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Antireflection thin-film coatings on Faraday rotators

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Reflection from magneto-optic (MO) substrates overcoated with quarterwave-thick isotropic layers is calculated analytically at normal incidence by the 2×2 extended Jones matrix method. Single-, double- and triple-layer coatings are considered. Simple relations are obtained for the reflection matrix elements. In the case of transparent MO media used in Faraday rotators, simple antireflection conditions for layer refractive indices are determined.

1. Introduction

The Faraday rotator, which is based on the Faraday effect, is the most important component in isolators and circulators. In those devices use is made of various kinds of transparent magneto-optic media of refractive indices which are greater than 2.2. Thus, the uncoated MO surfaces could reflect more than 10% of the incident light. This reflection limits the performances of Faraday rotators. Therefore, it is necessary to deposit antireflection coat on the transparent MO surfaces [1].

In the case of isotropic substrates, antireflection coatings consisting of quarter-wave-thick dielectric thin films are usually used [2]. For an anisotropic MO substrate, reflection from the film—substrate system is characterized by a matrix r which can be determined by the 2×2 extended Jones matrix method [3]—[5].

In this paper, we present simple analytical relations for the elements of the reflection matrix r from single-, double- and triple-layer coated MO surfaces at normal incidence. Dielectric quarterwave-thick, isotropic layers are considered. Antireflection conditions for layer refractive indices are determined in the case of transparent MO substrates.

2. Reflection matrix from magneto-optic surface in the ambient medium at normal incidence

The dielectric tensor for MO media can be written as [1], [6], [7]

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_x & -j\varepsilon_{xy} & 0 \\ j\varepsilon_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \tag{1}$$

with $j=(-1)^{1/2}$ and ε_{xy} proportional to the magnetization which is applied on the

z-direction. The coordinate system is chosen so that the x-y plane is the surface of the MO medium. We consider the MO medium isotropic in the x-y plane and $\varepsilon_x = \varepsilon_y = \varepsilon_z$. Let a monochromatic plane wave be normally incident from the ambient medium in the y-z plane. At the surface of the MO medium the incident wave is divided into a backward-propagating reflected wave and two forward-propagating circularly polarized waves denoted by α and β. The corresponding refractive indices of the MO medium [1], [7] are $n_\alpha = (\varepsilon_x + \varepsilon_{xy})^{1/2}$ and $n_\beta = (\varepsilon_x - \varepsilon_{xy})^{1/2}$. Let r_g be the 2×2 extended Jones reflection matrix from the MO surface in the ambient medium. At normal incidence the elements of the r_g matrix are [3]–[7]:

$$r_{gss} = (n_0^2 - n_\alpha n_\beta)/[(n_0 + n_\alpha)(n_0 + n_\beta)], \tag{2a}$$

$$r_{gsp} = jn_0(n_\beta - n_\alpha)/[(n_0 + n_\alpha)(n_0 + n_\beta)], \tag{2b}$$

with n_0 – the refractive index of the ambient medium, $r_{gps} = r_{gsp}$, and $r_{gpp} = -r_{gss}$, which is consistent in the limit of isotropy (at $\varepsilon_{xy} = 0$) with the Nebraska–Muller convention [8], [9].

3. Reflection matrix from the film–substrate system

Let us consider an isotropic multilayer film of quarterwave-thick layers coated on the MO surface. Reflection from the film–substrate system is much easier to calculate if an ambient gap of zero thickness is imaginatively inserted between the film and the substrate [10]. The complex amplitude reflection and transmission coefficients of the unsupported multilayer isotropic film surrounded by the ambient medium are determined by recurrence relations [11]. Reflection from the anisotropic MO substrate in the ambient gap is determined by the matrix r_g. Then, simple relations result for the elements of the reflection matrix r from the film–substrate system.

One obtains at normal incidence:

$$r_{ss} = (n_0^2 n_\alpha n_\beta - n_1^4)/[(n_1^2 + n_0 n_\alpha)(n_1^2 + n_0 n_\beta)], \tag{3a}$$

$$r_{sp} = jn_0 n_1^2(n_\alpha - n_\beta)/[(n_1^2 + n_0 n_\alpha)(n_1^2 + n_0 n_\beta)], \tag{3b}$$

for single-layer coated MO substrate,

$$r_{ss} = (n_0^2 n_2^4 - n_1^4 n_\alpha n_\beta)/[(n_0 n_2^2 + n_1^2 n_\alpha)(n_0 n_2^2 + n_1^2 n_\beta)], \tag{4a}$$

$$r_{sp} = jn_0 n_1^2 n_2^2(n_\beta - n_\alpha)/[(n_0 n_2^2 + n_1^2 n_\alpha)(n_0 n_2^2 + n_1^2 n_\beta)], \tag{4b}$$

for bilayer coated MO substrate, and

$$r_{ss} = (n_0^2 n_2^4 n_\alpha n_\beta - n_1^4 n_3^4)/[(n_1^2 n_3^2 + n_0 n_2^2 n_\alpha)(n_1^2 n_3^2 + n_0 n_2^2 n_\beta)], \tag{5a}$$

$$r_{sp} = jn_0 n_1^2 n_2^2 n_3^2(n_\alpha - n_\beta)/[(n_1^2 n_3^2 + n_0 n_2^2 n_\alpha)(n_1^2 n_3^2 + n_0 n_2^2 n_\beta)], \tag{5b}$$

for triple-layer coated MO substrate. The layers are counted from the ambient side towards the substrate. The other elements of the matrix r are $r_{ps} = r_{sp}$ and $r_{pp} = -r_{ss}$.

4. Antireflection conditions for layer refractive indices

Generally, the transparent MO media used in Faraday rotators have ε_{xy} very small, so that $n_\alpha \simeq n_\beta$ at normal incidence. Then, for these transparent MO media one obtains from $r_{ss} = -r_{pp} = 0$ the following antireflection conditions of refractive indices

$$n_1^4 = n_0^2 n_\alpha n_\beta, \tag{6}$$

for single-layer coated MO substrate,

$$n_0^2 n_2^4 = n_1^4 n_\alpha n_\beta, \tag{7}$$

for bilayer coated MO substrate, and

$$n_1^4 n_3^4 = n_0^2 n_2^4 n_\alpha n_\beta, \tag{8}$$

for triple-layer coated MO substrate. In the limit of isotropy, when $n_\alpha = n_\beta = n_g$, one obtains the known conditions of refractive indices for single-, double- and triple-layer antireflection quarterwave coatings on isotropic substrates [2]:

$$n_1^2 = n_0 n_g, \tag{9a}$$

$$n_0 n_2^2 = n_1^2 n_g, \tag{9b}$$

$$n_1^2 n_3^2 = n_0 n_2^2 n_g. \tag{9c}$$

5. Discussion

We determined the elements of the 2×2 reflection matrix r from thin-film coated MO substrates when the light wave is normally incident from the ambient medium side. Similar relations are obtained for light wave incident from the MO medium side. Even for single-layer coated MO surfaces the reflection matrix elements presented are much simpler than those given in [1].

The antireflection conditions for transparent MO media are deduced by assuming that because ε_{xy} is very small and $n_\alpha \simeq n_\beta$, then $r_{ps} = r_{ps} \simeq 0$. For example, when $\varepsilon_{xy} = 0.04$ [7], $\varepsilon_x = \varepsilon_y = \varepsilon_z = 2.2^2$, $n_1 = 1.45$, and $n_0 = 1$, Eq. (3b) gives $r_{sp} = j0.002$.

It should be noted that at small values of ε_{xy}, when $n_\alpha \simeq n_\beta \simeq n_g$, relations (9a)–(9c) for refractive indices of antireflection coatings on isotropic substrates are valid also for MO transparent substrates. This can be seen also from numerical results presented in [1].

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