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Recurrence relations for multilayer thin film coated anisotropic substrates

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Reflection from anisotropic substrates overcoated with multilayer isotropic thin films is considered. The film–substrate system is split by an imaginary ambient gap of zero thickness between the film and the substrate. Recurrence relations are used for complex amplitude reflection and transmission coefficients of unsupported multilayer film. Reflection of the anisotropic substrate in the ambient gap can be determined by the 2×2 extended Jones matrix method. Then, recurrence relations are obtained for reflection matrix elements of the film–substrate system. They are valid at normal and oblique incidence and are useful for specific problems without repeating a tedious algebra.

1. Introduction

Anisotropic materials are used in many optical devices, such as phase retarders, polarizers, optical isolators, and circulators. The performances of these devices depend on their reflection or transmission properties. These performances can be improved by specific thin-film coatings. Thus, it would be necessary to know the reflection or transmission properties of thin-film coated anisotropic materials.

In this paper, the problem of reflection from anisotropic substrates overcoated with multilayer isotropic thin films is considered. An ambient gap of zero thickness is imaginatively inserted between the multilayer film and the anisotropic substrate. Thus, the film–substrate system is split into an unsupported multilayer isotropic film in the ambient medium and the anisotropic substrate in the ambient gap [1]. The complex amplitude reflection and transmission coefficients of the unsupported multilayer film are determined by recurrence relations [2]. Reflection of the anisotropic substrate in the ambient medium can be determined by the 2×2 extended Jones matrix method [3]–[5]. Then, recurrence relations are obtained for the reflection matrix elements of the film–substrate system. These reflections are useful for solving specific reflection problems without repeating a large amount of tedious algebra. We assume that the ambient medium and the films are isotropic and all the materials are homogeneous.

2. Recurrence relations for multilayer unsupported thin films

Consider an unsupported thin film numbered by i of refractive index n_i and geometrical thickness d_i in the ambient medium of refractive index n_0. Let a monochromatic plane wave be incident at angle θ_0 on the film surface. The complex amplitude reflection and transmission coefficients of this film, r_{fi} and t_{fi}, are determined using the following relations for both s and p polarizations [2]:

$$r_{fi} = r_{0i}(1-X_i^2)/(1-r_{0i}^2 X_i^2), \tag{1}$$

$$t_{fi} = t_{0i} t_{i0} X_i/(1-r_{0i}^2 X_i^2) \tag{2}$$

where r_{0i} is the Fresnel reflection coefficient at the front interface, t_{0i} and t_{i0} are transmission coefficients at the front and bottom interface, respectively; $X_i = \exp(-j2\pi n_i d_i \cos\theta_i/\lambda)$ with $j=(-1)^{1/2}$, θ_i is the refraction angle which is determined by Snell's law, and λ is the light wavelength in vacuum. For a bilayer unsupported thin film which is split by an imaginary ambient gap of zero thickness between layers one obtains:

$$r_{f1\to 2} = [r_{f1} + r_{f2}(t_{f1}^2 - r_{f1}^2)]/(1-r_{f1}r_{f2}), \tag{3}$$

$$t_{f1\to 2} = t_{f1}t_{f2}/(1-r_{f1}r_{f2}), \tag{4}$$

when light is incident on layer 1, and

$$r_{f2\to 1} = [r_{f2} + r_{f1}(t_{f2}^2 - r_{f2}^2)]/(1-r_{f1}r_{f2}), \tag{5}$$

$t_{f2\to 1} = t_{f1\to 2}$, when light is incident on layer 2, where r_{fi} and t_{fi}, $i = 1, 2$, are given by Eqs. (1) and (2). Similarly, for a triple-layer unsupported film one obtains

$$r_{f1\to 3} = [r_{f1\to 2} + r_{f3}(t_{f1\to 2}^2 - r_{f1\to 2}r_{f2\to 1})]/(1-r_{f3}r_{f2\to 1}), \tag{6}$$

$$t_{f1\to 3} = t_{f3}t_{f1\to 2}/(1-r_{f3}r_{f2\to 1}), \tag{7}$$

when light is incident on layer 1, and

$$r_{f3\to 1} = [r_{f3\to 2} + r_{f1}(t_{f2\to 3}^2 - r_{f3\to 2}r_{f2\to 3})]/(1-r_{f1}r_{f2\to 3}),$$

$t_{f3\to 1} = t_{f1\to 3}$, when light is incident on layer 3.

Generally, for a multilayer unsupported thin film consisting of $N+1$ layers, the complex amplitude reflection and transmission coefficients are determined with recurrence relations

$$r_{f1\to N+1} = [r_{f1\to N} + r_{fN+1}(t_{f1\to N}^2 - r_{f1\to N}r_{fN\to 1})]/(1-r_{fN+1}r_{fN\to 1}), \tag{9}$$

$$t_{f1\to N+1} = t_{fN+1}t_{f1\to N}/(1-r_{fN+1}r_{fN\to 1}), \tag{10}$$

when light is incident on layer 1, and

$$r_{fN+1\to 1} = [r_{fN+1\to 2} + r_{f1}(t_{f2\to N+1}^2 - r_{fN+1\to 2}r_{f2\to N+1})]/(1-r_{f1}r_{f2\to N+1}), \tag{11}$$

$t_{fN+1\to 1} = t_{f1\to N+1}$, when light is incident on layer $N+1$. These relations are valid at normal and oblique incidence for s and p polarizations. They are obtained from the usual recurrence relations for multilayer thin films [2].

3. Reflection from multilayer coated anisotropic substrates

Let a plane wave be incident on an anisotropic material in the ambient medium. Denote by A_{0s}^{+} and A_{0p}^{+} the incident electric field amplitudes for s and p polarizations. Let A_{0s}^{-} and A_{0p}^{-} be the respective reflected amplitudes. They are related by the 2×2 extended Jones reflection matrix r_g as follows [3]–[5]:

$$\begin{pmatrix} A_{0s}^{-} \\ A_{0p}^{-} \end{pmatrix} = \begin{pmatrix} r_{gss} & r_{gsp} \\ r_{gps} & r_{gpp} \end{pmatrix} \begin{pmatrix} A_{0s}^{+} \\ A_{0p}^{+} \end{pmatrix}. \tag{12}$$

Consider an isotropic thin film coated on the anisotropic substrate. Let r_{fs1}, r_{fp1}, and t_{fs1}, t_{fp1} be the complex amplitude reflection and transmission coefficients of the usupported film for s and p polarizations which are determined by Eqs. (1) and (2). Using the standard boundary conditions at interfaces [3]–[5] one obtains the following elements of the 2×2 reflection matrix r for the thin-film coated anisotropic substrate:

$$r_{ss} = [F_s + \gamma r_{fp1}(t_{fs1}^2 - r_{fs1}^2)]/G, \tag{13a}$$

$$r_{pp} = [F_p + \gamma r_{fs1}(t_{fp1}^2 - r_{fp1}^2)]/G, \tag{13b}$$

$$r_{sp} = \gamma_{sp}[r_{fs1}F_s + (t_{fs1}^2 - r_{fs1}^2)](t_{fp1}/t_{fs1})/G, \tag{13c}$$

$$r_{ps} = \gamma_{ps}[r_{fp1}F_p + (t_{fp1}^2 - r_{fp1}^2)](t_{fs1}/t_{fp1})/G \tag{13d}$$

where:

$$F_q = [r_{fq1} + r_{gqq}(t_{fq1}^2 - r_{fq1}^2)]/(1 - r_{gqq}r_{fq1}), \quad q = s, p, \tag{14a}$$

$$\gamma_{sp} = r_{gsp}/(1 - r_{gpp}r_{fp1}), \tag{14b}$$

$$\gamma_{ps} = r_{gps}/(1 - r_{gss}r_{fs1}), \tag{14c}$$

$$\gamma = \gamma_{sp}\gamma_{ps}, \tag{14d}$$

$$G = 1 - \gamma r_{fs1}r_{fp1}. \tag{14e}$$

In the limit of isotropy, when $r_{gsp} = r_{gps} = 0$ one obtains $\gamma = 0$, $r_{qq} = F_q$ $(q = s, p)$, and we recover the relation for the single-layer coated isotropic substrate [1]. For a bilayer coated anisotropic substrate we obtain:

$$r_{ss} = [F_s + \gamma r_{fp2\to 1}(t_{fs1\to 2}^2 - r_{fs1\to 2}r_{fs2\to 1})]/G, \tag{15a}$$

$$r_{pp} = [F_p + \gamma r_{fs2\to 1}(t_{fp1\to 2}^2 - r_{fp1\to 2}r_{fp2\to 1})]/G, \tag{15b}$$

$$r_{sp} = \gamma_{sp}[r_{fs2\to 1}F_s + (t_{fs1\to 2}^2 - r_{fs1\to 2}r_{fs2\to 1})](t_{fp1\to 2}/t_{fs1\to 2})/G, \tag{15c}$$

$$r_{ps} = \gamma_{ps}[r_{fp2\to 1}F_p + (t_{fp1\to 2}^2 - r_{fp1\to 2}r_{fp2\to 1})](t_{fs1\to 2}/t_{fp1\to 2})/G \tag{15d}$$

where:

$$F_q = [r_{fq1\to 2} + r_{gqq}(t_{fq1\to 2}^2 - r_{fq1\to 2}r_{fq2\to 1})]/(1 - r_{gqq}r_{fq2\to 1}), \quad q = s, p, \tag{16a}$$

$$\gamma_{sp} = r_{gsp}/(1 - r_{gpp} r_{fp2\to1}), \tag{16b}$$

$$\gamma_{ps} = r_{gps}/(1 - r_{gss} r_{fs2\to1}), \tag{16c}$$

$$G = 1 - \gamma r_{fs2\to1} r_{fp2\to1}, \tag{16d}$$

with: $\gamma = \gamma_{sp}\gamma_{ps}$ and $r_{fq1\to2}$, $t_{fq1\to2}$, $r_{fq2\to1}$ $(q = s,p)$ determined by Eqs. (3)–(5). Generally, for an anisotropic substrate overcoated with N thin isotropic layers, we obtain the following recurrence relations for the elements of the reflection matrix r:

$$r_{ss} = [F_s + \gamma r_{fpN\to1}(t^2_{fs1\to N} - r_{fs1\to N} r_{fsN\to1})]/G, \tag{17a}$$

$$r_{pp} = [F_p + \gamma r_{fsN\to1}(t^2_{fp1\to N} - r_{fp1\to N} r_{fpN\to1})]/G, \tag{17b}$$

$$r_{sp} = \gamma_{sp}[r_{fsN\to1} F_s + (t^2_{fs1\to N} - r_{fs1\to N} r_{fsN\to1})](t_{fp1\to N}/t_{fs1\to N})/G, \tag{17c}$$

$$r_{ps} = \gamma_{ps}[r_{fpN\to1} F_p + (t^2_{fp1\to N} - r_{fp1\to N} r_{fpN\to1})](t_{fs1\to N}/t_{fp1\to N})/G \tag{17d}$$

where:

$$F_q = [r_{fq1\to N} + r_{gqq}(t^2_{fq1\to N} - r_{fq1\to N} r_{fqN\to1})]/(1 - r_{gqq} r_{fqN\to1}),\ q = s,p, \tag{18a}$$

$$\gamma_{sp} = r_{gsp}/(1 - r_{gpp} r_{fpN\to1}), \tag{18b}$$

$$\gamma_{ps} = r_{gps}(1 - r_{gss} r_{fsN\to1}), \tag{18c}$$

$$G = 1 - \gamma r_{fsN\to1} r_{fpN\to1}, \tag{18d}$$

with $\gamma = \gamma_{sp}\gamma_{ps}$, and $r_{fq1\to N}$, $t_{fq1\to N}$, $r_{fqN\to1}$ $(q = s,p)$ determined by recurrence relations (9)–(11).

4. Conclusions

Reflection from anisotropic materials overcoated with multilayer isotropic thin films is considered. The film–substrate system is split by an imaginary ambient gap of zero thickness between the film and the substrate. Recurrence relations (9)–(11) are used to determine the complex amplitude reflection and transmission coefficients of the unsupported multilayer film. Reflection from the anisotropic substrate in the ambient gap is represented by the 2×2 extended Jones matrix r_g as given by Eq. (12). Then, simple recurrence relations (17a)–(17d) are obtained for reflection matrix elements of the multilayer coated anisotropic substrate. These relations are friendly with computers and are well suited for solving specific reflection problems without repeating a tedious algebra. For example, they are applied to antireflection quarterwave thin-film coatings on magneto-optical substrates at normal incidence [6].

References

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