c\exp(\varkappa_c x + h_b), & x < -h_b, \end{cases} \tag{9}$$

with: $k_b^2 = \omega^2\varepsilon_b/c^2 - \beta^2$ and $\varkappa_c^2 = \beta^2 - \omega^2\varepsilon_c/c^2$. The continuity conditions across the interfaces determine the amplitudes E_b, H_b, E_c and H_c and the phases at boundaries Φ, Φ_E and Φ_H. Moreover, they also give the permittivity of the nonlinear film $\varepsilon(\beta, h_b)$ and the mode equation:

$$\varepsilon = \frac{\beta^2\varepsilon_b}{\dfrac{\omega^2\varepsilon_b}{c^2} + k_b^2 \mathrm{tg}(k_b h_b - \theta_E)\mathrm{tg}(k_b h_b - \theta_H)},$$

$$kh = 2\mathrm{arctg}\left(\frac{k}{k_b}\mathrm{ctg}(k_b h_b - \theta_E)\right) + m\pi, \quad m = 1, 2, \ldots \tag{10}$$

where: $\theta_E = \mathrm{arctg}(\varkappa_c/k_b)$, $\theta_H = \mathrm{arctg}(\varepsilon_b\varkappa_c/(\varepsilon_c k_b))$.

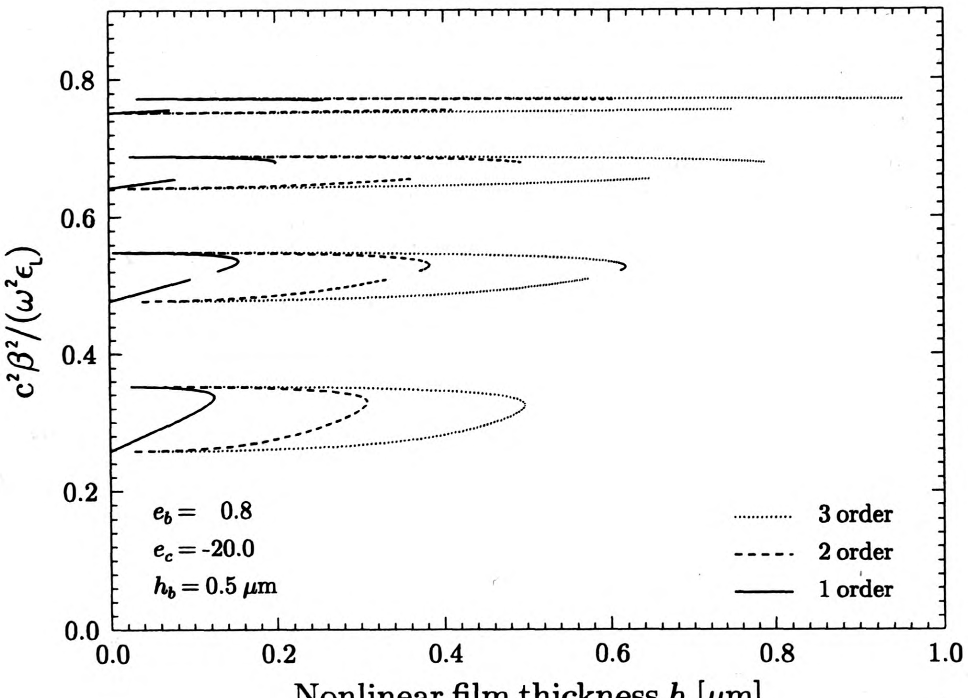

Fig. 2. Propagation constant against the thickness of the nonlinear slab for the lowest order hybrid balanced modes in a symmetric five-layered structure. The different fragments correspond to different orders in the buffer. $\varepsilon_L = 2.3716$, $\alpha = 6.4 \cdot 10^{-12}$ m^2/V^2

As in the pure TM case the mode equation determines the possible values of the propagation constant β in any given waveguide (Fig. 2). The thickness of the buffer h_b is not arbitrary – we observe certain intervals (gaps) for which hybrid balanced modes cannot propagate. Moreover, similarly to the TM case, one could expect in a vicinity of the balanced modes a certain kind of quasi-balanced fields, also possible to describe analytically.

4. Summary and conclusions

A few types of the hybrid fields in the isotropic nonlinear Kerr media are possible to describe analytically. The second order hybrid mode in an unbounded nonlinear medium, that is the electromagnetic field with antisymmetric electric component E_y and symmetric magnetic H_y can be approximated by analytical expressions (6), describing Sech-like waves with two field maxima. The approximation works well only for small fields (near the linear limit). Another analytical and exact solution corresponds to the hybrid field propagating in a five-layered structure. When the electric components balance together to produce the constant value of the nonlinear permittivity across the Kerr film, we can describe the electromagnetic fields by trigonometric functions (7), (8). The hybrid balanced fields exist in the structures with negative permittivity of the cover (and the substrate) and the proper thickness of the buffer. Analogously to the pure TM case one could expect certain quasi-balanced modes.

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